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TOPOLOGY AND THE COSMOLOGICAL PRINCIPLE

JUN ZHAO

NBC Universal Inc.
Mountain View, CA 94040
USA
e-mail: junzhao84@yahoo.com

Abstract

Modern cosmology rests on two pillars: general relativity (GR) and cosmological principle (CP). However, the CP is only vaguely defined. A close examination of its definition and applicability is provided, as well as a topological interpretation of them. The new interpretation of CP will have astronomical implications that may shed light on dark energy.

1. Introduction

The cosmological principle is the cornerstone of modern cosmology. A common definition of the cosmological principle is that the universe is isotropic and homogeneous on a large scale [1]. Based on this principle, cosmologists use spherical symmetry to simplify the Einstein field equation into the Friedmann equation. This has led to an expanding universe solution and big bang theory [1, 2]. This expanding universe was

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confirmed by observations [3]. Later observations revealed that the expansion rate is accelerating [4, 5], which inevitably leads to the dark energy hypothesis [4, 6].

However, universe expansion is noticeable only when we look far into the distance. Locally, no expansion was observed. For example, the spectrum of hydrogen atoms has remained the same for the past 13.7 billion years. The solar system has been of approximately the same size for the past 4.5 billion years. The Milky Way galaxy has been of approximately the same size for the past 13.5 billion years.

There are two possible outcomes of the expanding universe. The first is that dark energy will eventually overtake any physical forces, such that even the atomic nuclei will be ripped apart [7]. Second, the natural forces can still hold the galaxy intact, but galaxies will fly away from each other, so that eventually there is only one galaxy within our event horizon.

The first outcome was not supported by observations. The aforementioned three examples contradict this. The second outcome contradicts Mach's principle. Mach's principle was developed by Ernst Mach to counter Newton's absolute motion. Newton used the bucket of water thought experiment to demonstrate absolute motion [8, 9]. In this experiment, a bucket of water will have a flat surface if it stands still, but the water surface will curve when the bucket rotates around its axis. From this experiment, Newton conceived of both motion and space-time as absolute [8, 9]. Later, when Mach considered the same experiment, he drew a different conclusion, which eventually became Mach's principle. In Mach's principle, any motion, with or without acceleration, is relative; if the bucket of water is the only object in the universe, then its surface must be flat, even if it stands still, when all the surrounding objects, including distant stars, rotate around it, the water surface will curve [10, 11]. The central theme of Mach's principle is that we cannot talk about motion without a reference frame, and the reference frame must be defined by concrete objects. Therefore, if our galaxy is the only object in the observable universe, then there is no way to determine if the universe is expanding.

On a small scale, the cosmological principle is apparently not true because of the local motion of the observer and the local structure of the universe [12]. This principle seems to be true on a large scale. However, if we do not define an applicable scale quantitatively, this principle cannot be falsified. What is the applicable scale above which the cosmological principle is true, and why?

Observations have shown that the universe is full of cosmic filaments, ranging from a few hundred million light years to a few billion light years [13]. They are the largest structures in the universe. Recent observations reveal that the cosmic filament has spin [14, 15]. For example, most galaxies reside on cosmic filaments, and the spin direction of a spiral galaxy is aligned with the cosmic filament where it resides. This indicates that the cosmic filament is a physical structure that plays an important role in galaxy formation [16]. Can the scale of the largest cosmic filament be used to define the applicable scale of the cosmological principle? If so, why does the universe appear isotropic and homogeneous above this scale?

In hyperbolic geometry, a pair of parallel geodesics will diverge further apart when going to a distance [17]. This is true in both directions. If we use the Ricci tensor to track the volume along a bundle of geodesics, then this volume grows increasingly larger when we travel along the geodesics to a distance [17]. This certainly resembles cosmic expansion.

In topology, the surface of a multiholed torus forms a hyperbolic geometry [18]. If the universe is a finite, compact manifold with many holes, then we must use hyperbolic geometry to describe it. If this hypothesis is correct, can we find the holes in the universe? If a cosmic filament is a spinning tube that accretes matter to form galaxies [16], can this spinning tube be a hole in the compact manifold of the universe? If so, the scale of the largest cosmic filament can be used as the approximate size of the universe. If so, then our universe is probably only a few billion light years across; when we look through a light ray into the past for 13.7 billion years, we probably follow a geodesic on this compact manifold multiple times already.

If the universe is a multiholed torus, then when we look farther away, the universe appears to expand owing to hyperbolic geometry. This perfectly explains the conundrum that, on a small scale, the universe does not seem to expand, but on a large scale, the universe appears to expand globally.

If this hypothesis is correct, then when we look 13.7 billion years back, we will see the same universe repeatedly. If so, we should observe some pattern of repetition in deep-space observations. Unfortunately, such a pattern of repetition has not been observed [19, 20].

We must realize that our universe is a dynamic system. If we treat each galaxy as a particle, then these particles are constantly moving. Large-scale cosmic filaments constantly merge and morph. In such a dynamic system, finding the static pattern is not very useful because when we traverse the universe once through a light ray, a few billion years have already passed, so the shape and topology of the universe may have dramatically changed.

In the field of signal processing, when we search for a pattern of repetition in a time-varying signal, we look at the signal's power spectrum to identify resonances [21]. For example, a white noise signal is a stationary signal that looks statistically similar to a constant, but it is not a static signal; the signal rapidly oscillates at every frequency [21]. When we observe the universe on a large scale, we can use the same technique to search for repetition patterns. We observed resonances in the power spectrum of the received light. In this respect, CMB data are ideal for such studies. We have already found resonances in the CMB power spectrum [22]. If the resonances in the CMB power spectrum are the revelation of patterns of repetition for the universe, can we derive the exact size of the universe from it?

The CMB is believed to be a remnant of the big bang [23]. This is a major support of the big bang theory [23]. This is also a major support of the cosmological principle because the CMB temperature is almost the same in every direction. Moreover, the CMB signature, which is the CMB power spectrum, is the same in all directions [22]. However, this perfection raises doubts about the big bang theory. In the labs on Earth, it is impossible to create an explosion that is perfectly isotropic and homogeneous. If such a perfect explosion cannot be created locally on a small scale, how can it be created for the entire universe? To overcome this difficulty, inflation theory comes to rescue [24]. Inflation theory postulates that after the big bang, the universe went through a quick period of rapid and uniform inflationary expansion that smoothed everything out [24].

Inflation theory requires a hypothetical particle, the inflaton. These particles form a dark energy field, which is also responsible for accelerating the expansion of the universe [24]. However, neither inflaton nor dark energy is well understood. Hunting for inflatons also comes up empty [25].

On the other hand, if the universe is a stationary, finite, compact multiholed manifold, then its geometry is hyperbolic. This universe appears stationary locally but appears expanding globally. There are two standard terms to describe a manifold: the fundamental domain and universal cover [18, 19]. For example, the surface of a two-holed 2-torus can be represented as an octagon on a flat 2D surface. This octagon forms the fundamental domain of a two-holed 2-torus manifold. When we look through a geodesic on this manifold, the same octagon is repeated in every direction. These repeated octagons form a universal cover that extends to infinity [18, 19].

Even if the fundamental domain is not isotropic, the universal cover is isotropic and homogeneous. However, if we misinterpret the universal cover by considering that each copy of the fundamental domain is a unique piece, the universal cover can be mistreated as an infinite, flat manifold by itself.

In the universal cover, a pair of parallel geodesics diverges further and separates into infinity [17, 18]. This can easily lead people to think that the universal cover is expanding, but that the size of the fundamental domain may not change at all. If the universe is a finite, compact manifold, it must be stationary instead of static because every galaxy in the universe is moving. A stationary universe does not expand locally but appears to expand globally owing to hyperbolic geometry. To prove this hypothesis, we must identify repeated patterns in the universe. To search for patterns of repetition in a stationary universe, we cannot rely on any static patterns; we must look for resonances in its power spectrum, as is commonly done in the field of signal processing [21].

If the above hypothesis is correct, then we do not need dark energy to explain the accelerating expansion of the universe, and we do not need inflation to smooth the remnants of the big bang. As in any explosion, the universe after a big bang can be very rough. It is not even singly connected but has many holes in it. Each hole was a spinning tube that accreted matter to form galaxies. These holes have become cosmic filaments in the universe [16].

The cosmological principle will only be true for the universal cover when we traverse the universe through a light ray multiple times, and we will see many copies of the same universe. Each copy of the universe was statistically identical. We can look for harmonic signals in the power spectrum of the CMB data to look for patterns of repetition. From these harmonic signals, we can calculate the size of the universe accurately.

2. Mathematical Background

Now, let us briefly describe some mathematical tools that will be used in subsequent sections. In this section, most descriptions and figures are obtained from mathematical textbooks and reviews [17, 18, 19, 20, 21].

2.1. Topology

References [19, 20] provide an excellent review of the topology and its astronomical applications. We consider some terminology from these two review papers.

Because it is difficult to visualize an n- dimensional $(n > 2)$ manifold, we will use a popular 2D manifold, the two-holed 2-torus manifold, to illustrate some basic ideas.

On the surface of the two-holed 2-torus in Figure 1, if one travels along a geodesic orbit, the orbit is not necessarily a closed orbit. Most of the orbits on the surface are not closed. In a nonclosed orbit, a 2D surface can be traversed over and over without an intersection. If two persons travel on a pair of parallel nonclosed orbits, unlike in Euclidean geometry, the separation between the two persons becomes increasingly larger as they travel a distance. We refer to the geometry on this surface as hyperbolic geometry [17].

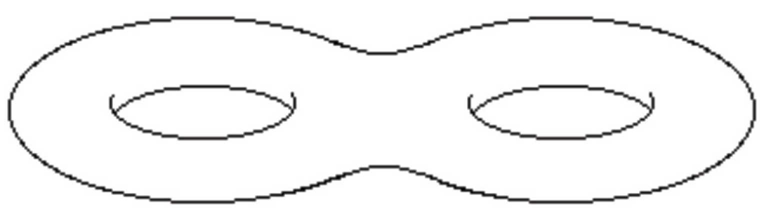

Figure 1.

We can find 4 closed orbits (a, b, c and d) on the surface and cut the surface along these orbits to form a flat octagon in Figure 2. This octagon forms the fundamental domain of the two-holed 2-torus manifold [18].

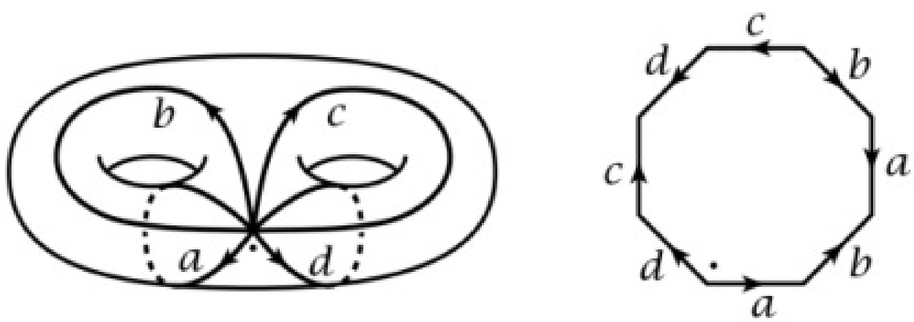

Figure 2.

We can reflect the octagon on each edge to form a new copy. Each copy of the octagon is a representation of the same fundamental domain. If this octagon is a regular octagon in Euclidean geometry, then its interior angle at each vertex is $\frac{3}{4}\pi.$ Therefore, we cannot pack 8 octagons at each vertex without overlap. However, in hyperbolic geometry, we can shrink the interior angle of each vertex to $\frac{1}{4}\pi$ by expanding the octagon, as in Figure 3, so that we can perfectly pack 8 octagons at each vertex without overlap, as in Figure 4 [18].

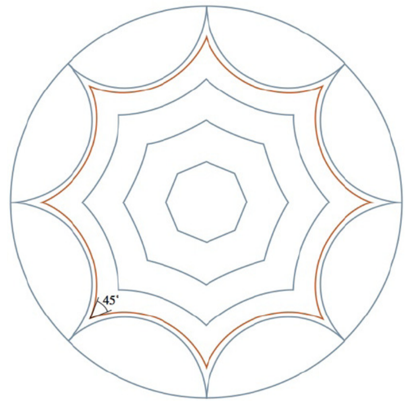

Figure 3.

These copies of the octagon form a perfect tessellation on the flat 2D hyperbolic disk in Figure 4. This hyperbolic disk is called the Poincare disk, whose boundary is the unit circle, which represents infinity [17].

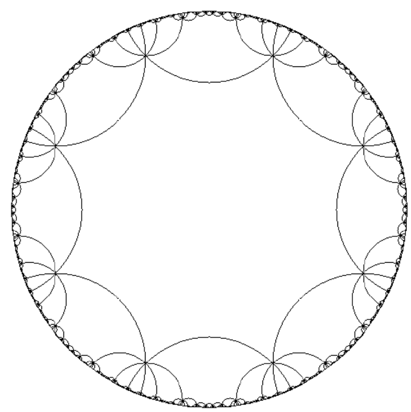

Figure 4.

This octagon tessellation covers the entire Poincare disk or the entire 2D hyperbolic space. It forms the universal cover of the two-holed 2-torus manifold [18]. In the universal cover, even if a copy of the octagon looks smaller and smaller when it approaches infinity, it has the same size and shape as the fundamental domain, so every copy of the octagon in the universal cover is an identical representation of the fundamental domain [18].

A geodesic orbit on the universal cover is either a straight line or semicircle perpendicular to the unit circle [17, 18]. The separation of any pair of nonintersecting geodesic obits becomes infinite when approaching infinity, as shown in Figure 5. This may give people the wrong impression that the space represented by the universal cover is expanding, even though the size of the fundamental domain or manifold itself remains the same.

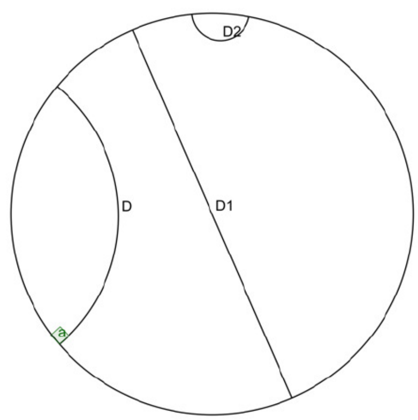

Figure 5.

When we shift from 2D manifolds to 3D manifolds, the fundamental domain will change from a polygon to a polyhedron, and the universal cover will change from the Poincare disk to the unit 3-ball with its boundary, the unit 2-sphere, representing infinity [26]. Because it is difficult for humans to visualize a 3D manifold, it is not easy to find the appropriate lower-dimensional surfaces to cut the manifold to form a polyhedron, the fundamental domain, and then glue the surfaces of neighboring copies of the fundamental domain to form a tessellation pattern in the universal cover. Fortunately, in the last few decades, mathematicians have made huge progress in the study of high-dimensional manifolds [26], and these results can be used to study the topology of the universe. Despite the difficulty in high-dimensional manifolds, the basic concepts of the fundamental domain and universal cover remain the same. For pedagogical reasons, in the rest of this paper, we will use Figure 4 to describe 3D manifolds instead of drawing a convoluted 3D picture.

Our universe is apparently much more complicated than a simple two-holed 2-torus. If the universe is a manifold, it can be viewed as either a 4D space-time manifold or a 3D space manifold with an evolutionary history in time. In the remainder of this paper, we take the second view.

If each cosmic filament in the universe is a spinning tube that accretes matter to form galaxies [16], and if each spinning tube is a hole in the universe, then the universe is a multiholed 3-torus that evolves in time. This manifold has a fundamental domain and universal cover in 3D. Its fundamental domain can be finite and compact; however, its universal cover is infinite.

Unlike the static two-holed 2-torus described above, the universe is constantly changing. For example, when cosmic filaments merge and morph, the number of holes in the universe also changes. Therefore, both the shape and topology of the multiholed 3-torus of the universe are changing. Even though the size of the fundamental domain remains the same, the tessellation on the universal cover is not static. Therefore, when we try to find the pattern of repetition in the universal cover, we cannot rely on any static pattern. Other methods must be used [21].

2.2. Hyperbolic geometry

It is very convenient to study the geometry in the universal cover because it is isotropic and homogeneous even if the underlying manifold has an exotic shape and topology. The universal cover is also infinite in size even though the underlying manifold is finite and compact.

There are three types of geometries: Euclidean, spherical and hyperbolic. The most important task in geometry is to measure length and angle. Thus, the metric tensor is used. For example, in Euclidean geometry, the metric tensor of the space-time manifold is the Minkowski metric described in Equation (1):

$$
\begin{aligned}
ds^2 = \eta_{\mu\nu} dX^{\mu} dX^{\nu} &= (cdt)^2 - dx^2 - dy^2 - dz^2 \\
&= (cdt)^2 - dr^2 - r^2 d\Omega \\
&= (cdt)^2 - dr^2 - r^2 (d\theta^2 + (\sin\theta)^2 d\phi^2). \qquad (1)
\end{aligned}
$$

In the last step of Equation (1), we use spherical coordinates owing to the spherical symmetry in the universal cover. The spherical symmetry originates directly from the isotropic and homogeneous properties of the universal cover [18].

In spherical geometry, we must use the spherical metric, and Equation (1) becomes Equation (2):

$$
ds^2 = (cdt)^2 - dr^2 - \left(\frac{D}{\pi}\sin\left(\pi\frac{r}{D}\right)\right)^2 (d\theta^2 + (\sin\theta)^2 d\phi^2). \qquad (2)
$$

In Equation (2), D is the diameter of the spherical manifold. Locally, on a small scale, when $\frac{r}{D} << 1$, Equation (2) is reduced to Equation (1); therefore, locally, the spherical geometry approximately becomes the Euclidean geometry.

In hyperbolic geometry, we must use the hyperbolic metric, and Equation (1) becomes Equation (3):

$$
ds^2 = (cdt)^2 - dr^2 - \left(D\sinh\left(\frac{r}{D}\right)\right)^2 (d\theta^2 + (\sin\theta)^2 d\phi^2). \qquad (3)
$$

In Equation (3), D is the size of the multiholed 3-torus manifold. Locally, on a small scale, when $\frac{r}{D} << 1,$ Equation (3) is reduced to Equation (1); therefore, locally, the hyperbolic geometry approximately becomes the Euclidean geometry.

In Equations (2) and (3), we assume that the manifold is finite and compact. However, if we believe that the universe is infinite so that $D \approx \infty,$ then both Equations (2) and (3) are reduced to Equation (1), and the universe will become a flat Euclidean space.

Equations (1), (2), and (3) differ slightly from the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric in GR textbooks [1]. More specifically, we replace the scale factor in the FLRW metric with the physical size of the space 3-manifold. This will be very useful when dealing with stationary, compact, and finite space manifolds.

2.3. Signal processing and Fourier analysis

According to Fourier theory, any periodic function can be expanded into a Fourier series [27]. For example, the periodic function in Equation (4) can be expanded into Equation (5):

$$f(x + nT) = f(x), \qquad n = 0, \pm 1, \pm 2, \dots , \tag{4}$$

$$f(x) = \sum_{n=0, \pm 1, \pm 2, \dots} a_n \sin(nkx) + b_n \cos(nkx), \qquad k = \frac{2\pi}{T}. \tag{5}$$

In Equation (4), function $f(x)$ is a static function. For a static function, we can either look for the repeated patterns in the coordinate space or look for harmonic resonances in the frequency space to determine if function $f(x)$ is a periodic function or not, and its period if it is [27].

For a time-varying function, finding periodicity is not straightforward. For example, searching for patterns of repetition in the coordinate space is difficult because the patterns are constantly changing.

However, we can still search for statistical resonance in the frequency space. In this respect, stochastic signal analysis is convenient [21].

For example, the white noise signal appears statistically similar to a constant, but the signal is not static; it oscillates rapidly at every frequency. In the coordinate space, the signal appears as random noise. In the frequency space, the signal is also very boring because its frequency spectrum is constant, which is why it is called white noise. There were no features of it. We call this type of signal a stationary signal to differentiate it from a static signal [21].

For a stationary signal, if its spectrum shows harmonic resonances, there are peaks at certain frequencies, and these frequencies are integer multiples of the base frequency, as in Equation (5), then, it is a manifestation of periodicity in the signal in the coordinate space. This type of analysis is widely used in signal processing [21].

Our universe is not static because every object in it is constantly moving, but it could be stationary, as Newton and Einstein originally thought, because locally, the universe does not seem to expand or contract. What about Hubble's law? Do we observe distant galaxies moving away from us [3]?

Can this seemingly expansion be considered an illusion of hyperbolic geometry? According to Figures 4 and 5, everything in the universal cover seems to move away from one another. Is it possible that we mistakenly treat the universal cover as our universe, but, in fact, is the universe just the size of its fundamental domain? If so, can we use the methods described in this section to identify patterns of repetition in the universal cover of the universe?

3. Astronomical Implications

Over the past century, physicists and astronomers have made significant advances in cosmology. Large telescopes and sophisticated astronomical instruments enable accurate astronomical measurements, and large databases have been compiled and made available for research. In theory, the developed ΛCDM standard model can provide accurate mathematical predictions, which are confirmed by observations [28].

However, several unsolved mysteries remain. The two biggest mysteries are dark matter and dark energy, which account for 95% of the material in the universe [19, 28]. In this study, we attempt to answer two questions: 1) the flat universe, which is revealed by CMB data [19], and 2) the accelerating expansion of the universe [4, 5]. We would like to demonstrate that these problems can be solved without dark matter or dark energy.

3.1. The cosmological principle and large-scale structures of the universe

The cosmological principle is a cornerstone of modern cosmology, and many theories are based on it [28]. However, the definition is vague. It states that, on a large scale, the universe is isotropic and homogeneous. However, this does not specify the scale size. Can we define a scale and explain why, above this scale, the universe looks isotropic and homogeneous?

Astronomers like to say that, on the CMB scale, the universe is quite uniform. The CMB temperature is almost the same in all directions. After correcting the dipole anisotropy that results from the solar motion with respect to the CMB [29, 30], the temperature fluctuation was only $0.0002\ \mathrm{K}$ [31].

Some astronomers use different radio galaxy sources to measure dipole anisotropy [12]. Their study shows that such dipole anisotropy cannot be solely kinematic but also structural [12, 32].

Astronomers have found that galaxies are not randomly distributed in the universe; they normally live on cosmic filaments, which are the largest structures in the universe. In the compiled cosmic filament catalog [13, 37], the size ranged from a few hundred million light years to a few billion light years. Large voids exist between the cosmic filaments, where the galaxy population is very low.

It is obvious that within the scale of the cosmic filaments, the cosmological principle cannot be true. Can we use the largest size from the cosmic filament catalog as the defining scale for the cosmological principle? When we do so, we must be very careful because we could erroneously combine two unrelated cosmic filaments to form a larger one. To avoid this problem, we can use the spin property of the cosmic filament to trim its catalog. For example, if we find one cosmic filament with opposite spin directions, it is most likely that we combined a cosmic filament with its mirror image. Nevertheless, this scale is only a few billion light years.

If we use this scale to define the cosmological principle, can we explain why the universe appears isotropic and homogeneous above this scale?

If our universe is a finite, compact manifold with many holes, each of which is a cosmic filament, then its topology is most likely a multiholed 3-torus that evolves in time. If so, the geometry is hyperbolic. This manifold can be described by its universal cover, as shown in Figure 4. The only difference is that both its fundamental domain and universal cover are 3D; however, the concept is the same.

From Figure 4, we can see that the shape of the fundamental domain can be very exotic, but when we look further, we see identical copies of the fundamental domain over and over in every direction. Therefore, the universal cover is infinite, isotropic, and homogeneous.

In 1995, Bob Williams and his team pointed the Hubble Space Telescope to a patch of dark sky previously known to have nothing for 100 hours [33]. Surprisingly, the image from the dark sky patch contains thousands of galaxies. Later, similar observations were carried out on other patches of dark sky, and the results were the same [34]: there were plenty of galaxies in every direction. This phenomenon is easily explained in Figure 4.

In Figure 4, when you look at any direction from the center, even in the direction of a large void, you will see copies of the fundamental domain in different orientations, so you are bound to see something if you look far enough.

Figure 1 shows that the size of the largest hole in our universe is a good approximation of the size of the fundamental domain of the universe. Inside the fundamental domain, the manifold is full of structures and is highly anisotropic. However, above this scale in the universal cover, different copies of the same fundamental domain smooth things out, and the universe appears increasingly isotropic when one looks further into the distance (or into the past).

3.2. CMB and its power spectrum

Accurate CMB measurements show that the CMB temperature is uniform for up to four decimals [19, 31]. This means that, on the CMB scale, the universe is isotropic and homogeneous. The small temperature fluctuations show that the CMB signal has features. These features manifest as harmonic peaks in the frequency spectrum [22]. The most astonishing fact is that this signature is the same in all directions.

The CMB is believed to be a remnant of the big bang [23]. The perfect homogeneity of the CMB also raises doubts on the big bang theory; not only is the CMB temperature almost identical everywhere, but its signature is also identical everywhere. No one has orchestrated such a perfect explosion in the laboratory. How is it possible to create such an explosion for the entire universe?

This homogeneity can be easily explained by the topology. If our universe is a finite multiholed 3-torus whose size is approximately a few billion light years (the scale of large cosmic filaments), then when we look 13.7 billion light years into the distance, we will see the same universe $4 \sim 6$ times. As shown in Figure 4, many copies of the fundamental domain are equally distributed on a sphere with a radius of 13.7 billion light years. These copies form a periodic signal whose frequency spectrum exhibits harmonic resonances. This is exactly what we see in the CMB spectrum in Figure 6 [22].

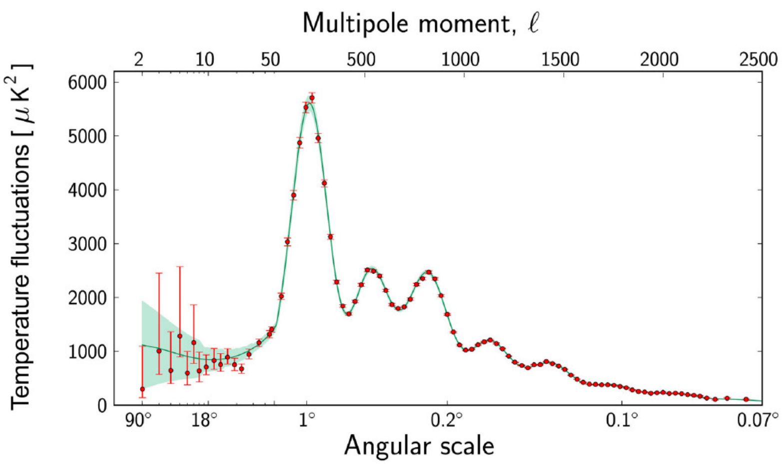

Figure 6.

If the CMB signal is a perfect periodic static signal, then according to Equation (5), we should see a sequence of pulses whose locations are integer multiples of the base frequency. However, our universe is a dynamical system, and every object in the universe is constantly moving, so the signal can only be stationary instead of static [21]. In such a signal, the sequence of pulses will become a sequence of peaks. This is very similar to the following process.

When we use the ground-based telescope to observe a point source in the universe, owing to the thermal motion in the Earth's atmosphere, the

CCD image of the point source becomes a distribution function instead of a point. Astronomers refer to this function as a point-spread function. They used this function to calibrate the observed image and produce a sharper image [35].

The CMB spectrum undergoes a similar process in which the random motion of galaxies causes a single frequency to spread into a frequency distribution; thus, the pulse train will become a smooth spectrum with multiple peaks whose locations are integer multiples of the base frequency.

If this analysis is correct, then the base frequency will indicate the size of the universe, the amplitudes of the harmonics will indicate the shape of the universe, and the width of the peaks will indicate the thermal character of the universe [21, 27].

From CMB data, astronomers concluded that our universe was very flat [19]. Such flatness requires 25% of the material in the universe to be dark matter, and 70% of the material is dark energy [19, 28]. Can we explain a flat universe without dark matter or dark energy?

Figure 4 shows that the universal cover of the universe is infinite, isotropic, and homogeneous. However, if we misinterpret the universal cover as the universe itself by treating each copy of the fundamental domain as a unique piece, our universe will be infinite and flat. For example, in Equation (3), if we take $D \approx \infty$, then Equation (3) becomes Equation (1); hyperbolic geometry becomes Euclidean geometry.

People may argue that if what we observe is the universal cover of our universe, then we should see the pattern of repetition when we do deep-sky observations, but such repeated patterns are missing [19]. This is because our universe is a dynamic system in which every object is moving. Its topology also changes when cosmic filaments merge and morph; therefore, the tessellation pattern in the universal cover is also constantly changing. Although our universe is finite, it is still very large. Thus, when traversing the universe once through a light ray, a few billion years have already passed, and any patterns will be totally different.

As discussed in Section 2.3, in such a dynamical system, finding static patterns is not very useful; we need to look for harmonic resonances in its spectrum to find the pattern of repetition. This is exactly what we observed in the CMB spectrum in Figure 6 [22].

3.3. Size of the universe

In 240 BC, the Greek astronomer Eratosthenes used the shadow of the Sun and simple geometry to calculate the circumference of the Earth to be 250000 stadia, which is between 24000 miles and 29000 miles [36]. The modern-day measurement of the circumference around the equator was 24900 miles. Can we perform the same feats for the universe?

In Euclidean geometry, an object of size l at distance r will have an angular size, as in Equation (6):

$$\theta = \frac{l}{r}. \tag{6}$$

In hyperbolic geometry, according to Equation (3), the angular size of the object is given by Equation (7):

$$\theta = \frac{l}{D \sinh\left(\frac{r}{D}\right)}. \tag{7}$$

In Equation (7), D is the size of the universe.

According to Figure 6, if the base frequency $\theta = 0.9° = \frac{0.9\pi}{180}$ $= 0.0157$ corresponds to the size of the universe, then, according to Equation (7),

$$0.0157 = \frac{D}{D\sinh\left(\frac{13.7}{D}\right)} = \frac{1}{\sinh\left(\frac{13.7}{D}\right)} \approx \frac{2}{e^{\left(\frac{13.7}{D}\right)}}. \tag{8}$$

In Equation (8), 13.7 billion light years is the distance to the CMB image. Solving Equation (8), the size of the universe is:

$$D = \frac{13.7}{\ln\left(\frac{2}{0.0157}\right)} = 2.826. \tag{9}$$

Therefore, our universe is only 2.826 billion light years in size, if it is a finite, compact manifold. This size is much smaller than the standard model prediction but agrees with the scale of large cosmic filaments [13, 37]. In the compiled cosmic filament catalog, only four out of 46 exceeded the above size; in the compiled cosmic void catalog, only one out of 117 exceeded this size [37].

There is still some debate regarding the size of the largest cosmic filaments [38, 39]. From Figure 4, we can see that it is very easy to combine structures from neighboring copies of the fundamental domain in the universal cover if they are very close. If we can use the spin property of the cosmic filament to triage the catalog of large cosmic structures, it will remove some of the controversies.

3.4. Expanding universe

A century ago, when Albert Einstein first applied his famous field equation to the universe, he could not believe that the universe was expanding, so he added a cosmological constant to his equation to create a stationary universe [1, 28]. After Edwin Hubble discovered that all distant galaxies receded from us, Einstein admitted that the cosmological constant was his biggest mistake and removed it from his equation [1, 28].

At the end of the last century, astronomers astonishingly found that the expansion of our universe was accelerating [4, 5]. Therefore, they returned the cosmological constant and attributed it to dark energy [4, 6].

As discussed earlier, the expanding universe has dire consequences, such as a big rip when everything is ripped apart by dark energy [7]. However, locally, we did not observe any expansion. Can this seemingly expansion be attributed to a hyperbolic geometry? From Figures 4 and 5, everything in the hyperbolic geometry seems to fly away from each other when approaching infinity.

In astronomy, when light is used to measure distance, time and distance are used interchangeably. In cosmology, we know that the Hubble constant, which measures the expansion rate of the universe, is not constant and is a function of time [1]. The Hubble constant is defined in Equation (10) [3]:

$$H_0 = \frac{3.42 \times 10^5 \text{mph}}{10^6 \text{light years}} = \frac{1}{1.963 \text{ billion years}}. \tag{10}$$

If the expansion of the universe is due to hyperbolic geometry, then from Equations (1) and (3), the expansion rate can be defined as in Equation (11):

$$\tilde{H} = \left| \frac{D \sinh\left(\frac{ct}{D}\right)}{ct} \right|, \quad t \in [0, -\infty]. \tag{11}$$

In Equation (11), when we look far into distance, we look back in time. Locally, when $\left|\frac{ct}{D}\right| << 1$, $\tilde{H} \approx 1$, there is no expansion. Globally, when $\left|\frac{ct}{D}\right| >> 1$, $\tilde{H} \approx \left|\frac{D}{2ct}\right| e^{\left|\frac{ct}{D}\right|}$, the expansion becomes exponential. The exponent rate is

$$H = \frac{c}{D} = \frac{c}{2.826 \text{ billion light years}} = \frac{1}{2.826 \text{ billion years}}. \qquad (12)$$

In Equation (12), c is the speed of light and D is the size of the universe given by Equation (9).

These results are consistent with observations: locally, both the solar system and the Milky Way galaxy have maintained their size throughout their lifetime; globally far into the distance, the expansion is accelerating exponentially [4, 5]. If the universe is a stationary, finite, and compact manifold, then the size of its fundamental domain will not change, but its universal cover will be infinite and expanding. In this view, cosmic expansion is just a hyperbolic illusion in the universal cover; physically, the universe maintains its size within the fundamental domain.

When trying to solve the GR equation for the universe, we must solve it in the fundamental domain. If our universe is a multiholed 3-torus instead of a spherical 3-ball, the cosmological principle will not be applicable in the fundamental domain, so we cannot use spherical symmetry to simplify the Einstein field equation into the Friedmann equation. Thus, the dynamics do not necessarily lead to simple expanding or contracting solutions.

3.5. Crucial observation tests

Any scientific theory should be falsifiable. We propose a crucial test that can either prove or falsify the new theory presented in this study.

The standard interpretation of the CMB power spectrum in Figure 6 is that it is the spectrum of the acoustic waves frozen in the last scattering epoch [42]. To fit the observed spectrum, one must rely on two large unknowns: dark matter and dark energy [19]. We interpret it as a manifestation of the pattern of repetition in the universal cover, as shown in Figure 4. In this interpretation, the CMB map is composed of all ghost images of the fundamental domain on the sphere with a radius of 13.7

billion light years (bly).

If we choose a smaller sphere with a radius of 8 billion light years, according to Figure 4, the ghost images of the fundamental domain on that sphere will also form a stationary periodic signal. Name the signal as the cosmic microwave foreground (CMF). According to Equation (11), for the CMB:

$$\frac{\lambda_{CMB}}{\lambda_0} = \widetilde{H}(13.7) = \left| \frac{D \sinh\left(\frac{13.7}{D}\right)}{13.7} \right|. \tag{13}$$

For the CMF:

$$\frac{\lambda_{CMF}}{\lambda_0} = \widetilde{H}(8) = \left| \frac{D \sinh\left(\frac{8}{D}\right)}{8} \right|. \tag{14}$$

In Equations (13) and (14), D is the size of the fundamental domain given by Equation (9). From Equations (13) and (14):

$$\frac{T_{CMB}}{T_{CMF}} = \frac{\lambda_{CMF}}{\lambda_{CMB}} = \left| \frac{13.7 \sinh\left(\frac{8}{D}\right)}{8 \sinh\left(\frac{13.7}{D}\right)} \right| \approx \frac{13.7}{8} \frac{e^{\frac{8}{2.826}}}{e^{\frac{13.7}{2.826}}} = 0.228. \tag{15}$$

Therefore, the CMF temperature is:

$$T_{CMF} = \frac{T_{CMB}}{0.228} = \frac{2.725}{0.228} = 11.96 \text{ K}. \tag{16}$$

According to Equation (7), the harmonic base frequency of the CMF is:

$$\theta = \frac{\mathrm{D}}{D \sinh\left(\frac{8}{D}\right)} \approx 2e^{-\frac{8}{D}} = 2e^{-\frac{8}{2.826}} = 0.118 = 6.76°. \tag{17}$$

The new theory predicts that if we look for a CMF at a temperature of

11.96 K, then the harmonics of its power spectrum will have a base frequency of 6.76°.

According to Figure 4, the ghost image of the fundamental domain at the CMB is much more focused than that at the CMF owing to cosmic lensing; therefore, the CMF is more transparent than the CMB. Nevertheless, the CMF is detectable with existing telescopes.

When I stand on the shore of Lake Tahoe, which is only a few miles across, I love to stare at water waves propagating on the lake surface. Patches of different wave patterns can be easily seen in different regions of the lake surface. According to the big bang theory, at the epoch of the last scattering, the universe was a lake of hot plasma with a size of 42 million light years. It is mind-boggling that the acoustic waves in this giant lake were not only uniform but also uniform at every single frequency. On the other hand, if the CMB is composed of all ghost images of the background radiation in the universe on a sphere with a radius of 13.7 billion light years, then the harmonic resonances in its power spectrum become easy to understand.

With the advent of James Webb telescope, more and more high redshift galaxies will be observed [40, 41]. The high redshift will push the age of these galaxies to approach the age of the universe. This will pose a serious threat to the big bang theory. The high redshift can be easily explained by the new theory. According to Equations (11) and (9), the redshift z is

$$z+1=\frac{D\sinh\left(\frac{r}{D}\right)}{r}=\frac{2.826\sinh\left(\frac{r}{2.826}\right)}{r}\cong\frac{2.826}{2r}\,e^{\frac{r}{2.826}}. \tag{18}$$

So, $z=12.52$ when $r=13.8\,\text{bly}$. For $z>12.52$, $r>13.8\,\text{bly}$. This is allowed by the new theory; the light ray only needs to traverse the finite stationary universe in more iterations.

3.6. Comparison with previous works

In the past, many physicists explored the subject of cosmic topology. References [19, 20] provide excellent reviews on this subject. The most common way for people to test finite universe models is to look for patterns of repetition in the observable universe. For example, they searched for ghost images or ran correlations on the CMB map. As explained in Section 2.3, our universe is a dynamical system described by a stationary signal instead of a static signal [21]. Identifying static patterns in such systems is futile. The correct way to look for patterns of repetition in such a system is to look for harmonic resonances in the signal's power spectrum, which is a standard procedure in stochastic signal processing [21].

People prefer to use flat or positively curved manifolds to model the universe. Part of the reason is that these 3-manifolds are better classified than the negatively curved 3-manifolds, and part of the reason is that observations point slightly toward the flat or positively curved space, as well as favorable mathematical properties, such as permitting a larger collection of Killing vectors in these models [26]. For example, in reference [43], Jean-Pierre Luminet chose a manifold with positive curvature, namely, the Poincare dodecahedral space (PDS) model. The metric of this model, which is a spherical geometry, is described by Equation (2). According to this metric, if the universe is stationary, then when we look far into the distance, the universe expands, contracts, alternates on and on. This contradicts the observations unless the diameter is physically expanding.

Another popular cosmic model is the single-holed 3-torus, which is the simplest 3-torus model. Almost all the torus models in the literature are single-holed [20]. The space represented by this model was a flat Euclidean space. In this model, the universe must be physically expanded to agree with observations. In paper [16], we demonstrated that a cosmic filament is a spinning hole that accretes matter to form galaxies. If a cosmic filament is a hole in the universe, then according to the catalog [37], there must be more than one hole in the universe.

We chose the multiholed 3-torus manifold, which has a negative curvature. In our model, this metric is described by Equation (3), which is a hyperbolic geometry. According to this metric, the universe appears to expand when we look far into the distance, even though its physical size remains the same. As previously explained, the expanding universe has dire consequences, such as a big rip or contradicting Mach's principle. We postulate that cosmic expansion is just a hyperbolic illusion in the universal cover, and that the physical size of the fundamental domain remains the same.

It is well known that positively curved manifolds have fewer symmetries [17, 18]. Therefore, only five platonic solids were present. Tessellation patterns in the universal cover are very limited. On the other hand, a negatively curved manifold has more symmetries, and there are many tessellation patterns to choose from [17, 18]. Pedagogically, this is best illustrated in Figure 3, where the interior angle of the polygon can be shrunk to the size so that more copies of the polygons can be fitted at a vertex.

We postulate that cosmic filaments are a manifestation of holes in the manifold of our universe. These physical structures are similar to blood veins in the body of the universe. A complicated cosmic web means that the tessellation in the universal cover is not only exotic but also changing. This certainly favors a hyperbolic geometry over a spherical geometry or Euclidean geometry. The multiholed 3-torus model is more realistic than the PDS model, single-holed 3-torus, or many other models.

4. Conclusion

By using the scale of large cosmic filaments, we quantitatively define the cosmological principle and explain why, above this scale, the universe appears isotropic and homogeneous. We propose a stationary, finite, and compact universe whose topology is a multiholed 3-torus manifold that evolves over time.

Using topology, hyperbolic geometry, and signal processing, we explain the flatness of the universe, which is revealed by the CMB data without dark matter and dark energy. Using the CMB frequency spectrum, we calculated the size of the universe accurately. Using hyperbolic geometry, we quantitatively explain the accelerating expansion of the universe without dark energy. We propose a crucial test to prove or falsify the proposed theory.

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