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THE QUANTUM TEMPERATURES BASED ON THE BCS THEORY AND PLUMBENE-AU

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Abstract

The newly introduced "Swamp Model" is first proposed to simplify the complex issues surrounding the failure of the LK-99 superconductor and its magnetic susceptibility. Although the superconductivity of LK-99 has been disproved, the findings from the study still hold significant value. This paper broadens the investigation by exploring LK-99-like materials

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(such as Copper (II) oxides like YBCO). The analysis is based on Density Functional Theory (DFT) calculations, which represent an initial theoretical approach without simulating experimental data, aligning with other studies in the field. The core work of this article follows the energy-level equations for Cooper pairs proposed by Bogoliubov, mediated by the BCS theory. Ultimately, it concludes that the critical point of LK-99-like superconductors can reach a maximum of 272 K in high-temperature regions. By extending the BCS theory, we explore the potential for superconductivity at higher temperatures and introduce the concept of quantized temperatures.

Introduction

LK-99 (named after Lee and Kim's 1999 research) [1], also referred to as PCPOSOS [2], is a gray-black, polycrystalline compound identified as copper-doped lead oxyapatite. A research team from Korea University, led by Lee Sukbae and Kim Ji-Hoon, began investigating this material as a potential superconductor in 1999 [3]. In July 2023, the scientists published preprints claiming that LK-99 exhibited room-temperature (RT) superconductivity, operating at temperatures of up to 400 K (127°C; 260°F) under ambient pressure [4, 5]. Following these claims, many researchers attempted to replicate the findings, achieving preliminary results within weeks, as the synthesis process was relatively simple. However, by mid-August 2023, a consensus had formed that LK-99 is not a superconductor at any temperature and is, in fact, an insulator in its pure form [6-9].

As of 12 February 2024, no replication of the LK-99 claims had undergone the peer review process in any journal, though some had been reviewed by materials science laboratories. Several replication attempts identified non-superconducting causes for the observations that suggested superconductivity, such as ferromagnetic and diamagnetic effects. A notable issue was the presence of copper sulfide impurities [10] during the synthesis process, which can lead to resistance drops, a lambda transition in heat capacity, and magnetic responses in small samples [11-

14]. After the initial preprints were published, Lee admitted that the papers were incomplete, while coauthor Kim Hyun-Tak acknowledged flaws in one of the papers.

Although this paper was submitted later (2024), we have employed more convincing methods to deduce a theory of superconductivity at high temperatures. This work focuses on the concept of LK-99-like materials, expanding on the BCS Theory to explain Type-II superconductors in high-temperature regimes. While room-temperature superconductors have been widely speculated, they have not materialized in any LK-99-like materials, likely due to the temperature barrier around 272 K (with a 1.15 K gap from the freezing point at 273.15 K).

Method

A. The first principle: The Density Functional Theory

Based on density functional theory (DFT), the Hamiltonian for LK-99-like superconductor systems is given by:

$$\hat{H}\psi(\hat{r}, t) = (\hat{K} + V(\hat{r}))\psi(\hat{r}, t), \tag{1}$$

where $V(\hat{r})$ represents the magnetic energy stored within the system, and the kinetic energy term is accounted for by all corresponding terms in the Hartree-Fock equation (i.e., DFT: $3N \to 3$). The forms of power are expressed as[1]

$$\frac{\partial \hat{K}}{\partial t} = \left[\frac{\partial}{\partial r_{ij}} V(\hat{r}_{ij})\right]\dot{r}_{ij}, \; i = 3, \; j > i. \tag{2}$$

[1]In Eq. (2), the electron density is taken as 1, and in Eq. (4), it is considered as 2. Thus, the total variable count is 3. The concept of electron density provides a simpler approach compared to the Hartree-Fock method for solving problems.

The conditions for superconductivity phenomena are fulfilled when:

$$0 = \left[\frac{\partial}{\partial y} V(\hat{y})\right] \dot{y}. \tag{3}$$

Considering a constant potential along the y-axis, but leaving magnetic energy in B-fields on the xy-plane:

$$\frac{\partial \hat{K}}{\partial t} = \sum_{i=1}^{2} \sum_{j>1}^{2} \left[\frac{\partial}{\partial r_{ij}} V(\hat{r}_{ij})\right] \cdot \dot{r}_{ij}. \tag{4}$$

At temperature T_C, this formula implies that an object can move on the xy-plane if variables exist in $\partial \hat{K}/\partial t$ (i.e., small changes such as $\partial \hat{E}/\partial t\big|_{T=T_c} = \partial \hat{K}/\partial t$). Experimental observations occur at instantaneous moments, thus $dK = dE$ is allowed. In this paper, derivatives are regarded as perturbative contributions from electron-electron interactions in liquid-like states, simplifying the calculation.

B. The Swamp Model and Eddy Currents

In swamps, one typically cannot escape, as the density is greater than that $\rho = 1\,\mathrm{g/cm^3}$ water. How does this make scientific sense in this paper? The concept of "Swamp Density" provides an important hint in the form of a mathematical relationship that[2]

$$\sqrt{|x|} - 1 > 1, \tag{5}$$

where x is smooth and continuous, Eq. (5) reveals that objects move downward in swamps, utilizing $|dx| f(x) < 0$ because of the inherent

[2] $\sqrt{|x|} > 2$, $|dx| > 0$, $dx < 0$ ($dx' > 0$ is banned). Such leads in $x < 0$, $f(x) < 0$, $\Delta f(x) < 0$.

nature of swamps. To accurately connect this to Eddy Currents (EC) in superconductors, we examine the behavior in LK-99-like materials. The theoretical physics expressions outlined here align with the phenomena observed in superconductors, where ECs are closely linked to the Meissner effect. Based on this logical framework, the deductions made in this paper are well-supported and convincing.

If $f(y)$ could be interpreted as the energy in the Meissner effect, then $\Delta E < 0$ represents diamagnetism[2]. This approach within the swamp model offers a unique and precise method for uncovering the underlying principles of LK-99-like superconductors during theoretical investigations.

C. The magnetization and the Type II superconductors

In general, B-fields $B = \mu_0(M + H) = 0$, where $M = -H$ and the well-known $\chi_{ij} = \partial M_i / \partial H_j \equiv -1$ represent a superconductor in the Meissner effect. In the case of a vortex $(B > 0)$ and $M < -H$, as shown in Figure 1, we employ the swamp model to verify that the LK-99-like superconductor exhibits diamagnetism (with LK-99 possessing this property partially).

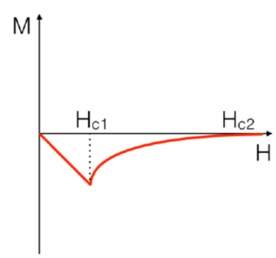

Figure 1. Type II superconductors: Notice that the curve at (H_{C1}, H_{C2}) is exactly the same form as $\sqrt{|x|} - 1 > 1$ on the x- axis, as indicated in Eq. (5). The magnetic field penetrates the superconductor in vortex form, characteristic of Type II superconductors in vortex states. The swamp

model (see Section B) is a new discovery introduced for the first time in theoretical studies. Based on these properties, we hypothesize that LK-99-like materials exhibit at least two critical temperatures.

Results and Discussion

A. Quantum mechanics and solid states physics

The points indicate that the BCS theory, as stated by Former U.S.S.R. scientist Bogoliubov through the Bogoliubov transformation, provides the following expression:

$$E = \left\langle \hat{E} \right\rangle = \left\langle \hat{K} \right\rangle = 3.52 k_B T_C \sqrt{1 - \frac{T}{T_c}} = K(T) \neq \text{Const.} \tag{6}$$

Since LK-99-like superconductivity occurs at high temperatures, the precise temperature $T \approx T_C$ contributes to the y-component and maintains the diamagnetism of LK-99-like. It is widely known that quantum fluctuations require T to be a variable running point when B-fields are applied.

$$dE = 3.52 k_B T_C d\sqrt{1 - \frac{T}{T_C}},$$

$$dE = \frac{3.52 k_B T_C}{2} \frac{1}{\sqrt{1 - \frac{T}{T_C}}} \left(- \frac{dT}{T_C} \right). \tag{7}$$

Let $x \equiv \frac{T}{T_C} = (\sqrt{T/T_C})^2$ and is involved in the parabola. Therefore $\frac{dT}{T_C} = dx,$ and the new $dE' = dE$ will be expressed as

$$dE' \equiv dE = - \frac{3.52 k_B T_C}{2} \frac{dx}{4\sqrt{1 - x}} < 0. \tag{8}$$

Assuming that Cooper pairs are collected at the upper bound of temperature T_C, the conductor can be represented in the BCS plot of energy levels of electrons versus temperature. Here, the parabola $(\sqrt{T/T_C})^2$ is thrown upwards (where $dE < 0$), meaning the maximum value of the first derivative is precisely at the peak of the parabola. Due to the properties of the parabola with the factor as 4, the lower limit of the integral is considered to be zero.

$$\int_{E_0}^{E} dE = -\frac{3.52 k_B T_C}{2} \int_{x=0}^{x} \frac{-d(1-x)}{4\sqrt{1-x}},$$

$$E - E_0 = \frac{3.52 k_B T_C}{2} \frac{2}{4} \sqrt{1-x}\Big|_{x=0}^{x},$$

$$\underbrace{E - E_0}_{\equiv \Delta E} = \frac{3.52 k_B T_C (\sqrt{1-x} - 1)}{4}. \tag{9}$$

Placing LK-99-like into B-fields, the energy change in the system is theoretically calculated as follows:

$$\Delta E = 0.88 k_B T_C \left(\sqrt{1 - \frac{T}{T_C}} - 1 \right). \tag{10}$$

In the case of $T \approx T_C$ represents the result of $\Delta E < 0$ and the negative sign indicates anti-magnetism. This suggests that the superconductor is in a vortex state, generating a negative surface energy as the magnetic field penetrates into the superconductor. This is represented as:

$$(H_{C1}, H_{C2}). \tag{11}$$

In this scenario, superconductivity is observed experimentally when $\Delta E > 0$ indicates para-magnetism. The critical temperatures for such a material may be located in "*lucky places*",[3] implying unique points on the temperature scale where superconducting behavior could occur.

Notably, the presence of -1 at the end of Eq. (10) reinforces the necessity of negative energy contributions, suggesting that if -1 is absent, LK-99-like behavior would not be established. This emphasizes the critical role of diamagnetism in the behavior of such superconducting materials.

B. The failure data of LK-99 By Lee/Kim (2023)

In Eq. (10), one may ask the question: can we regard -1 as the magnetic susceptibility of -1 in the Meissner effect? Does the above approach generalize the BCS theory to high temperatures?

Table 1. The different conditions for the superconductors

The Superconductor as an Example	Modification in Eq. (10)	Notes
YBCO	$\Delta E = 3.52 k_B T_C \left(\sqrt{1 - \frac{T}{T_c}}\right) > 0$ (12)	$T < T_C$ (Diamagnetism) (13)

The electronic structures for LK-99, $Pb_{10-x}Cu_x(PO_4)_6O$ $(0.9 < x < 1.1)$, were determined based on reference papers [4-5], with figures illustrating the individual positions of Cu and Pb atoms. These visualizations suggest the possibility of replacing these atoms under previously unexplored conditions. Initially, Figure 2(a) shows that the

[3]The plot of electron energy levels versus temperature in BCS theory shows the relationship between the energy gap of electrons and temperature, crucial for understanding superconductivity.

compound $Pb_{10}(PO_4)_6O$ forms a hexagonal-like structure. It is well-known that the partial diamagnetism of LK-99 is attributed to the presence of CuS_2 impurities. When a Cu atom substitutes for a Pb atom at the Pb(1) site, it suggests that Cu takes the place of a lead atom within the hexagonal coordination, resulting in the mixed configuration $Pb_9Cu(PO_4)_6O$_Pb(1) shown as Figure 2(b). This indicates a coexistence of copper and lead atoms within the bulk material. Figure 2(c), depicting $Pb_9Cu(PO_4)_6O$_Pb(2) describes the substitution of a Cu atom for a Pb atom at the Pb(2) site. To visualize the atomic arrangements in $Pb_9Cu(PO_4)_6O$_Pb(1), Figures 3(a), 3(b), and 3(c) represent the site views along the a-axis, b-axis, and c-axis, respectively. The Cu atom substitution may localize or alter the electronic structure at the Pb(1) site, particularly along the b-axis, where it may influence neighboring layers, affecting properties such as conductivity or magnetism.

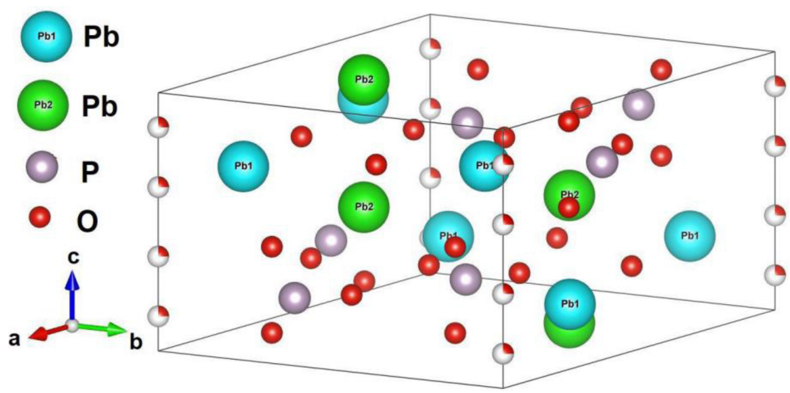

Figure 2(a). The compound $Pb_{10}(PO_4)_6O$ forms a hexagonal-like structure at the atomic level, characterized by the arrangement of lead (Pb), phosphate (PO_4) and oxygen (O) atoms. In this structure, the phosphate groups (PO_4) are likely arranged in a network that helps stabilize the overall lattice, with lead atoms playing a crucial role in maintaining the bulk material's stability.

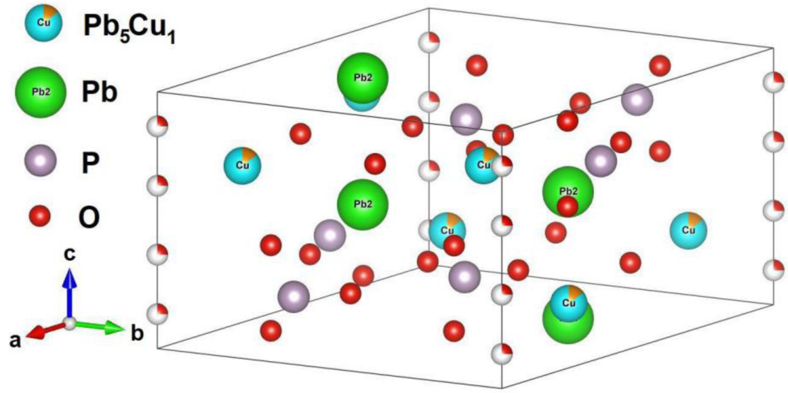

Figure 2(b). In the $Pb_9Cu(PO_4)_6O$_Pb(1) structure, the copper (Cu) atom is substituted at the Pb(1) site within a hexagonal-like lattice.

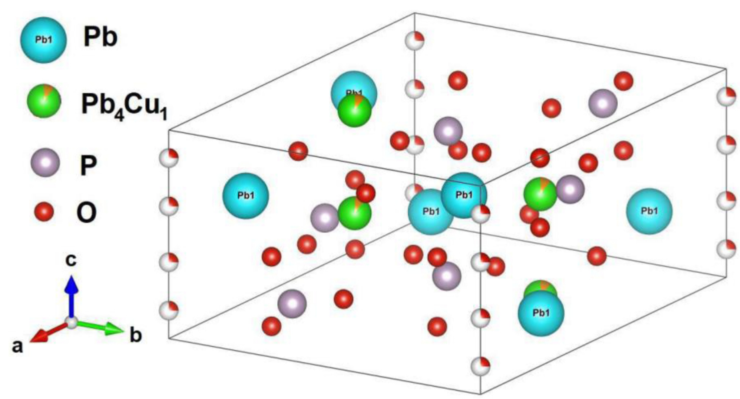

Figure 2(c). In the $Pb_9Cu(PO_4)_6O$_Pb(2) structure, the copper (Cu) atom is substituted at the Pb(2) site within a hexagonal-like lattice.

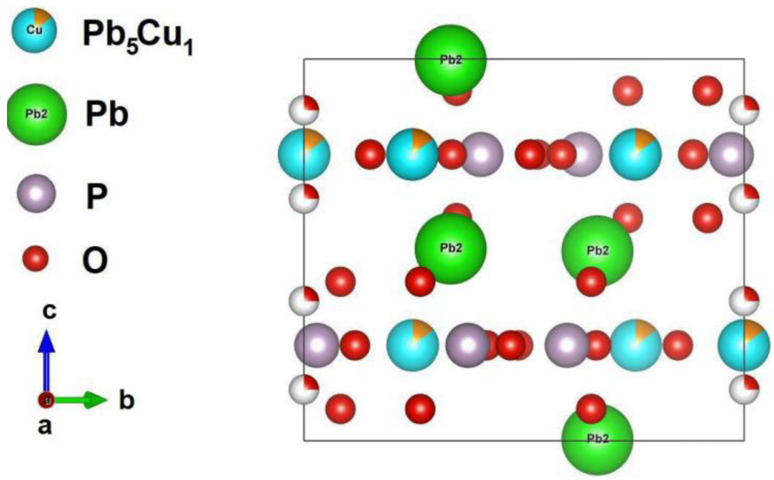

Figure 3(a). In the a-axis view of the $Pb_9Cu(PO_4)_6O$_Pb(1) structure, the copper (Cu) atom, having been substituted at the Pb(1) site, is positioned within a hexagonal-like lattice. Viewing the structure along the c-axis allows one to see the layered arrangement of atoms within the lattice.

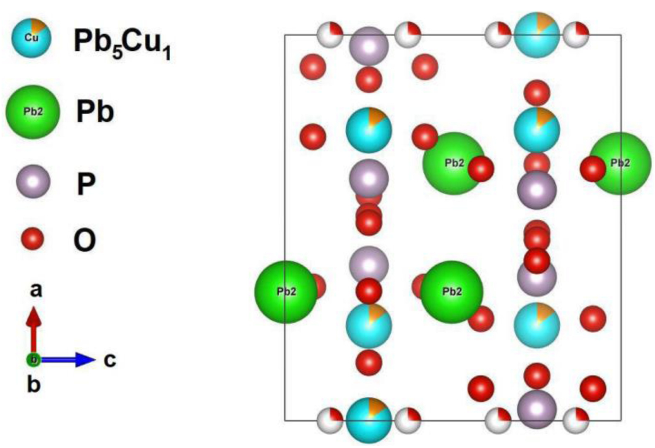

Figure 3(b). In the b-axis view of the $Pb_9Cu(PO_4)_6O$_Pb(1) structure.

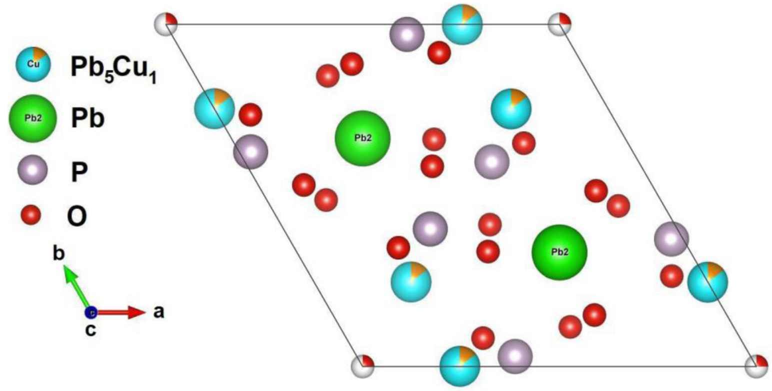

Figure 3(c). In the c-axis view of the $Pb_9Cu(PO_4)_6O\_Pb(1)$ structure.

C. The relative nature of temperatures of LK-99-like

Similar to the diagrams of M versus H, where mathematical behavior of $\sqrt{x}-1$ occurs at $[H_{C1}, H_{C2}]$, with $x \equiv T/T_C$ instead of M/H [magnetization (i.e., the magnetic moment per unit volume over H)] being regarded as $\log x \approx \sqrt{x}-1$, $x \approx 1$, note that 1 cannot be considered for the physical reason related to Type II superconductors. In such cases, one can directly take the logarithm of Eq. (10). Based on the continuity of logarithmic functions, it follows that

$$\log \Delta E \approx 0.88 k_B T_C \log(\sqrt{1-x}-1) \approx \log dE,$$

$$(\log \Delta E)' \approx -0.88 k_B T_C \frac{1/(2\sqrt{1-x})}{\sqrt{1-x}-1} \equiv \log d^2 E, \tag{14}$$

where $0.88 k_B$ is the slope of H_{C2} at T_C.

Magnetic susceptibility:

$$\chi_{ij} = \frac{\partial M_i}{\partial H_j}, \quad \Delta\chi_{ij} \sim \sqrt{x}-1, \quad \frac{\partial \Delta\chi_{ij}}{\partial x} = \left(\frac{1}{2}\frac{1}{\sqrt{x}}\right)_{ij} = \gamma \frac{\Delta\chi_{ij}}{2}, \quad 1 \le \gamma \le 2$$

(e.g., Ferromagnetism in $CuOFe_2O_3$* and the set of data complies with extensive experimental results) precisely

$$k_B \chi_{ij} = \frac{\partial E_i}{\partial T_j}. \tag{15}$$

Consider the form of Eq. (14), and then

$$k_B \chi_{ij} = \frac{\log \partial E_i}{\partial T_j}. \tag{16}$$

In case of $i \neq j$, $\partial T_i \neq \partial T_j \neq T_C$, $\partial T_i = \partial T$, $\partial T_j = \partial T$. Suppose that $1/\chi_{ij}$ depends on high temperatures. Hence in short-range variations within T-regions:

$$k_B \frac{d\chi_{ij}}{dT} = \frac{\log d^2 E}{dT^2}\bigg|_{T=T_C} = \frac{\log d^2 E}{T_C^2}. \tag{17}$$

Therefore, by substituting Eq. (14) into Eq. (17), we clearly obtain:

$$k_B \frac{d\chi_{ij}}{dT} = -0.88 \frac{k_B}{T_C} \frac{1/(2\sqrt{1-x})}{\sqrt{1-x}-1},$$

$$\frac{d\chi_{ij}}{dT} = -\frac{0.88}{T_C} \frac{1/(2\sqrt{1-x})}{\sqrt{1-x}-1}. \tag{18}$$

And by utilizing the properties of implicit functions accurately, we obtain:

$$T_C = 0.88 \frac{dT}{d\chi_{ij}} \frac{1/(2\sqrt{1-x})}{\sqrt{1-x}-1}. \tag{19}$$

The critical temperature results from an instantaneous change in LK-99-like, such leads us to derive the forms of ΔT

$$T_C = 0.88 \frac{\Delta T}{\Delta\chi_{ij}} \frac{1/(2\sqrt{1-x})}{\sqrt{1-x}-1}. \tag{20}$$

By definitively ensuring that LK-99-like belongs to Type II superconductors (i.e., high-temperature superconductivity), one can directly observe the interval of $[H_{C1}, H_{C2}]$, thus obtaining two rates of change using the second derivative at $[H_{C1}, H_{C2}]$, as indicated by Eq. (18)-(19). In the examinations for LK-99 (2023), the observable $\chi = 2.5 \times 10^{-3} \equiv \Delta\chi_{ij}$, $T \approx 400\,\text{K}$ (for LK-99-like at high temperatures (HT)) shows that in the case of HT being near the critical temperature, we obtain a form of $1/0$. Applying L'Hospital's rule, we can resolve this by taking the derivative into Eq. (19) or Eq. (20) once again.

$$T_C \overset{\text{(L.H.)}}{=} 0.88 \frac{\frac{dT}{dT}}{\left(\frac{d}{dx}\Delta\chi_{ij}\right)} \frac{\frac{d}{dx}\frac{1}{2\sqrt{1-x}}}{\frac{d}{dx}(\sqrt{1-x}-1)},$$

$$T_C = 0.88 \frac{1}{\gamma \cdot \frac{\Delta\chi_{ij}}{2}} \frac{\frac{1}{2}\left(-\frac{1}{2}\right)(1-x)^{-\frac{3}{2}}}{\frac{1}{2}(1-x)^{-\frac{1}{2}}}, \quad 1 \le \gamma \le 2,$$

$$T_C = -\frac{0.88}{\gamma}\frac{1}{\Delta\chi_{ij}}(1-x)^{-1}, \quad 1 \le \gamma \le 2, \tag{21}$$

where $x \equiv T/T_C$. Hence

$$T_C = -\frac{0.88}{\gamma}\frac{1}{\Delta\chi_{ij}}\left(1-\frac{T}{T_C}\right)^{-1}, \quad 1 \le \gamma \le 2. \tag{22}$$

The advanced calculation (substituting $\Delta\chi_{ij} = 2.5 \times 10^{-3}$ and considering $127°\text{C} \approx 400\,\text{K}$, the diamagnetism of LK-99-like) leads to the following arrangement and result:

$$T_C = -\frac{0.88}{\gamma \cdot (2.5 \times 10^{-3})}\left(1-\frac{T}{T_C}\right)^{-1},$$

$$T_{C1} = \frac{\gamma \cdot (2.5 \times 10^{-3})T - 0.88}{\gamma \cdot (2.5 \times 10^{-3})} = T - \left.\frac{0.88}{\gamma \cdot (2.5 \times 10^{-3})}\right|_{T = 400\ \mathrm{K}}$$

$$= 48\ \mathrm{K}, \quad \gamma = 1 \tag{23}$$

and

$$T_{C2} = T - \left.\frac{0.88}{\gamma \cdot (2.5 \times 10^{-3})}\right|_{T = 400\ \mathrm{K}} = 224\ \mathrm{K}, \quad \gamma = 2. \tag{24}$$

In the case of the widely-known LK-99-like material, with fixed components of its atoms or materials, the resulting yield presents two typical critical temperatures as shown in Eq. (23)-(24). The universal formula is given as follows:

$$T_{C,\gamma} = 400 - \frac{0.88}{\gamma \cdot (2.5 \times 10^{-3})} [\mathrm{K}], \quad 1 \le \gamma \le 2, \tag{25}$$

where

$$T_{C1} = 48\ \mathrm{K}, \quad \gamma = 1; \quad T_{C2} = 224\ \mathrm{K}, \quad \gamma = 2. \tag{26}$$

Recently, Southeast University of China obtained its own result datasets of

$$T_{C'} = 110\ \mathrm{K}, \quad R = 0. \tag{27}$$

While zero-resistance is observed, one could substitute the suitable $\gamma \approx 1.35 \sim 1.37$ to obtain the results as claimed. The most important point is that critical temperatures occur within the entire range of γ and can vary with the particular components of the LK-99-like alloy. The logic is consistent, based on mathematics, but the difference between Eq. (26) and the upcoming Eq. (28) lies in the central limit theorem in statistics: the closer to the center, the more accurately it reflects the actual situation in scientific terms (i.e., normal distributions). Therefore, we can use averages to explain it. Namely

$$\frac{|T_{C1} - T_{C2}|}{2} \leq T_{C'} \leq \frac{T_{C1} + T_{C2}}{2}. \tag{28}$$

Actually

$$\frac{|48 - 224|\,\mathrm{K}}{2} \leq T_{C'} \leq \frac{(48 + 224)}{2}, \quad 88\,\mathrm{K} \leq T_{C'} \leq 136\,\mathrm{K}. \tag{29}$$

So, we can conclude that the data obtained by Southeast University of China for $T_C = 110\,\mathrm{K}$ is actually located within the temperature regions predicted by Eq. (29), which is a significant discovery. These temperature regions $(92\,\mathrm{K} \sim 138\,\mathrm{K})$ of Copper Oxides in superconductivity have long been known, but the underlying reason was unclear. However, our work in this paper provides an explanation. In the realm of quantum mechanics, the suggested interpretation is that the probability density, indicated by the temperature discrepancy, depends on doped Cu and the replacement of Pb clearly, this is a quantum mechanical effect. Detailed explanations can be found in the literature or from other experimental results (2023). In relation to the Kondo Effect [15], if various atoms are doped:

$$(10 - x,\ x), \quad 0.9 < x < 1.1 \tag{30}$$

which is in a crystalline, confined in the small regions of temperature. The difficulty in achieving room-temperature superconductivity $(\text{e.g.}, > 272\,\mathrm{K})$ may be attributed to the Kondo Effect, which presents a barrier gap of $1.15\,\mathrm{K}.$ In this context, reduced resistance is accompanied by an increase in temperature due to magnetic impurities (see Figures 2 and 3). As a result, LK-99 transforms into LK-99-like, and the LK-99-like material undeniably exhibits superconductivity.[4]

[4]Note that the equations in this section do not provide a standard framework for supporting LK-99 directly. The equations presented are meant to insert the analogy of "temperatures versus magnetizations" in the context of LK-99-like materials.

D. Susceptibilities and the nature of relativities

We calculate the reversal solutions for temperature T_C using the susceptibility $\chi_{ij} \equiv -1$ as shown below (individually at $48\,\text{K}$ and $224\,\text{K}$). LK-99-like materials exhibit superconductivity (i.e., the investigated $\chi_{ij} \equiv -1$):

(I) In case of $T_C = 48\,\text{K}$. Here, we use the reversal method based on Eq. (21):

$$\Delta\chi_{ij} = \frac{-0.88}{\gamma \cdot T_C}\left(1 - \frac{T}{T_C}\right)^{-1},$$

$$\Delta\chi_{ij} = \frac{-0.88}{(1)\cdot(48)}\left(1 - \frac{T}{48}\right)^{-1} \equiv -1,$$

$$T = 48 - 0.88 = 47.12\,(\text{K}),$$

$$T < T_C. \tag{31}$$

(II) In case of $T_C = 224\,\text{K}$. Again, we use the reversal method by Eq. (21)

$$\Delta\chi_{ij} = \frac{-0.88}{\gamma \cdot T_C}\left(1 - \frac{T}{T_C}\right)^{-1},$$

$$\Delta\chi_{ij} = \frac{-0.88}{(2)\cdot(224)}\left(1 - \frac{T}{224}\right)^{-1} \equiv -1,$$

$$T \approx 224(1 - 0.002) \approx 223.55\,[\text{K}],$$

$$T < T_C. \tag{32}$$

Namely, LK-99-like performs superconductivity in the condition of $T < T_C$. Based on this, we conclude that LK-99-like is a high-temperature superconductor, not a room-temperature (RT) one. The indicated temperatures $T < T_C$ are significant because they reveal that the magnetic field $H_{C1} < H < H_{C2}$ (the Meissner effect, associated with Type II superconductors) occurs in LK-99-like materials. This means that superconductivity (zero resistance) would be observed at high-temperature regions. Therefore, we can state that LK-99-like exhibits the Meissner effect at the specified temperatures $88\,\mathrm{K} \leq T_{C'} \leq 136\,\mathrm{K}$ (as in Eq. (29)) and not at RT $(\sim 300\,\mathrm{K})$. However, we must emphasize that this conclusion is relative, not absolute.[5]

Conclusion

The superconductors of LK-99-like materials could be validated through the examinations presented in this paper, and we have introduced new concepts for extending the BCS theory to high temperatures. This generalization to Type-II superconductors has been well executed, with the electron energy levels expressed in an integral form for high temperatures and in a discrete series form for low temperatures. Additionally, replacing Pb with Cu to modify the magnetic susceptibilities of $1 \leq \gamma \leq 2$ LK-99-like materials has been identified as a viable option, as detailed in Eq. (14). We conclude that replacing Pb/Cu atoms in suitable proportions through chemical engineering or experimental methods can achieve a maximum temperature of $272\,\mathrm{K}$ for LK-99-like materials. Furthermore, we have introduced the idea of quantum temperatures at low temperatures (see Appendix C).

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[5]Define $(1 - T/T_C)^{-1}$ as a relative factor. This definition is based on the mathematical properties of curves that exhibit suppression. Additionally, the Kondo Effect can contribute to this relativity. This research, aimed at deriving the critical temperature of a super conductor, is conducted under the condition of standard atmospheric pressure, $P = 1\,\mathrm{atm}$.

Acknowledgement

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Appendix

A. Proportions of Pb/Cu

The graph of $\log(10-x)$ demonstrates that as the temperature increases, the resistance decreases (see Figure S1). In the graph, the values in the x -axis column where 9 can be adjusted to 0.9 and 10 can be adjusted to 1 simultaneously. The initial concentrations of Pb/Cu, calculated through these methods, have been proven feasible. The main focus here is to determine whether the trend aligns with the Kondo effect. If it is established that high-temperature superconductivity (e.g., around $272\,\mathrm{K}$) is achievable by adjusting the proportions of doped Pb/Cu, one could apply an optimal method for determining the best Pb/Cu ratios. If the concentration of Pb is increased in Eq. (29), the new expression $2\mathrm{Pb}_{10\text{-}x}\mathrm{Cu}_{2x}$ will be broadened, where $1.8 < 2x < 2.2$ corresponds to lower resistance. In this case, Au could be used to modify the material accurately, while $2\mathrm{Pb}_{10-x}\mathrm{Cu}_x\mathrm{Au}_x$ [6] and Pb remain at double the initial concentrations. As indicated, $88\,\mathrm{K} \le T_C \le 136\,\mathrm{K}$ in Eq. (29) naturally becomes $176\,\mathrm{K} \le T_C \le 272\,\mathrm{K}$, and we can obtain a definite temperature around $272\,\mathrm{K}$ (close to room temperature). By adjusting the proportions of Pb/Cu, as suggested in this paper, one can fine-tune the

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[6]If doped- Au (as indicated in this paper) is introduced into a superstructure of a film-layer of atoms, $\mathrm{Pb_2Au}$ exhibits the proximity effect. This effect undoubtedly supports the doubling of the initial concentrations of Pb. The work on Plumbene- Au done by Ref. [16] fully supports the ideas presented in this paper. [It is important to note that while our work is independent of theirs, it supports the individual study.]

superconducting properties of the material.

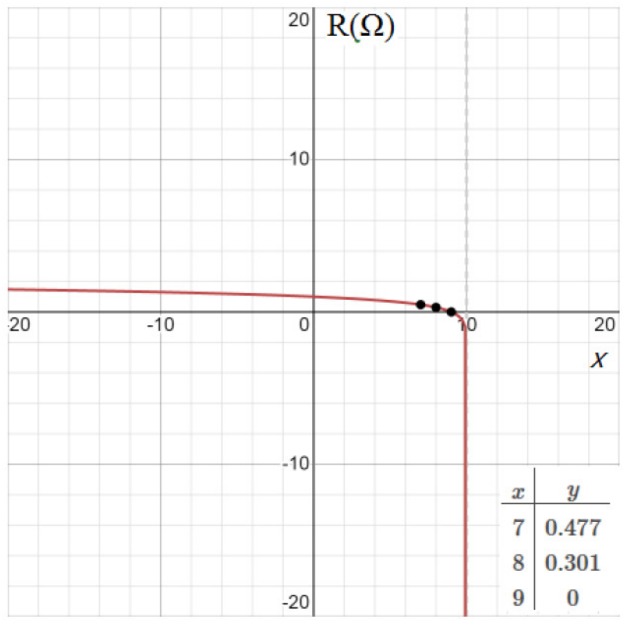

Figure S1. $Pb_{10-x}Cu_x$, $0.9 < x < 1.1$ where R represents the resistance.

The notation denoted by $x \approx 0.9$, $R \to 0$, as defined by Lee/Kim, is already widely known. The x-axis represents proportions, while the y-axis represents resistance. This plot aligns with the behavior predicted by the Kondo effect.

B. Exchanges of electron wave-functions in orbitals s and d

Based on quantum mechanics, the tunneling effect allows electrons in the s-orbital and d-orbital to pass through, as the d-orbital possesses second excited states (see wave-functions alternating between s-wave and d-wave). The resistance is reduced through the mechanism of exchange between s- and d-electrons, producing electron channels with zero resistance. This phenomenon, known as the “orbital Kondo Effect,” has been previously verified in a study by Dr. Stefan Kirchner, Dr.

Johann Kroha, Dr. Farzaneh Zamani, and Professor Lin J. J. at NCTU, Taiwan (2020).

C. Is the quantum temperature possible?

Using Eq. (30) and the condition of $Pb_{10-x}Cu_x$, $0.9 < x < 1.1$ provided, we can proceed with the following deductions. First, it is important to consider the results from Appendix A. The concentration limit of Pb doping is clearly twice the initial concentration, which is derived at the critical temperature of 272 K. Based on this, the challenge in realizing room-temperature superconductors (RT superconductors) becomes evident. It is primarily due to the barrier-temperature mechanism at 272 K, with a barrier width of 1.15 K, which originates from the Landau Level ($1.15\text{ K} << \hbar\omega_c/k$). This temperature barrier serves as a significant constraint, making RT superconductors extremely difficult to achieve. As one can observe, achieving room-temperature superconductivity (RT-superconductor) seems impossible, while the concept of quantum temperature remains plausible. For a fully doped 2Pb in a crystalline alloy $2Pb_{10-x}Cu_{2x}$, $T_C = 272\text{ K}$, the temperature cannot exceed the limit of 273.15 K, regardless of increasing the numerical Pb concentration. This suggests a hard limit. One possible solution involves replacing Cu with Au atoms, which remarkably follows the large n expansions theory by t'Hooft, within the temperature range $2Pb_{10-x}Cu_{2x}$, $T_C = 272\text{ K}$. This modification is convincing due to the impurity of doped Cu -specifically the presence of CuS_2 -that leads to diamagnetism, preventing the expected superconductivity, as is well-known. Therefore, Eq. (30) becomes:

$$n \log(10 - x) \approx n \log 10 - \log x^n, \tag{C.1}$$

where $\log x^n$ is indeed a finite constant. Rearranging Eq. (C.1), we discover that:

$$n \log(10 - x) \approx \log 10^{n} - n' \log x. \tag{C.2}$$

For, n, x are continuous and $x << n$ where n' is unknown.

In the alloy crystalline, $x{\cdot}\mathrm{Cu}$ represents a motional point around 1, and n' is contributed by neighboring atoms, but not by Cu therefore $n' = 1$ (nearby $x{\cdot}\mathrm{Cu}$). Taking into consideration the 3 K cosmic background radiation (CBM) and the last scattering surface for all alloy materials, we derive:

$$3\,\frac{\log 10^{n} - \log x}{e}, \quad 0.9 < x < 1.1,$$

$$\lim_{x \to 0.9} 3\,\frac{\log 10^{n} - \log x}{e} = 3\,\frac{\log 10^{n} - \log 0.9}{e}, \tag{C.3}$$

where the number “3” denotes the degrees of freedoms of electromagnetic (EM) waves, which can be manifested in objects exposed to the 3 K CBM on both the right-hand side (RHS) and left-hand side (LHS) of the equation, undoubtedly. Under this recognition, it is evident that the temperature barrier is determined by the ground-state barrier, as expressed in the following form:

$$3\mathrm{K}\,\frac{\log 10^{n} - \log 0.9}{e} = 3\,\mathrm{K}\,\frac{1 - \log 0.9}{e} \approx 1.15\,\mathrm{K}, \qquad n = n' = 1. \tag{C.4}$$

Note that in the case of $n = n' = 1$, the barrier is quantized. The variable n must be an integer due to the nature of the 3 K CBM, as the temperature itself changes slowly and does not correspond to any fractional values of n. Below is a useful table listing several superconducting materials, alongside their quantum temperatures. The majority of the calculations were performed by the first author, Hung-Te Henry Su, while the second author, Po-Han Lee, was responsible for supervision. This temperature barrier is extremely difficult to overcome since all materials are exposed to the cosmic $3\,\mathrm{K}$ CBM.

Table S1. Some "quantum temperatures" have been discovered, derived using the useful formula of $3\,\mathrm{K}(\log 10^n - \log 0.9)/e$. It is assumed that the number n is quantized under specific conditions

Representative Materials	Temperature ($T \neq 3\,\mathrm{K}$ CBM)	Quantum Numbers of Temperatures	Space-time Ripple (Bubbles)
Vacuum	0.00206 K (Casimir temperature) [17]	$n \equiv 0$	Scalar Fields φ_i
Ripples of Graphenes	1.15 K (RT gap)	$n = 1 + \varepsilon^i = 1$	$\varepsilon^i = 0$
^{4}He II	2.17 K	$n = 1.92 + \varepsilon^i = 2$	$\varepsilon^i = 0.08$
^{3}He	3.35 K	$n = 2.99 + \varepsilon^i = 3$	$\varepsilon^i = 0.01$
^{4}He I	4.20 K (The boliling point)	$n = 3.76 + \varepsilon^i = 4$	$\varepsilon^i = 0.24$
^{4}He I	5.20 K (T_C)	$n = 4.67 + \varepsilon^i = 5$	$\varepsilon^i = 0.33$
LN_2	77 K (Barrier)	$n = 69.81 + \varepsilon^i = 70$	$\varepsilon^i = 0.19$
BSCCO	110 K	$n = 99.7 + \varepsilon^i = 100$	$\varepsilon^i = 0.30$
Atmosphere	300 K	$n = 272 + \varepsilon^i = 272$	$\varepsilon^i = 0$

Additionally, we must consider that space-time ripples-caused by the last scattering surface-are permitted, as evidenced by the well-known irregular fine-line formations. The initial materials were once located on this surface, and their properties, influenced by these ripples, contribute to the quantization of temperature in superconducting systems. Obviously, the $n = 1, 2, 3, ..., 70, ...$ temperatures (not the exact values but the quantum number n) are quantized. In this context, the relationship between T and n fits a Poisson distribution, as $P(X = n) = \frac{e^{-T/1K}(T/1K)^n}{n!}$ can be verified through calculation. This means that the occurrence of certain temperature states follows a probabilistic pattern, where the quantum number n behaves similarly to discrete events in a Poisson process.

Thus far, for a convincing presentation, the Planck temperature can be replaced by our concept of quantum temperatures at several distinct temperature points. Notably, the wave-function of barriers $(n = 1, 2, 3, ...)$ is expressed as a *sinc* function, calculated accordingly. Additionally, the width of the temperature barrier of Pb (111) is mentioned in Ref. [16], where $T_C = 6.9\,\mathrm{K}$ is indicated. This value evidently possesses a rational connection to the $1.15\,\mathrm{K}$ barrier discussed in this paper. Specifically, the width is six times greater than the value promoted in this study. The BCS theory $T_C = 1.14 T_D e^{\frac{1}{-VN(E_F)}}$ and the significance of “six” suggest a potential correlation with the hexagonal structure of superconductivity, reinforcing the feasibility of this structure for superconductive properties.

†To acquire access to the data stated in this paper, please send a request to the first author.

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