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WEAKLY P (URYSOHN) SPACES AND PROPERTIES, AND EQUIVALENT SEPARATION AXIOMS

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Abstract

In 1975, it was proven that a space is R_1 iff its T_0-identification space is Hausdorff. The 1975 work motivated the introduction and investigation of weakly Po properties, which led to the introduction and investigation of weakly P1 and weakly P2 properties. Within this paper, the weakly P properties are expanded to include weakly P(Urysohn). Relationships between weakly P(Urysohn) and the above weakly P spaces and properties are investigated, other properties of weakly P(Urysohn) spaces and properties are given, and for weakly P(Urysohn) spaces, it is shown that T_0, T_1, T_2, and Urysohn are equivalent.

1. Introduction and Preliminaries

Urysohn spaces were introduced in 1925 [18].

Definition 1.1. A space (X, T) is Urysohn iff for distinct elements x and y in X,

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there exist open sets U and V such that $x \in U$, $y \in V$, and $Cl(U) \cap Cl(V) = \phi$.

In 1943 [16], the R_0 separation axiom was introduced.

Definition 1.2. A space (X, T) is R_0 iff for each closed set C and each $x \notin C$, $Cl(\{x\}) \cap C = \phi$.

In 1961 [1], the R_0 separation axiom was rediscovered and used to further characterize T_1 spaces and the R_1 separation axiom was introduced and used to further characterize T_2 spaces.

Definition 1.3. A space (X, T) is R_1 iff for x and y in X for which $Cl(\{x\}) \neq Cl(\{y\})$, there exist disjoint open sets U and V such that $x \in U$ and $y \in V$.

Theorem 1.1. *A space is T_1 iff it is $(R_0$ and $T_0)$ and a space is T_2 iff it is $(R_1$ and $T_1)$.*

In 1975 [15], R_1 spaces, which were proven to be equivalent to weakly Hausdorff spaces, were characterized using T_0-identification spaces, which were introduced in 1936 [17].

Definition 1.4. Let (X, T) be a space, let R be the equivalence relation on X defined by xRy iff $Cl(\{x\}) = Cl(\{y\})$, let X_0 be the set of R equivalence classes of X, let $N : X \to X_0$ be the natural map, and let $Q(X, T)$ be the decomposition topology on X_0 determined by (X, T) and the natural map N. Then $(X_0, Q(X, T))$ is the T_0-identification space of (X, T) [17].

Theorem 1.2. *A space (X, T) is weakly Hausdorff, or equivalently R_1, iff its T_0-identification space is Hausdorff* [15].

In 1988 [2], using weakly Hausdorff $= R_1$ as a model, weakly Urysohn spaces were defined.

Definition 1.5. A space (X, T) is weakly Urysohn iff for x and y in X for which $Cl(\{x\}) \neq Cl(\{y\})$, there exist open sets U and V such that $x \in U$, $y \in V$, and $Cl(U) \cap Cl(V) = \phi$.

In 2015 [3], the question of whether T_0-identification spaces could be used to uniquely define other weakly *P* properties behaving in the same manner as weakly Hausdorff led to the introduction and investigation of weakly Po properties.

Definition 1.6. Let *P* be a topological property for which Po $= (P$ and $T_0)$ exists. Then (X, T) is weakly Po iff its T_0-identification space $(X_0, Q(X, T))$ has property *P*. A topological property Po for which weakly Po exists is called a weakly Po property [3].

In the 2015 paper [3], it was proven that for a topological property *P* for which weakly Po exists, weakly Po is a unique, topological property. In addition, since for each space (X, T), $(X_0, Q(X, T))$ is T_0 [17], then, as given in the 2015 paper [3], a space is weakly Po iff its T_0-identification space has property Po. The investigation of weakly Po properties revealed that for a weakly Po property Qo, a space is weakly Qo iff its T_0-identification space is weakly Qo [3]. Combining that result with the fact that other topological properties are simultaneously shared by a space and its T_0-identification space led to the introduction of T_0-identification *P* properties.

Definition 1.7. Let *S* be a topological property. Then *S* is a T_0-identification *P* property iff *S* is simultaneously shared by a space and its T_0-identification space [4].

Within the paper [5], it was shown that weakly Urysohn is a T_0-identification *P* property and Urysohn is a weakly Po, weakly $P1$, and weakly $P2$ property with weakly Urysohn $=$ T_0-identification (weakly Urysohn) $=$ weakly (weakly Urysohn)o $=$ weakly (weakly Urysohn)1 $=$ weakly (weakly Urysohn)2.

Since for weakly Po properties, the T_0 separation axiom plays a major role, the question of what would happen if in the definition of weakly Po, T_0 was replaced by T_1 or by T_2 arose leading to the introduction and investigation of weakly $P1$ [6] and weakly $P2$ [7] properties.

Definition 1.8. Let *P* be a topological property for which $P1 = (P$ and $T_1)$ exists. Then a space (X, T) is weakly $P1$ iff its T_0-identification space

$(X_0, Q(X, T))$ is $P1$. A topological property $P1$ for which weakly $P1$ exists is called a weakly $P1$ property.

Definition 1.9. Let P be a topological property for which $P2 = (P$ and $T_2)$ exists. Then a space (X, T) is weakly $P2$ iff its T_0-identification space $(X_0, Q(X, T))$ is $P2$. A topological property $P2$ for which weakly $P2$ exists is called a weakly $P2$ property.

Within this paper, the study of weakly P properties is expanded to include weakly *P*(Urysohn) properties.

2. Weakly *P*(Urysohn) Properties and Other Weakly *P* Properties

Definition 2.1. Let P be a topological property for which *P*(Urysohn) = (*P* and Urysohn) exists. Then a space (X, T) is weakly *P*(Urysohn) iff its T_0-identification space $(X_0, Q(X, T))$ is *P*(Urysohn). A topological property *P*(Urysohn) for which weakly *P*(Urysohn) exists is called a weakly *P*(Urysohn) property.

Since Urysohn(Urysohn) = Urysohn, then Urysohn is a weakly *P*(Urysohn) property.

Theorem 2.1. *Let Q be a topological property for which weakly Q*(*Urysohn*) *exists*. *Then weakly Q*(*Urysohn*) *implies each of weakly* Qo, *weakly* $Q1$, *and weakly* $Q2$; *and* Qo *is a weakly* Po *property*, $Q1$ *is a weakly* $P1$ *property*, *and* $Q2$ *is a weakly* $P2$ *property*.

Proof. Let (X, T) have property weakly *Q*(Urysohn). Then $(X_0, Q(X, T))$ has property *Q*(Urysohn). Thus, $(X_0, Q(X, T))$ has property Qo, which implies (X, T) has property weakly Qo. Thus weakly Qo exists, weakly *Q*(Urysohn) implies weakly Qo, and Qo is a weakly Po property. In similar manner, the remainder of the theorem can be proved.

Theorem 2.2. *Let Q be a topological property for which weakly Q*(*Urysohn*) *exists*. *Then Q*(*Urysohn*) *is a weakly* $P2$, *a weakly* $P1$, *and a weakly* Po *property*.

Proof. Since *Q*(Urysohn) = (*Q*(Urysohn))2, then *Q*(Urysohn) is a weakly $P2$

property. In similar manner Q(Urysohn) is a weakly $P1$ and a weakly Po property.

Since weakly Po spaces are neither T_0 nor "not-T_0" [3], then, by the results above, neither are weakly P(Urysohn) spaces.

Theorem 2.3. *Let Q be a topological property for which weakly Q(Urysohn) exists. Then Q(Urysohn) = ($Q2$ and Urysohn) = ($Q1$ and Urysohn) = (Qo and Urysohn).*

Proof. Since Q(Urysohn) is a weakly $P2$ property, then Q(Urysohn) = (Q(Urysohn))2 = ((Q and Urysohn) and T_2) = ((Q and T_2) and Urysohn) = ($Q2$ and Urysohn). Since $Q2$ = ($Q1$ and R_1) = (Qo and R_1) [7] and Urysohn implies T_2, which implies R_1, then Q (Urysohn) = ($Q1$ and Urysohn) = (Qo and Urysohn).

Theorem 2.4. *Let Q be a topological property for which weakly Q*(Urysohn) *exists. Then weakly Q(Urysohn) = (weakly $Q2$ and weakly Urysohn) = (weakly $Q1$ and weakly Urysohn) = (weakly Qo and weakly Urysohn).*

Proof. Let (X, T) be weakly Q(Urysohn). Then $(X_0, Q(X, T))$ is Q(Urysohn) = ($Q2$ and Urysohn). Since $(X_0, Q(X, T))$ is $Q2$, then (X, T) is weakly $Q2$ and since $(X_0, Q(X, T))$ is Urysohn, then (X, T) is weakly Urysohn. Thus weakly Q(Urysohn) implies (weakly $Q2$ and weakly Urysohn).

Suppose (X, T) is (weakly $Q2$ and weakly Urysohn). Then $(X_0, Q(X, T))$ is ($Q2$ and Urysohn) = Q(Urysohn) and (X, T) is weakly Q(Urysohn). Hence (weakly $Q2$ and weakly Urysohn) implies weakly Q(Urysohn). Therefore weakly Q(Urysohn) = (weakly $Q2$ and weakly Urysohn).

Since weakly $Q2$ = (weakly $Q1$ and R_1) = (weakly Qo and R_1) and weakly Urysohn implies R_1 [2], as above, then weakly Q(Urysohn) = (weakly $Q1$ and weakly Urysohn) = (weakly Qo and weakly Urysohn).

Corollary 2.1. *Let Q be a topological property for which weakly Q(Urysohn) exists. Then weakly Q(Urysohn) is a unique topological property.*

Theorem 2.5. *Let Q be a topological property for which weakly Q(Urysohn)* *exists. Then weakly Q(Urysohn) is a* T_0 -identification *P* property.

Proof. Let (X, T) be a space. Suppose (X, T) is weakly *Q*(Urysohn). Then $(X_0, Q(X, T))$ is *Q*(Urysohn). Since $(X_0, Q(X, T))$ and $((X_0)_0, Q(X_0, Q(X, T)))$ are homeomorphic [3], then $((X_0)_0, Q(X_0, Q(X, T)))$ is *Q*(Urysohn), which implies $(X_0, Q(X, T))$ is weakly *Q*(Urysohn).

Suppose $(X_0, Q(X, T))$ is weakly *Q*(Urysohn). Then $((X_0)_0, Q(X_0, Q(X, T)))$ is *Q*(Urysohn), which, by the homeomorphism given above, implies $(X_0, Q(X, T))$ is *Q*(Urysohn) and (X, T) is weakly *Q*(Urysohn). Hence weakly *Q*(Urysohn) is a T_0 -identification *P* property.

Thus the question of whether for a topological property *Q* for which weakly *Q*(Urysohn) exists, is (weakly *Q*(Urysohn))o, a weakly *P*o property arose.

Within the paper [3], it was shown that for a topological property *P* for which weakly *P*o exists, *P*o = (weakly *P*o)o. In the paper [8], it was shown that for a topological property *P* for which weakly *P*1 exists, *P*1 = (weakly *P*1)1 and in the paper [9], it was shown that for a topological property *P* for which weakly *P*2 exists, *P*2 = (weakly *P*2)2. Below these results are used in the continued investigation of weakly *Q*(Urysohn) spaces and properties and to give a position response to the question above.

Theorem 2.6. *Let Q be a topological property for which weakly Q(Urysohn) exists. Then Q(Urysohn) = (weakly Q(Urysohn))o = (weakly Q(Urysohn))1 = (weakly Q(Urysohn))2.*

Proof. Since *Q*(Urysohn) is a weakly *P*o property, then *Q*(Urysohn) = *Q*(Urysohn)o = (weakly *Q*(Urysohn)o)o = (weakly *Q*(Urysohn))o. The remainder of the proof is straight forward using that *Q*(Urysohn) is a weakly *P*1 and a weakly *P*2 property, and an argument similar to that above and is omitted.

Corollary 2.2. *Let Q be a topological property for which weakly Q(Urysohn) exists. Then weakly Q(Urysohn) = weakly (weakly Q(Urysohn))o = weakly (weakly Q(Urysohn))1 = weakly (weakly Q(Urysohn))2, and weakly*

Q(Urysohn)o is a weakly Po property, weakly Q(Urysohn)1 is a weakly P1 property, and weakly Q(Urysohn)2 is a weakly $P2$ property.

Theorem 2.7. *Let Q be a topological property for which weakly Q(Urysohn) exists. Then (weakly Q(Urysohn)) (Urysohn) exists.*

Proof. Let (X, T) be weakly Q(Urysohn). Then $(X_0, Q(X, T))$ has property Q(Urysohn), which implies $(X_0, Q(X, T))$ is Urysohn. Since $(X_0, Q(X, T))$ is homeomorphic to its T_0-identification space, then its T_0-identification space has property Q(Urysohn), which implies $(X_0, Q(X, T))$ has property weakly Q(Urysohn). Thus $(X_0, Q(X, T))$ is weakly Q(Urysohn) and Urysohn, and (weakly Q (Urysohn)) (Urysohn) exists.

Theorem 2.8. *Let Q be a topological property for which weakly Q(Urysohn) exists. Then Q(Urysohn) = (weakly Q(Urysohn)) (Urysohn).*

Proof. Let the space (X, T) have property Q(Urysohn). Then (X, T) is T_0 and the natural map $N : (X, T) \to (X_0, Q(X, T))$ is a homeomorphism [10] and $(X_0, Q(X, T))$ has property Q(Urysohn). Thus $(X_0, Q(X, T))$ is Urysohn. Since $(X_0, Q(X, T))$ and its T_0-identification space are homeomorphic, then the T_0-identification space of $(X_0, Q(X, T))$ has property Q(Urysohn), which implies $(X_0, Q(X, T))$ has property weakly Q(Urysohn). Hence $(X_0, Q(X, T))$ has property (weakly Q(Urysohn)) (Urysohn) and since the natural map N is a homeomorphism, then (X, T) has property (weakly Q(Urysohn)) (Urysohn). Thus Q(Urysohn) implies (weakly Q(Urysohn)) (Urysohn).

Let the space (X, T) have property (weakly Q(Urysohn)) (Urysohn). Then (X, T) is T_0, and the natural map $N : (X, T) \to (X_0, Q(X, T))$ is a homeomorphism [10], and $(X_0, Q(X, T)$ has property (weakly Q(Urysohn)) (Urysohn). Then $(X_0, Q(X, T)$ has property weakly Q(Urysohn) and its T_0-identification space has property Q(Urysohn). Since $(X_0, Q(X, T)$ and its T_0-identification space are homeomorphic, then $(X_0, Q(X, T)$ has property Q(Urysohn) and since the natural map is a homeomorphism, then (X, T) has property Q(Urysohn). Hence (weakly Q(Urysohn)) (Urysohn) implies Q(Urysohn). Therefore Q(Urysohn) = (weakly Q(Urysohn)) (Urysohn).

Corollary 2.3. *Let Q be a topological property for which weakly Q(Urysohn) exists. Then weakly Q(Urysohn) = weakly (weakly Q(Urysohn)) (Urysohn) and (weakly Q(Urysohn)) (Urysohn) is a weakly P(Urysohn) property.*

Within the paper [7], it was shown that for weakly $P2$ properties $Q2$ and $W2$, weakly $Q2$ = weakly $W2$ iff $Q2 = W2$ raising the corresponding questions for weakly P(Urysohn) properties.

Theorem 2.9. *Let Q(Urysohn) and W(Urysohn) be weakly P(Urysohn)* properties. *Then weakly Q(Urysohn)* = weakly *W(Urysohn)* iff *Q(Urysohn) = W(Urysohn).*

Proof. Suppose weakly Q(Urysohn) = weakly W(Urysohn). Then weakly Q(Urysohn)2 = weakly Q(Urysohn) = weakly W(Urysohn) = weakly W(Urysohn)2 and Q(Urysohn) = Q(Urysohn)2 = W(Urysohn)2 = W(Urysohn).

Clearly, if Q(Urysohn) = W(Urysohn), then weakly Q(Urysohn) = weakly W(Urysohn).

Below the investigation of weakly P(Urysohn) spaces and properties continues with the investigation of product spaces, subspaces, decompositions, and the equivalence of T_0, T_1, T_2, and Urysohn in weakly P(Urysohn) spaces.

3. Product Spaces, Subspaces, Decompositions and Equivalent Separation Axioms

In the introductory weakly P paper [3], the search for a topological property that failed to be a weakly Po created a need and a use for the topological property "not-T_0" motivating further investigations of topological properties "not-P", where P is a topological property and "not-P" exists. Within those investigations, it was discovered that there is a least topological property L given by $L = (T_0$ or "not-T_0"), which is also given by $L = (P$ or "not-P"), where P is a topological property different from L [11]. As established in the papers [12] and [13], the existence of L created problems in the definitions of both product properties [12] and subspace properties [13] requiring the exclusion of L from both properties.

Definition 3.1. Let P be a topological property different from L. Then P is a product property iff a product space, with the Tychonoff topology, has property P iff each factor space has property P [12].

In the paper [4], it was proven that for a product property P for which weakly $P\text{o}$ exists, weakly $P\text{o}$ is a product property. Combining this result with the fact that for a topological property Q for which weakly Q(Urysohn) exists, Q(Urysohn) = $Q(\text{Urysohn})\text{o}$ gives the following result.

Corollary 3.1. *Let* $\mathcal{P} = \{Q \mid$ *such that* Q *is a product property and weakly* Q(*Urysohn*) exists$\}$, *let* $Q \in \mathcal{P}$, *let* (X_α, T_α) *be a space for each* $\alpha \in A$, *let* $X = \prod_{\alpha \in A} X_\alpha$, *and let* W *be the Tychonoff topology on* X. *Then* (X, W) *is weakly* Q(*Urysohn*) *iff* (X_α, T_α) *is weakly* Q(*Urysohn*) *for each* $\alpha \in A$.

Definition 3.2. A topological property P different from L for which a space has property P iff each subspace of the space has property P is called a subspace property [13].

In the paper [4], it was proven that for a subspace property S for which weakly $S\text{o}$ exists, weakly $S\text{o}$ is a subspace property, which when combined with results above, as for Corollary 3.1, gives the following result.

Corollary 3.2. *Let* $\mathcal{S} = \{Z \mid Z$ *is a subspace property and weakly* Z(*Urysohn*) *exists*$\}$ *and let* $Z \in \mathcal{S}$. *Then a space has property weakly* Z(*Urysohn*) *iff each subspace of the space has property weakly* Z(*Urysohn*).

In the paper [3], it was proven that for a topological property Q for which weakly $Q\text{o}$ exists, weakly $Q\text{o} = Q\text{o}$ or (weakly $Q\text{o}$ and "not-T_0"), where both $Q\text{o}$ and (weakly $Q\text{o}$ and "not-T_0") exist; neither of which are weakly $P\text{o}$ properties. Thus questions concerning decompositions of weakly Q(Urysohn) spaces arose.

Theorem 3.1. *Let* Q *be a topological property for which weakly* Q(*Urysohn*) *exists. Then weakly* Q(*Urysohn*) $= Q$ (*Urysohn*) *or* (*weakly* Q(*Urysohn*) *and* "*not-*T_0"), *where both* Q(*Urysohn*) *and* (*weakly* Q(*Urysohn*) *and* "*not-*T_0") *exist*; *neither of which are weakly* P(*Urysohn*) *properties*.

Proof. Since weakly Q(Urysohn) = weakly Q(Urysohn)o and Q(Urysohn) = Q(Urysohn)o, then weakly Q (Urysohn) = weakly Q(Urysohn)o = (Q(Urysohn)o or (weakly (Q Urysohn)o and "not-T_0")) = (Q(Urysohn) or (weakly Q(Urysohn) and "not-T_0")), where both Q(Urysohn) and (weakly Q(Urysohn) and "not-T_0") exist; neither of which are weakly Q(Urysohn) properties.

Theorem 3.2. *Let Q be a topological property for which weakly Q(Urysohn) exists. Then weakly Q(Urysohn) = (Q(Urysohn) or (weakly Q(Urysohn) and "not-Urysohn")), where (Q(Urysohn) and (weakly Q (Urysohn) and "not-Urysohn")) does not exist.*

Proof. Since (weakly Q(Urysohn) and L) = weakly Q(Urysohn), then (weakly Q(Urysohn) and (Urysohn or "not-Urysohn")) = ((weakly Q(Urysohn)) (Urysohn) or (weakly Q(Urysohn) and "not-Urysohn")) = (Q (Urysohn) or (weakly Q(Urysohn) and "not-Urysohn")), where (Q(Urysohn) and (weakly Q(Urysohn) and "not-Urysohn")) does not exist.

Theorem 3.3. *Let Q be a topological property for which weakly Q(Urysohn) exists and let (X, T) be a space with property weakly Q(Urysohn). Then (X, T) is "not-T_0" iff (X, T) is "not-Urysohn".*

Proof. Since weakly Q(Urysohn) = (Q (Urysohn) or (weakly Q(Urysohn) and "not-T_0")) = (Q (Urysohn) or (weakly Q(Urysohn) and "not-Urysohn")), where each pair Q(Urysohn) and (weakly Q(Urysohn) and "not-T_0"), and Q(Urysohn) and (weakly Q(Urysohn) and "not-Urysohn") do not exist, then (weakly Q(Urysohn) and "not-T_0") = weakly Q(Urysohn)$\backslash Q$(Urysohn) = (weakly Q(Urysohn) and "not-Urysohn"). Thus (X, T) is "not-T_0" iff (X, T) is "not-Urysohn".

Corollary 3.3. *Let Q be a topological property for which weakly Q(Urysohn) exists and let (X, T) be a space with property weakly Q(Urysohn). Then (X, T) is T_0 iff (X, T) is Urysohn.*

Combining Corollary 3.3 with the facts that Urysohn implies T_2, which implies T_1, which implies T_0 gives the last two results in this paper.

Corollary 3.4. *Let Q be a topological property for which weakly Q(Urysohn) exists and let (X, T) be a space that has property weakly Q(Urysohn). Then (X, T) is Urysohn iff (X, T)* is *T_2 iff (X, T) is T_1 iff (X, T) is T_0.*

Corollary 3.5. *For a weakly Urysohn space (X, T), (X, T) is Urysohn iff (X, T) is T_2 iff (X, T) is T_1 iff (X, T) is T_0.*

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