Fundamental Journal of Mathematics and Mathematical Sciences
Vol. 4, Issue 1, 2015, Pages 69-82
This paper is available online at http://www.frdint.com/
Published online December 1, 2015

SYNCHRONIZATION OF THE AGE-DEPENDENT STOCHASTIC LOTKA-VOLTERRA SYSTEM

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Abstract

In this paper, we formulate and investigate the stability of the error system which guarantees the synchronization for a class of the stochastic Lotka-Volterra system. Different from the existing models, an observer for the considered Lotka-Volterra system in this paper is modeled with age-structured and without the restriction of local Lipschitz condition. Some synchronization criteria are derived and these criteria are convenient to be

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Keywords and phrases: synchronization control, Lyapunov functions, Brownian motion, Itô's formula.

2010 Mathematics Subject Classification: 92B25.

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Received October 3, 2015; Accepted November 3, 2015

used for concision. Finally, a numerical example is provided to illustrate the effectiveness of the method proposed in this paper.

1. Introduction

The nonlinear age-dependent Lotka-Volterra population dynamic can be written in the following form [1-3]:

$$\begin{cases} \dfrac{\partial p_i}{\partial t} + \dfrac{\partial p_i}{\partial a} = f(t, a, p_i) - \mu(a, t)p_i + \displaystyle\sum_{j=1}^{n} \lambda_{ij} p_j(t) p_i, & \text{in} \quad Q = (0, A) \times (0, T), \\ p_i(0, t) = \beta_i(t) \displaystyle\int_{a_1}^{a_2} m_i(a, t) p_i(a, t) da, & \text{in} \quad [0, T], \\ p_i(a, 0) = p_{i0}(a), & \text{in} \quad [0, A], \end{cases} \tag{1.1}$$

where $p_i = p_i(a, t)$ is the density of ith population of age a at time t; $p_j(t) = \int_0^A p_j(a, t)da$; $\mu(a, t)$ denotes the average mortality of population; $\beta_i(t)$ denotes the average fertility of ith population; $m_i(a, t)$ is the ratio of females in ith population; $p_{i0}(a)$ is the initial age distribution of ith population; A is the life expectancy, $0 < A < +\infty$. Here, without loss of generality, we assume that the n populations have the same life expectancy. $f(t, a, p_i)$ denotes influence of external environment for population system, such as emigration and earthquake and so on. The effects of external environment has the deterministic and random parts which depend on t, a and p_i.

In the past decades, much progress on system (1.1) has been made. For example, authors in [1] proved the existence and uniqueness of positive solutions for a kind of Lotka-Volterra system by using sub-supersolution method. In view of [2, 3], we know that the system (1.1) has a unique nonnegative solution.

In the present investigation, the random behavior of the birth-death process is carefully incorporated into the continuous-time age-structured population equations to obtain a system of stochastic differential equations that model age-structured population dynamics. This age-structured population model is of theoretical interest.

However, an application of the stochastic age-structured model is to study how age-structured influences estimated persistence time of a population where extinction is influenced by random fluctuations in the birth-death process. We also note that ecosystems in the real world are continuously distributes by unpredictable forces which can result in changes in the biological parameters such as survival rates. In ecology, we know that the practical question of interest is just whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of time. Therefore it is worth pointing out that parameter uncertainties, noises are ubiquitous in both nature and man-made systems, and the stochastic effects on population systems have drawn particular attention. Hence, the effects of the stochastic environmental noise considerations lead to stochastic age-structures population systems, which are more realistic. On the other hand, synchronization is a most identical phenomenon that can always be found among the population system, what’s more, studying the synchronicity of population plays an important role in the protection of endangered species, pest control and eliminating infectious diseases. Recently, synchronization is one of the most important and interesting problems in the analysis of stochastic age-structured population equations. To the best of our knowledge, synchronization has received little attention in the field of mathematical biology just about dynamical networks (see [4-11]). Therefore, the aim of this paper is to close this gap. In this article, we shall investigate the stochastic multi-species age-dependent Lotka-Volterra system, that is

$$\begin{cases} \dfrac{\partial p_i}{\partial t} + \dfrac{\partial p_i}{\partial a} = f(t, a, p_i) - \mu(a, t)p_i + \displaystyle\sum_{j=1}^{n} \lambda_{ij} p_j(t) p_i & \\ \qquad\qquad + \varphi_i(t)\dot{\omega}_i(t), & \text{in } Q = (0, A) \times (0, T), \\ p_i(0, t) = \beta_i(t) \displaystyle\int_{a_1}^{a_2} m_i(a, t) p_i(a, t) da, & \text{in } [0, T], \\ p_i(a, 0) = p_{i0}(a), & \text{in } [0, A]. \end{cases} \tag{1.2}$$

This paper can be organized as follows. In Section 2, we introduce the model of Lotka-Volterra model with feedback controls and some definitions and lemmas to be used later are presented. In Section 3, the synchronization criteria are derived. In Section 4, an illustrative example is provided to show the result and give the map to illustrate.

2. Model Formulation and Preliminaries

The system (1.2) can be described by the Itô equation

$$\begin{cases} d_t p_i = \left[-\dfrac{\partial p_i}{\partial a} - \mu(a, t) p_i + \displaystyle\sum_{j=1}^{n} \lambda_{ij} p_j(t) p_i + f(t, a, p_i) \right] dt \\ \qquad + \varphi_i(t) d\omega_i(t), & \text{in } Q = (0, A) \times (0, T), \\ p_i(0, t) = \beta_i(t) \displaystyle\int_{a_1}^{a_2} m_i(a, t) p_i(a, t) da, & \text{in } [0, T], \\ p_i(a, 0) = p_{i0}(a), & \text{in } [0, A], \end{cases} \tag{2.1}$$

where $d_t p_i = (\partial p_i / \partial t) dt$, $\omega_i(t)$ is a Brownian motion defined on $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, P)$, $\varphi_i(t)$ denotes the corresponding noise intensities.

In this paper, we give another species as follows:

$$d_t s_i = \left[-\frac{\partial s_i}{\partial a} - \mu(a, t) s_i + \sum_{j=1}^{n} \lambda_{ij} s_j(t) s_i + f(t, a, s_i) \right] dt + \phi_i(t) d\sigma_i(t). \tag{2.2}$$

Denoting $e_i(t, a) = p_i(t, a) - s_i(t, a)$, then

$$\begin{aligned} d_t e_i = \Bigg[& -\frac{\partial e_i}{\partial a} + f(t, a, p_i) - f(t, a, s_i) - \mu(a, t) e_i \\ & + \sum_{j=1}^{n} \lambda_{ij} (p_j(t) p_i - s_j(t) s_i) \Bigg] dt + \varphi_i(t) dw_i(t) - \phi_i(t) d\sigma_i(t), \end{aligned} \tag{2.3}$$

for $i = 1, 2, \ldots, n$.

Remark 1. $p_i(t, a)$ and $s_i(t, a)$ denote the density of two species population of age a at time t. We suppose they have no interaction relations, but they have the same community.

The objective of this paper is to prove all of the species in the Lotka-Volterra systems (2.1) synchronize with target species (2.2) for all initial state, i.e., all of the states satisfy

$$\lim_{t \to \infty} \| e_i(t, a) \| = 0 \quad \forall i = 1, 2, \ldots, n.$$

In what follows, we will provide some assumptions and definitions.

Definition 1. The zero solution of error system (2.3) is said to be

(i) globally stable in probability, if for any $\varepsilon \in (0, 1)$, there exists a class $\mathcal{K}$ function r^2 such that for $\forall e_i(0, a) \in R^n$, every weak solution $e_i(t, a)$ of system (2.3) satisfies

$$\mathbb{P}^{e_i}\{\sup_{t \geq t_0} \|e_i(t, a)\| < r^2(\|e_i(0, a)\|)\} \geq 1 - \varepsilon; \quad i = 1, 2, \ldots, n$$

(ii) globally asymptotically stable in probability, if it is globally stable in probability and for $\forall e_i(0, a) \in R^n$, every weak solution $e_i(t, a)$ of error system (2.3) satisfies

$$\mathbb{P}_i^e\{\lim_{t \to \infty} e_i(t, a) = 0\} = 1, \quad i = 1, 2, \ldots, n.$$

3. Main Results

In this section, the objective is under the conditions described above. Now we are in position to establish the following main results.

Theorem 1. *For an n-dimensional stochastic differential system*

$$\frac{\partial x}{\partial t} + \frac{\partial x}{\partial a} = F(t, x)dt + G(t, x)dv(t) \tag{3.1}$$

if the Borel measurable functions $F(t, a, x)$ *and* $G(t, a, x)$ *are continuous on* $R^{\geq T_0} \times R \times R^n$, *then for any initial distribution* μ *on measurable space* $(R^n, \mathfrak{B}(R^{(n)}))$, *system* (3.1) *has a weak solution* $x(t, a)$ *with initial distribution* μ, i.e., $\mathbb{P}^x\{x(t_0, a) \in \mathfrak{B}\} = \mu(\mathfrak{B})$ *for any* $B \in \mathfrak{B}(R^n)$.

Proof. The proof is immediately follows from p. 167 of [12] and hence omitted.

Theorem 2. *For the error system* (2.3), *suppose that there exists a function* $V \in C^2(R^n, R^+)$ *such that*

(i) $\mathcal{L}V(x) \leq 0$;

(ii) *not only for* $e_i(0) = 0_0$, *but also for* $e_i(0)$ *having any other distribution*, *no* *nonzero weak solution of system* (2.3) *completely belongs to* $\{s \in R^n | \mathcal{L}V(e) = 0\}$ a.s..

Then the zero solution of system (2.4) *is globally asymptotically stable in probability*.

Proof. From Theorem 1, it follows that system (2.3) has a weak solution, denoted by $e_i(t, a)$, for any deterministic initial condition. We first prove that every weak solution of system (2.3) is defined on $[0, +\infty]$ a.s.. Otherwise, for a weak solution $e_i(t, a)$, there holds, $\mathbb{P}^{e_i}\{\sigma_\infty^{e_i} < +\infty\} > 0$, where $\sigma_\infty^{e_i} = \lim_{r \to +\infty} \sigma_r^i$, and $\sigma_r^{e_i}$ is stopping time defined as

$$\sigma_r^{e_i} = \inf\{t \geq 0 | \, \|e_i(t, a)\| \geq r\}.$$

Then there exist constants $\varepsilon > 0$ and $T^{e_i} > 0$ such that

$$\mathbb{P}^{e_i}\{\sigma_\infty^{e_i} \leq T^{e_i}\} > 2\varepsilon.$$

Furthermore, there exists a sufficiently large constant $N > 0$ such that

$$P^{e_i}\{\sigma_\infty^{e_i} \leq T^{e_i}\} \geq \varepsilon, \, \forall r \geq N. \tag{3.2}$$

By the continuity of weak solution, we have

$$E^{e_i}(\|e_i(t \wedge \sigma_r^{e_i}, a)\|^2) \geq E^{e_i}(I_{\{\sigma_r^{e_i} \leq t\}} \|e_i(\sigma_r^{e_i})\|^2 \geq \mathbb{P}^{e_i}\{\sigma_r^{e_i} \leq t\} \cdot r^2, \, \forall t \geq 0. \tag{3.3}$$

By (ii) and Itô formula, we obtain

$$E^{e_i}(\|e_i(t \wedge \sigma_r^{e_i}, a)\|^2) \leq \|e_i(0, a)\|^2, \, \forall t \geq 0. \tag{3.4}$$

Combining (3.2), (3.3) and (3.4) yields that

$$\varepsilon r^2 \leq \mathbb{P}^{e_i}\{\sigma_r^{e_i} \leq T^{e_i}\} r^2 \leq \|e_i(0, a)\|^2, \, \forall r \geq N.$$

From this, it follows that

$$+\infty = \lim_{r \to +\infty} \varepsilon r^2 \leq \|e_i(0, a)\|^2 < +\infty,$$

which is clearly a contradiction. Hence $\mathbb{P}^{e_i}\{\sigma_\infty^{e_i} < +\infty\} = 0$, and every weak solution of system (2.3) is defined on $[0, +\infty)$ a.s..

Next, we prove that the zero solution of system (2.4) is globally stable in probability. In fact, in the case of $e_i(0, a) \neq 0$, for any $\varepsilon \in (0, 1)$, $r > 0$ and every weak solution $e_i(t, a)$ of system (2.3), by (3.3) and (3.4), we obtain that, for all

$\mathbb{P}^{e_i}\{\sigma_r^{e_i} \leq t\} \leq \frac{\|e_i(0, a)\|^2}{r^2}$.

By this and letting $t \to \infty$, we derive $\mathbb{P}^{e_i}\{\sigma_r^{e_i} < +\infty\} \leq \frac{\|e_i(0, a)\|^2}{r^2}$. Thus, choosing $r^2 = \frac{\|e_i(0, a)\|^2}{\varepsilon}$ gives

$$\mathbb{P}^{e_i}\{\sup_{t\geq 0}\|e_i(t, a)\| \leq r^2\} \geq 1 - \varepsilon. \tag{3.5}$$

As for the case $e_i(0, a) = 0$, by (3.4), we have $E^{e_i}(e_i^2(t \wedge \sigma_r^{e_i})) = 0$ for all $t \geq 0$. From $V \geq 0$, it follows $E_i^2(t \wedge \sigma_r^s, a) = 0$ for all $t \geq 0$ a.s., that is, $e_i(t \wedge \sigma_r^{e_i}, a) = 0$ for all $t \geq 0$ a.s.. Letting $r \to +\infty$ gives $e_i(t, a) \equiv 0$ a.s.. This implies that, in this case, (3.5) holds for any $\varepsilon \in (0, 1)$. Hence, the zero solution of system (2.3) is globally stable in probability.

In the remainder of the proof, we prove that every weak solution $e_i(t, a)$ of the system (2.3) satisfies

$$\mathbb{P}_i^e\{\lim_{t\to\infty} e_i(t, a) = 0\} = 1, \quad i = 1, 2, \ldots, n.$$

In fact, since $V(e_i(t, a)) = \|e_i(t, a)\|^2 \geq 0$ and by (i), combine the continuity of weak solution and Lemma 2.4 in p. 163 of [13], for any $r = 1, 2, \ldots,$ $\{e_i(t \wedge \sigma_r^{e_i}, a)\}_{t\geq 0}$ is a nonnegative continuous supermartingale. That is, for any $r = 1, 2, \ldots$ and $i > s \geq 0$,

$$E(\|e_i(t \wedge \sigma_r^{e_i}, a)\|^2 | \mathcal{F}_s^{e_i}) \leq \|e_i(s \wedge \sigma_r^{e_i}, a)\|^2 \quad \text{a.s.},$$

which, together with $\lim_{r\to\infty} \sigma_r^{e_i} = +\infty$ a.s. and Fatou lemma in p. 22 of [15], implies that, for any $t > a \geq 0$,

$$
\begin{aligned}
E(\|e_i(t, a)\|^2 | \mathcal{F}_s^{e_i}) &= E(\liminf_{r\to+\infty} \|e_i(t \wedge \sigma_r^{e_i}, a)\|^2 | \mathcal{F}_s^{e_i}) \\
&\leq (\liminf_{r\to+\infty} E\|e_i(t \wedge \sigma_r^{e_i}, a)\|^2 | \mathcal{F}_s^{e_i}) \\
&\leq \liminf_{r\to+\infty} \|e_i(s \wedge \sigma_r^{e_i}, a)\|^2 \\
&= \|e_i(s\ a)\|^2 \quad \text{a.s..} \qquad (3.6)
\end{aligned}
$$

Hence, $\{\|e_i(t, a)\|^2\}$ is a nonnegative continuous supermaritingale. Furthermore, by Theorem 2.1 in p. 163 of [13], we see that $V_\infty = \lim_{t\to\infty} V(e_i(t, a))$ exists and is finite a.s., and $E^{e_i}(V_\infty) = \lim_{t\to\infty} E^{e_i} V(e_i(t, a))$ is finite and nonnegative. This, together with Corollary 4 in p. 100 of [16], implies that $\{V(e_i(t, a))\}_{t\geq 0}$ is uniformly integrable.

Now we prove $E^{e_i}(V_\infty) = 0$. For contradiction we suppose $E^{e_i}(V_\infty) > 0$. Choose an increasing time sequence $\{t_k\}_{k\in\mathcal{N}}$ satisfying $\lim_{k\to+\infty} t_k = +\infty$. Define

$$X_{i,k}(t, a) = e_i(t + t_k, a),\ \forall t \geq 0,\ \forall k \in \mathcal{N}.$$

By the continuity of weak solution, $X_k(t, a)$ is continuous for $\forall k \in \mathcal{N}$. Moreover, it is clear that $X_k(t, a)$ satisfies the following equation

$$X_k(t, a) = X_k(0, a) + \int_0^t F(X_k(s, a))ds + \int_0^t G(X_k(s, a))dw_k(s), \quad \forall t \geq 0, \qquad (3.7)$$

where $w_k(s) = w^{e_i}(s + t_k) - w^{e_i}(t_k)$ is a Brownian motion as well, and $X_k(t, a)$ and $w_k(t)$ are defined on the same probability space as that of the solution $e_i(t, a)$.

In terms of the supermartingale property, we know that

$$E^{e_i}(V(X_k(t, a))) = E^{e_i}(\|e_i(t + t_k, a)\|^2) \leq e_i(0, a),\ \forall t \geq 0,\ \forall k \in \mathcal{N}.$$

From this, together with supermartingale inequality, we obtain that, for any $T \geq 0$

and $m > 0$,

$$\sup_{k\in\mathcal{N}} \mathbb{P}_{e_i}\{\sup_{0\le t\le T}\|X_{i,k}(t, a)\| \ge m\}$$

$$\le \sup_{k\in\mathcal{N}} \mathbb{P}_{e_i}\{\sup_{0\le t\le T}\|e_i(t+t_k, a)\| \ge m^2\} \le \frac{e_i(0, a)}{m^2}. \tag{3.8}$$

Then we have

$$\lim_{m\to\infty}\sup_{k\in\mathcal{N}} \mathbb{P}^{e_i}\{\|X_k(0, a)\| \ge m\} = 0. \tag{3.9}$$

By (3.7) and (3.8), we can prove that, for any $T \ge 0$ and $\varepsilon > 0$, there holds

$$\lim_{\delta\to 0^+}\sup_{k\in\mathcal{N}} \mathbb{P}^{e_i}\{\max_{\substack{|t-s|\le\delta\\ t,s\in[0,T]}}\|X_k(t, a) - X_k(s, a)\| > \varepsilon\} = 0. \tag{3.10}$$

By (3.9), (3.10) and Theorem 4.2 in p. 17 of [12], there exists a subsequence $\{X_{k_j}\}_{j\in\mathcal{N}}$ of $\{X_k\}_{k\in\mathcal{N}}$, a probability space $(\hat{\Omega}, \hat{\mathcal{F}}, \hat{P})$ and n-dimensional continuous processes $\hat{e}_i(\hat{e}_{ik_j})$, $j \in \mathcal{N}$ defined on this probability space, such that $\hat{e}_{i,k_j}$ converges to $\hat{e}_i$ as $j \to +\infty$ a.s., and $\hat{e}_{i,k_j}$ and X_{k_j} are equivalent in distribution, written as

$$\hat{e}_{i,k_j} \overset{\mathcal{D}}{=} X_{k_j}, \ \forall j \in \mathcal{N}. \tag{3.11}$$

From this and (3.7), and similar to the proof of Theorem 2.2 in p. 169 of [8], we derive that, for every $h \in C_b^2(R^n, R)$ and every $l \in \mathcal{N}$, $M(t) := h(\hat{e}_i(t\wedge\hat{\sigma}_l) - h(\hat{e}_i(0, a)) - \int_0^{t\wedge\hat{\sigma}_l}\mathcal{L}h(\hat{e}_i(s, a))ds$ is a martingale, where $\hat{\sigma}_l = \inf\{t \ge 0 \mid \|\hat{e}_i(t, a)\| \ge l\}$. Hence, by Proposition 2.1 on p. 169 of [12], $\hat{e}_i(t, a)$ is a weak solution of system (2.3) with $\hat{e}_i(0, a)$ having certain distribution.

Since $X_{k_j}(0, a) = e_i(t_{k_j}, a)$, $j \in \mathcal{N}$, and $\{V(e_i(t, a))\}_{t\ge 0}$ is uniformly integrable, we have that $V(X_{k_j}(0, a))_{j\in\mathcal{N}}$ is uniformly integrable. By (13), we get that $\{V(\hat{e}_{i,k_j}(0, a))\}_j \in \mathcal{N}$ is uniformly integrable as well. By Theorem 1.3 in p. 16 of [16], we derive

$$E^{e_i}(V_\infty) = E^{e_i}(\lim_{j\to+\infty} V(X_{k_j}(0, a))) = \lim_{j\to+\infty} E^{e_i}(V(X_{k_j}(0, a))),$$

$$\hat{E}(V(\hat{e}_i(0, a))) = \hat{E}(\lim_{j\to+\infty} V(\hat{e}_{i,k_j}(0, a))) = \lim_{j\to+\infty} \hat{E}(V(\hat{e}_{i,k_j}(0, a))),$$

which, together with (3.11), implies that $\hat{E}(V(\hat{e}_i(0, a))) = E^{e_i}(V_\infty) > 0.$ So $\hat{e}_i(t, a)$ is a nonzero weak solution.

For any $t \geq 0,$ since $X_{k_j}(t, a) = e_i(t + t_{k_j}, a),\ j \in \mathcal{N},$ and $V(e_i(t, a))_{t\geq 0}$ is uniformly integrable, we derive that $\{V(X_{k_j}(t, a))\}_{j\in\mathcal{N}}$ is uniformly integrable. By (13), we get that $\{V(\hat{e}_{i,k_j}(t, a))\}_{j\in\mathcal{N}}$ is uniformly integrable as well. By Theorem 1.3 in p. 16 of [14] again, we have

$$E^{e_i}(V_\infty) = E^{e_i}(\lim_{j\to+\infty} V(X_{k_j}(t, a))) = \lim_{j\to+\infty} E^{e_i}(V(X_{k_j}(t, a))),$$

$$\hat{E}(V(\hat{e}_i(t, a))) = \hat{E}(\lim_{j\to+\infty} V(\hat{e}_{i,k_j}(t, a))) = \lim_{j\to+\infty} \hat{E}(V(\hat{e}_{i,k_j}(t, a))),$$

which, together with (3.11), implies that $\hat{E}(v(\hat{e}_i(t, a))) = E^{e_i}(V_\infty)$ for all $t \geq 0.$ This means that $\mathcal{L}V(\hat{e}_i(t, a)) \equiv 0$ a.s., which contradicts (iii). So we see $E^{e_i}(V_\infty) = 0,$ we have $V_\infty = 0$ a.s., that is,

$$\lim_{t\to+\infty} V(e_i(t, a)) = 0 \quad \text{a.s.},$$

which, contradicts (ii). So we see $E^{e_i}(V_\infty) = 0.$

Since $V_\infty \geq 0$ a.s. and $E^{e_i}(V_\infty) = 0,$ we have $V_\infty = 0$ a.s., that is,

$$\lim_{t\to+\infty} V(e_i(t, a)) = 0 \quad \text{a.s.},$$

which, together with (i), implies that (3.6) holds. This completes the proof.

Remark 2. Form Theorem 1, we can see all the species can synchronize each other if only the nonlinear function $f(x(t), t)$ satisfied Lipschitz condition. Different from the results in [7-9], we only need to verify the inequality (3.1) according to the parameters of the system (2.1) and not to solve some complex linear or nonlinear matirx inequalities depended on some unknown variables. Which also shows that synchronization is an essential phenomenon of the population system and only depends on its parameters.

4. A Numerical Example

Let us consider a stochastic age-structures population equation of the form

$$\begin{cases} p_i(0, t) = \int_0^1 \left(\frac{1}{1-a}\right)^2 p_i(a, t)da, & \text{in } (0, 1) \\ p_i(a, 0) = e^{-\frac{1}{1-a}}, & \text{in } (0, 1) \end{cases} \tag{4.1}$$

In (2.1) and (2.2), we give the parameters as follows: $f(t, x(t)) = \frac{x(t)}{1+t^2}$, $\mu(t, a) = \frac{1}{(1-a)^2}$, $\varphi_i(t) = \frac{\ln(t+1)}{t+1}$, $\phi(t) = \frac{\sqrt{t}}{1+t^2}$ for $i = 1, 2, 3,$ respectively. We set

$$\lambda = \begin{bmatrix} -0.5 & 0 & -0.3 \\ 0 & -0.2 & -0.1 \\ -0.3 & -0.1 & -0.8 \end{bmatrix}.$$

We give the pictures above with fixed step sizes $\Delta t = 0.005$, $\Delta a = 0.05$. The maximum value of the error is not greater than 0.005.

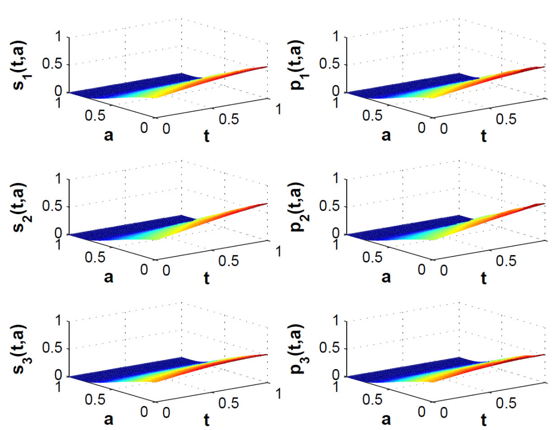

Figure 1. Transient response of state variables $(s_1, s_2, s_3, p_1, p_2, p_3)$.

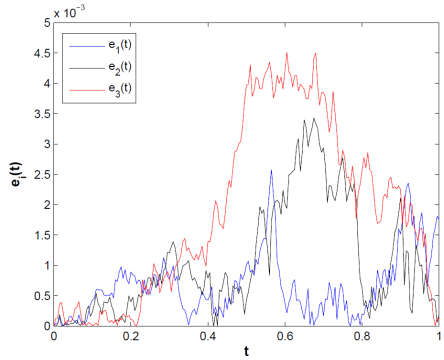

Figure 2. Synchronization error $e = (e_1, e_2, e_3)$.

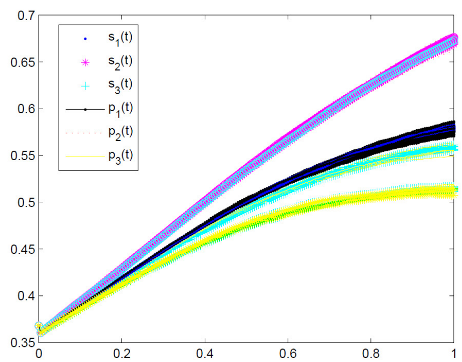

Figure 3. Transient response of state variables $s_1(t), s_2(t), s_3(t), p_1(t), p_2(t), p_3(t)$ at $a = 1$.

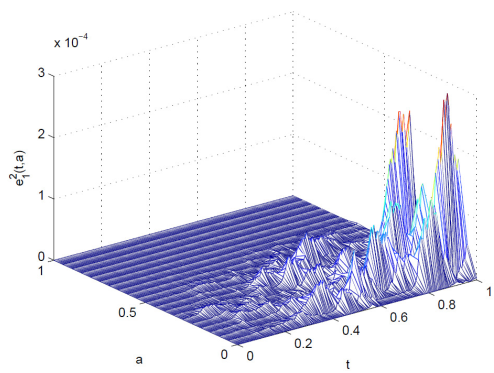

Figure 4. Synchronization error e_1^2 between the target state s_1 and the state p_1.

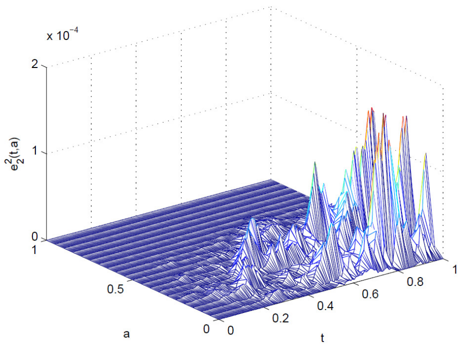

Figure 5. Synchronization error e_2^2 between the target state s_2 and the state p_2.

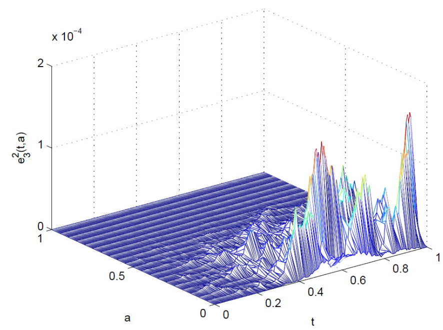

Figure 6. Synchronization error e_3^2 between the target state s_3 and the state p_3.

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