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WEAKLY NEARLY-R_0 SPACES

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Abstract

The object of the present paper is to define and study weakly nearly-R_0 spaces which comes out to be weaker than the nearly-R_0 axiom due to Jain [2].

1. Introduction

In a topological space X, a set A is called regular open [6] if $\operatorname{int}(\operatorname{cl} A) = A$. The complement of a regular open set is called regular closed [6]. Obviously a set A is regular closed if $A = \operatorname{cl}(\operatorname{int} A)$. Since the intersection of any two regular open sets is regular open, the family of all regular open sets of a space (X, τ) form a base for some topology τ^*. It is clear that $\tau^* \subseteq \tau$. A space (X, τ) is called semi-regular [6]

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iff $\tau = \tau^*$. Velicko [6] has defined δ-open sets. A set A is δ-open if for each $x \in A$, there exists a regular open set U such that $x \in U \subseteq A$. The complement of a δ-open set is called δ-closed [6]. The closure of a set A in (X, τ^*) is denoted by $\delta\mathrm{cl}(A)$. A subset A of X is δ-closed iff $A = \delta\mathrm{cl}(A)$.

N. A. Shanin [5] first defined the R_0 axiom. A space (X, τ) is called R_0 [1] iff for each $U \in \tau$, $x \in U$ implies $\mathrm{cl}(x) \subseteq U$.

A. S. Davis [1] rediscovered the axiom R_0 and gave several interesting characterizations. Jain [2] defined a space X to be nearly-R_0 if for each δ-open set O and each $x \in O$, $\delta\mathrm{cl}\{x\} \subseteq O$.

The purpose of the present paper is to introduce weakly nearly-R_0 spaces and obtain its basic properties.

Definition 1. A space X is said to be nearly-R_0 [2] if for each δ-open set O and each $x \in O$, $\delta\mathrm{cl}\{x\} \subseteq O$.

Definition 2. A space X is said to be weakly nearly-R_0 iff $\bigcap_{x \in X} \delta\mathrm{cl}\{x\} = \phi$.

It is evident that every nearly-R_0 space is weakly nearly-R_0. But the converse need not be true as can be seen from the following example:

Example 1. Let $X = \{a, b, c, d\}$ and let $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then the space $\{X, \tau\}$ is weakly nearly-R_0 as $\bigcap_{x \in X} \delta\mathrm{cl}\{x\} = \phi$. But it is not nearly-$R_0$. For, $\{a, c\}$ is δ-open and $a \in \{a, c\}$, but $\delta\mathrm{cl}\{a\} = \{a, c, d\} \not\subset \{a, c\}$.

Definition 3. A space X is said to be weakly-R_0 [3] iff $\bigcap_{x \in X} \mathrm{cl}\{x\} = \phi$.

Theorem 1. *A space (X, τ) is weakly nearly-R_0 iff (X, τ^*) is weakly-R_0.*

Proof. Obvious.

Theorem 2. *Every weakly nearly*-R_0 *space is weakly*-R_0.

Proof. The proof is obvious in view of the fact that

$$\bigcap_{x\in X} \mathrm{cl}\{x\} \subseteq \bigcap_{x\in X} \delta\mathrm{cl}\{x\} \text{ for any } x \in X.$$

The converse of the above Theorem is not true as is shown by the following example:

Example 2. Let $X = \{a, b, c, d\}$ and let $\tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{b\}, \{a, b\}\}$. Then the space (X, τ) is weakly-R_0 as $\bigcap_{x\in X} \mathrm{cl}\{x\} = \phi$. But it is not weakly nearly-R_0 since $\bigcap_{x\in X} \delta\mathrm{cl}\{x\} = \{d\}$.

Definition 4 [4]**.** Let X be a topological space and $x \in X$. Then the δ-kernel of x is $\delta\ker\{x\} = \bigcap\{U : U$ is δ-open and $x \in U\}$.

Theorem 3. *A space* X *is weakly nearly*-R_0 *iff* $\delta\, ker\{x\} \neq X$ *for any* $x \in X$.

Proof. Necessity: Let $a \in X$ such that $\delta\ker\{a\} = X$. This means that a is not contained in any proper δ-open subset of X. Thus a belongs to the δ-closure of every singleton. Hence $a \in \bigcap_{x\in X} \delta\mathrm{cl}\{x\}$, which is a contradiction.

Sufficiency: Assume that $\delta\ker\{x\} \neq X$ for any $x \in X$. If there is a point $a \in X$ such that $a \in \bigcap_{x\in X} \delta\mathrm{cl}\{x\}$, then every δ-open set containing a must contain every point of X. Therefore, the unique δ-open set containing a is X. Hence $\delta\ker\{x\} = X$, which is a contradiction . Thus X is weakly nearly-R_0.

Definition 5. A function $f : X \to Y$ is said to be pre-δ-closed if the image of every δ-closed set in X is δ-closed in Y.

Theorem 4. *Let* $f : X \to Y$ *is a pre*-δ-*closed injection. If* X *is weakly* *nearly* - R_0, *then so is* Y.

Proof. We have

$$\bigcap_{x \in X} \delta\mathrm{cl}\{y\} \subseteq \bigcap_{x \in X} \delta\mathrm{cl}\{f(x)\} \subseteq f\left(\bigcap_{x \in X} \delta\mathrm{cl}\{x\}\right) = f(\phi) = \phi.$$

Hence Y is weakly nearly - R_0.

Theorem 5. *If a space* X *is weakly nearly* - R_0, *then for any space* Y, *the product space* $X \times Y$ *is weakly nearly* - R_0.

Proof. We have

$$\bigcap_{(x,\, y) \in X \times Y} \delta\mathrm{cl}\{(x,\ y)\} \subseteq \bigcap_{(x,\, y) \in X \times Y} \{\delta\mathrm{cl}\{x\} \times \delta\mathrm{cl}\{y\}\}$$

$$\subseteq \bigcap_{x \in X} \delta\mathrm{cl}\{x\} \times \bigcap_{y \in Y} \delta\mathrm{cl}\{y\} \subseteq \phi \times Y = \phi.$$

Hence the product space $X \times Y$ is weakly nearly - R_0.

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