

SYNCHRONISATION OF MOVING CLOCKS AND INTERNAL DETECTION OF UNIFORM TRANSLATIONAL MOTION

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Abstract

Assuming the existence of an inertial frame, S , in which light propagates isotropically with a uniform speed, it is shown how measurements of time intervals between the epochs of transmission and reception of light signals by a single uniformly moving clock can be used to measure the velocity of the clock in S . Methods to synchronise two or more such moving clocks both with and without the observation of light signals are described. It is also shown that the Galilean definition of velocity and relativistic time dilation are incompatible with the Einstein postulate, and the prediction of Maxwell's electromagnetic theory of light, that the speed of light is the same in all inertial frames. Flaws in Einstein's arguments claiming consistency of predictions of the space-time Lorentz transformations with light speed frame independence are pointed out. The 'Conventionality of Clock Synchronisation' concept and Poincaré's related assertion of the impossibility of internal detection of

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uniform translational motion are shown to be untenable. A consistent description of all known optical phenomena is given by identifying light with massless particles obeying the laws of relativistic kinematics - the 'light quanta', now called photons, discovered by Einstein in 1905.

1. Introduction

The space-time Lorentz Transformations (LT) were originally obtained by Lorentz [1], Larmor [2, 3] and Poincaré [4] as those that left the form of Maxwell's electromagnetic field equations invariant when transformed from a frame in which a putative 'luminiferous aether' was at rest, into any frame (an 'inertial' frame) moving uniformly with respect to the aether frame. The same demonstration is found in Einstein's original special relativity paper [5] except that the aether frame is replaced by a frame where, by hypothesis, light propagates with uniform speed c , independent of the velocity of its source - Einstein's 'stationary' frame.

Since Maxwell's equations in free space lead to the prediction of 'electromagnetic waves' moving at a definite speed c , it is concluded that, on identifying light with these electromagnetic waves, that the speed of light must be the same in the aether frame, in Einstein's stationary frame and in an arbitrary inertial frame. This prediction from classical electromagnetic theory of the constancy of the speed of light was promoted by Einstein to a postulate (the second postulate of the special theory of relativity) and employed in Ref. [5] to derive, from first principles, the LT.

Perhaps surprisingly it will be demonstrated in the present paper that such frame-independence of the speed of light is incompatible with constraints of space-time geometry in Einstein's stationary frame and the existence of the experimentally-verified time dilation (TD) effect for a moving clock, as derived by Einstein from the LT, in Ref. [5].

In Ref. [5] a method was proposed to synchronise two spatially-

separated clocks A and B, by exchange of light signals between them. If the clocks are at rest in a frame in which the speed of light is isotropic, one method to carry out the procedure suggested by Einstein is as follows. Initially both clocks are stopped and their epochs¹ set to zero. When the light signal is transmitted from A, this clock is started. When the signal arrives at B, where it is promptly reflected back towards A, clock B is started. The signal arrives back at A at epoch t_A . Since the clock B will be slow relative to A by the time interval for the light to pass from A to B (or from B to A), which is $t_A/2$, clock B is synchronized with A by adding $t_A/2$ to its epoch. If it is indeed true that the speed of light is the same in all inertial frames, (as suggested by the application of the LT to Maxwell's Equations) this procedure would be a valid one in all inertial frames - and not subject to any kind of ambiguity. On the other hand if the to-be-synchronised clocks were at rest in a frame in which light speed is anisotropic (Einstein himself considers just such a case in the section of Ref. [5] following the one in which his light signal synchronisation procedure is described; this will be discussed in Section 8 below) then allowance would have to be made for the anisotropy in order to use exchange of light signals to synchronise the clocks. Just this point was made by Poincaré [6]:

But this method of operation (Einstein's procedure) assumes that light takes the same time to travel from A to B and to return from B to A. This is true if the observers are motionless, but no longer true if they are involved in a common transposition, because in this case A, for instance will be meeting the light that comes from B while B is retreating from the light that comes from A.

In order to correctly synchronise the clocks therefore, proper account

¹An 'epoch', t , is the number registered by a clock at any instant. Time intervals are defined as the difference between two epochs: $\Delta t \equiv t_2 - t_1$, $t_2 > t_1$.

must be taken of the light speed anisotropy. Just this is what will be done in the calculations presented below in the present paper. Instead of adding $t_A/2$ to the epoch of clock B it will be necessary to add ϵt_A where $0 < \epsilon < 1$ and the value of ϵ is calculated from the known light speed anisotropy in the proper frame of the clocks A and B. In spite of this quite evident generalisation of his light signal exchange synchronisation procedure Einstein makes in Ref. [5] the following assertion *before* describing the procedure:

*We have not defined a “common time” for A and B, for the latter cannot be defined at all unless we establish **by definition** that the “time” required by light to travel from A to B equals the “time” it requires to travel from B to A. (Einstein’s emphasis)*

Just the contrary of this assertion will be demonstrated by the calculations presented below in the present paper.

Following Reichenbach [7], many authors, particularly of philosophical literature, have argued that the parameter ϵ introduced above to correct for the effect of light speed anisotropy has no physical significance, being of a purely ‘conventional’ nature. This claim will be examined in Section 8 below in the light of the calculations presented in the previous sections.

Another issue addressed in the present paper is Poincaré’s formulation of the special relativity principle as a statement (a generalisation of ‘Galileo’s ship’ [8]) of the impossibility by means of any ‘internal’ measurements whatever to detect uniform translational motion [9]:

The principle of relativity according to which the laws of physical phenomena should be the same whether for an observer fixed, or for an observer carried along in a uniform movement of translation; so that we have not and could not have any means of discovering whether or not we

are carried along in such motion.

Counter examples to this statement of the special relativity principle are given by the calculations presented in Sections 3, 4 and 6 below.

This paper is organised as follows: In the next section, time intervals for exchange of light signals between two clocks at rest in an arbitrary inertial frame are derived. The calculation is based on three postulates that are also given in Ref. [5]. In the following Sections 3 and 4, the results obtained in Section 2 are used, firstly, to determine the relative motion of the inertial frames S and S' , given isotropy of light speed in the frame S , and, secondly, to synchronise two spatially-separated clocks at rest in the frame S' . This is done at order v/c in Section 3 and exactly (to all orders in v/c) in Section 4. In Section 5 various methods of synchronising, without the use of light signals, spatially-separated clocks at rest in an arbitrary inertial frame are described. Section 6 describes a method to detect the motion of an inertial frame by observation of time intervals recorded by clocks moving in a known manner relative to the frame, as well as another method to synchronise clocks at rest in the frame. In Section 7, the general problem of the existence of preferred frames for particular physical phenomena such as particle propagation or observations of clocks, is considered, with particular reference to experiments carried out in the vicinity of the Earth. In Section 8, the discrepancies between the results obtained in the present paper and standard special relativity theory are discussed. Particular emphasis is placed on comparison with Ref. [5] where identical initial postulates to those employed in the present paper lead to very different predictions. Section 9 gives conclusions. The Appendix describes a method to measure the parameters D and θ specifying the separation and orientation, in the frame S , of the to-be-synchronised clocks in uniform motion in this frame.

2. Exchange of Light Signals between Moving Clocks

The calculations of the present section are based on three postulates:

(i) The existence of an inertial frame S in which light propagates isotropically in free space with speed c .

(ii) The Galilean definition of uniform velocity in the frame S:

$$\text{velocity} \equiv \frac{\text{displacement of object}}{\text{elapsed time}} \equiv \frac{s}{\Delta t}.$$

The time interval Δt is recorded by a clock at rest in S registering an epoch t .

(iii) The validity of the interval Lorentz transformations:

$$\Delta x' = \gamma(\Delta x - v\Delta t) = 0, \quad (2.1)$$

$$\Delta y' = \Delta y = 0, \quad (2.2)$$

$$\Delta z' = \Delta z = 0, \quad (2.3)$$

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right), \quad (2.4)$$

where $\gamma \equiv 1/\sqrt{1 - (v/c)^2}$, relating the time interval Δt to that $\Delta t'$ recorded by a clock at rest in the frame S' that moves with speed v along the common x, x' coordinate axis of the frames. Intervals along the world line of any clock at rest in S' respect the conditions $\Delta x' = \Delta y' = \Delta z' = 0$ in S' and $\Delta x = v\Delta t, \Delta y = \Delta z = 0$ in S. Postulates (i) and (ii) were explicitly given in Einstein's seminal special relativity paper [5] where the space-time LT (2.1)-(2.4) were also derived.

As shown in Figure 1, two clocks, C'_0 and C'_1 at rest in the frame S', are separated by a distance D in the frame S and the line segment $C'_0 C'_1$

is at an angle θ relative to the x -axis in the same frame. When the clock C'_0 , placed at the origin of coordinates in the frame S' , is aligned with the origin O of the frame S , a light signal is transmitted from C'_0 and arrives at C'_1 after the time interval Δt_+ in the frame S . The path length

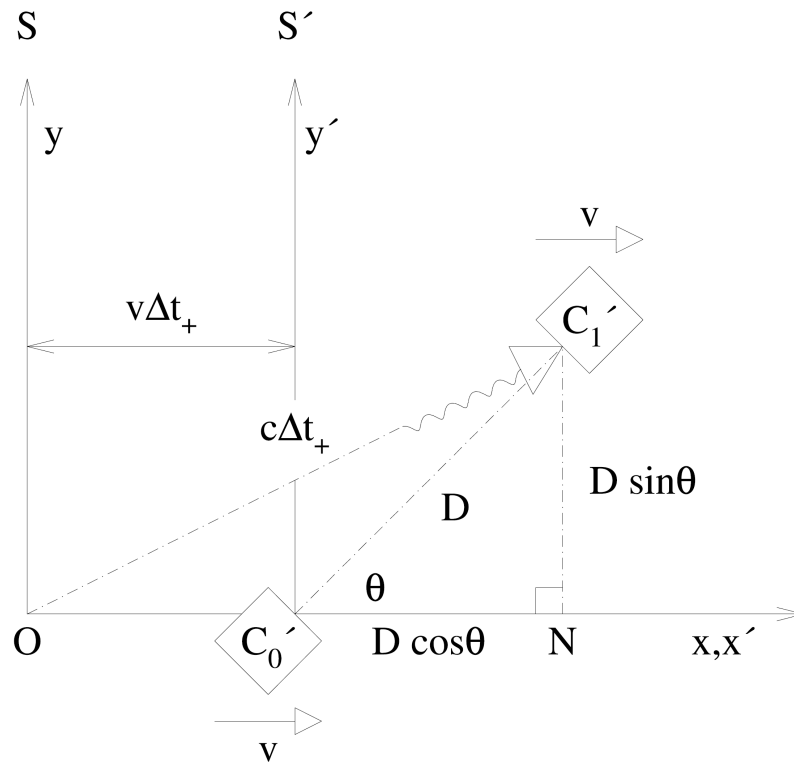


Figure 1. Space time geometry in the frame S where light propagates isotropically with speed c . The clocks C'_0 and C'_1 are at rest in the frame S' that moves with speed v relative to S . A light signal transmitted from the clock C'_0 is received by the clock C'_1 after the time interval Δt_+ , during which period the clock C'_0 moves a distance $v\Delta t_+$. See text for discussion.

of the signal in the frame S is, by postulates (i) and (ii), $s_+ = c\Delta t_+$, while the definition of the frame S' and postulate (ii) require that the displacement OC'_0 of C'_0 in Figure 1 during the time interval Δt_+ is $v\Delta t_+$. The Theorem of Pythagoras applied to the triangle OC'_1N in Figure 1 gives:

$$s_+ = c\Delta t_+ = D\gamma^2[\sqrt{1 - \beta^2 \sin^2 \theta} + \beta \cos \theta], \quad (2.5)$$

where $\beta \equiv v/c$. The corresponding time interval, Δt_- , of a signal transmitted by C'_1 and received by C'_0 is given by setting $\beta \rightarrow -\beta$ in Eq. (2.3):

$$s_- = c\Delta t_- = D\gamma^2[\sqrt{1 - \beta^2 \sin^2 \theta} - \beta \cos \theta]. \quad (2.6)$$

The time interval for a signal transmitted from C'_0 to C'_1 and promptly reflected back to C'_0 , Δt_{010} , is, from (2.5) and (2.6):

$$\Delta t_{010} = \Delta t_+ + \Delta t_- = \frac{2D\gamma^2}{c} \sqrt{1 - \beta^2 \sin^2 \theta}. \quad (2.7)$$

Also

$$\Delta t_{101} = \Delta t_- + \Delta t_+ = \Delta t_{010}. \quad (2.8)$$

Only motion in the frame S has been considered in deriving Eqs. (2.5)-(2.8) and all time intervals are those recorded by a single clock at rest in the frame S. To find the corresponding time intervals in the frame S', i.e., those actually recorded by the clocks C'_0 and C'_1 , during the passage of the light signal shown in Figure 1 postulate (iii) is used. Substituting the interval world line equation $\Delta x = v\Delta t$ given by (2.1) into (2.4) so as to eliminate Δx , it is found that:

$$\Delta t' = \frac{\Delta t}{\gamma}. \quad (2.9)$$

Since this time dilation relation contains no spatial coordinates, it is applicable to any clock at rest in S' , independently of its spatial position. It then follows from (2.9) on introducing the epochs $t'_1(t)$, $t'_2(t)$ recorded by C'_0 , C'_1 respectively, at epoch t that:

$$\Delta t'_1 \equiv t'_1(t) - t'_1(t_0) = \Delta t'_2 \equiv t'_2(t) - t'_2(t_0) = \frac{\Delta t}{\gamma} \equiv \frac{t - t_0}{\gamma} \quad (2.10)$$

so that if C'_0 and C'_1 are synchronised at $t = t_0$: $t'_1(t_0) = t'_2(t_0) = t'(t_0)$, they remain so at all later times for any value of t : $t'_1(t) = t'_2(t) = t'(t)$. There is no 'relativity of simultaneity' effect for spatially-separated clocks as in conventional special relativity theory. For further discussion of the spurious nature of 'relativity of simultaneity' and the correlated 'length contraction' effect see Refs. [10-12].

The time intervals recorded by the clocks C'_0 and C'_1 are therefore, on combining (2.5), (2.6) and (2.9):

$$\Delta t'_\pm(\beta, \theta) = \frac{D\gamma}{c} [\sqrt{1 - \beta^2 \sin^2 \theta} \pm \beta \cos \theta], \quad (2.11)$$

$$\Delta t'_{010} = \Delta t'_{101} = \frac{2D\gamma}{c} \sqrt{1 - \beta^2 \sin^2 \theta}. \quad (2.12)$$

In the following two sections, it is shown how observations of the time intervals given by Eqs. (2.11) and (2.12) can be used, firstly to determine the parameters β , θ defining the motion of C'_0 and C'_1 in the frame S , and secondly to synchronise these clocks by exchange of light signals.

Since only measurements internal to the system of moving clocks are considered, use of (2.11) and (2.12) enables the motion of the frame S' relative to S to be detected by such purely 'internal' measurements. This is in contradiction with the widely-used definition (following Poincaré [9]), mentioned above, of the special relativity principle, as the assertion of the impossibility of such a detection.

3. Light-Signal-Exchange Clock Synchronisation at Order v/c

Retaining only order v/c terms in Eqs. (2.11) and (2.12), they simplify to:

$$\Delta t'_{\pm}(\beta, \theta) = \frac{D}{c} [1 \pm \beta \cos \theta], \quad (3.1)$$

$$\Delta t'_{010} = \Delta t'_{101} = \frac{2D}{c}. \quad (3.2)$$

At this order of approximation a single 'echo delay' measurement of $\Delta t'_{010}$ or $\Delta t'_{101}$ enables measurement of the speed of light in free space, c , without any considerations of clock synchronisation:

$$c = \frac{2D}{\Delta t'_{010}} = \frac{2D}{\Delta t'_{101}}. \quad (3.3)$$

Three distinct series of operations are required to determine β and θ from observations of $\Delta t'_{\pm}(\beta, \theta)$. It is assumed at the outset that the values of c - possibly measured by use of Eq. (3.3) - and the separation, D , of the clocks in the frame S are known. A space-time experiment to measure the parameters v , θ and D , using an array of synchronised clocks at rest in the frame S, is described in the Appendix. The operations are:

Ia: Stop C'_0 and C'_1 and set the epoch of C'_0 to t'_0 and that of C'_1 to $t'_0 + D/c$.

Ib: Start C'_0 and send a light signal to C'_1 .

Ic: Start C'_1 on receipt of the light signal.

If $\beta = 0$, this procedure synchronises C'_0 and C'_1 . Since the signal from C'_0 actually arrives at C'_1 at C'_0 epoch:

$$t'_0 + \Delta t'_+(\theta) = t'_0 + D(1 + \beta \cos \theta) / c$$

it follows that step Ic actually results in setting C'_1 slow by the time interval $(D\beta \cos \theta) / c$ relative to C'_0 .

IIa: The line joining the two clocks is rotated anticlockwise through $\pi/2$ radians in the xy plane of Figure 1. During this procedure, C'_0 and C'_1 may have different velocities in the frame S, so that there will be different time dilation effects that will change the relative synchronisation of the clocks. Since however all such effects are at least of order $(v/c)^2$, they may be neglected at the level of approximation of the present calculation.

IIb: Send a signal from C'_1 to C'_0 at known C'_1 epoch $(t'_1)^A$ (i.e., at C'_0 epoch $(t'_1)^A + (D\beta \cos \theta) / c$).

IIc: The signal arrives at C'_0 at C'_0 epoch²:

$$\begin{aligned} (t'_0)^A &= (t'_1)^A + (D\beta \cos \theta) / c + \Delta t'_-(\theta + \pi/2) \\ &= (t'_1)^A + \frac{D}{c} [1 + \beta(\cos \theta + \sin \theta)]. \end{aligned}$$

IIIa: The line joining the two clocks is rotated clockwise through π radians in the xy plane of Figure 1 (i.e., clockwise by $\pi/2$ radians relative to their original orientation).

IIIb: Send a signal from C'_1 to C'_0 at known C'_1 epoch $(t'_1)^C$, i.e., at C'_0 epoch $(t'_1)^C + (D\beta \cos \theta) / c$.

²The epoch labels $A(C)$ denote, respectively, anti-clockwise (clockwise) rotations by $\pi/2$ in the xy plane of the line joining the two clocks.

IIIc: The signal arrives at C'_0 at C'_0 epoch:

$$\begin{aligned} (t'_0)^C &= (t'_1)^C + (D\beta \cos \theta)/c + \Delta t'_-(\theta - \pi/2) \\ &= (t'_1)^C + \frac{D}{c} [1 + \beta(\cos \theta - \sin \theta)]. \end{aligned}$$

With the definitions:

$$(\Delta t'_0)^A \equiv (t'_0)^A - (t'_1)^A - D/c = \frac{D\beta}{c} [\cos \theta + \sin \theta], \quad (3.4)$$

$$(\Delta t'_0)^C \equiv (t'_0)^C - (t'_1)^C - D/c = \frac{D\beta}{c} [\cos \theta - \sin \theta], \quad (3.5)$$

the parameters β and θ are given, in terms of time intervals measured uniquely by the clock C'_0 , as:

$$\beta = \frac{c}{2D} \sqrt{[(\Delta t'_0)^A]^2 + [(\Delta t'_0)^C]^2}, \quad (3.6)$$

$$\theta = \arctan \left[\frac{(\Delta t'_0)^A - (\Delta t'_0)^C}{(\Delta t'_0)^A + (\Delta t'_0)^C} \right]. \quad (3.7)$$

Finally, to synchronise the clocks, both C'_0 and C'_1 are stopped and the epoch of C'_0 set to t'_0 and that of C'_1 to:

$$t'_1 = t'_0 + \Delta t'_+(\theta - \pi/2) = t'_0 + D(1 + \beta \sin \theta)/c.$$

Clock C'_0 is started, and simultaneously a signal is sent to C'_1 . When C'_1 is started on receipt of the signal, the clocks C'_0 and C'_1 are synchronised up to corrections of order β^2 .

4. Exact Light-signal-echo Synchronisation of three Clocks in the Same Inertial Frame

In this case, three clocks at rest in the frame S' : C'_0 , C'_1 and C'_2 are

considered, disposed in the xy plane in the frame S , as shown in Figure 2. The echo time-delays of signals sent from C'_0 to C'_1 or C'_2 and promptly reflected back, are given by Eq. (2.8) as:

$$\Delta t'_{010} = \Delta t'_+(\theta) + \Delta t'_-(\theta) = \frac{2D\gamma}{c} \sqrt{1 - \beta^2 \sin^2 \theta}, \quad (4.1)$$

$$\Delta t'_{020} = \Delta t'_+(\theta + \pi/2) + \Delta t'_-(\theta + \pi/2) = \frac{2D\gamma}{c} \sqrt{1 - \beta^2 \cos^2 \theta}. \quad (4.2)$$

Squaring and adding (4.1) and (4.2) gives:

$$(\Delta t'_{010})^2 + (\Delta t'_{020})^2 = \left(\frac{2D}{c}\right)^2 \frac{2 - \beta^2}{1 - \beta^2}. \quad (4.3)$$

Solving (4.3) for β gives:

$$\beta = \sqrt{\frac{\alpha - 2}{\alpha - 1}}, \quad (4.4)$$

where

$$\alpha \equiv \left(\frac{c}{2D}\right)^2 [(\Delta t'_{010})^2 + (\Delta t'_{020})^2]. \quad (4.5)$$

Taking the ratio of (4.1) to (4.2) gives:

$$R \equiv \frac{\Delta t'_{010}}{\Delta t'_{020}} = \sqrt{\frac{1 - \beta^2 + \beta^2 \cos^2 \theta}{1 - \beta^2 \cos^2 \theta}}. \quad (4.6)$$

Solving (4.6) for θ gives:

$$\theta = \arccos \frac{1}{\beta} \sqrt{\frac{R^2 - 1 + \beta^2}{R^2 + 1}}. \quad (4.7)$$

Once the values of β and θ have been determined from (4.4) and (4.7), respectively, the clocks C'_0 , C'_1 and C'_2 may be synchronised by a procedure similar to that used for the clocks C_0 and C_1 in the previous

section:

(i) All three clocks are stopped and their epochs set to the values:

$$C'_0: t'_0, \quad C'_1: t'_0 + \Delta t'_+(\beta, \theta) \quad C'_2: t'_0 + \Delta t'_+(\beta, \theta + \pi/2),$$

where $\Delta t'_+(\beta, \theta)$ and $\Delta t'_+(\beta, \theta + \pi/2)$ are given by Eqs. (2.5) and (2.10).

(ii) Light signals are sent to C'_1 and C'_2 from C'_0 at the instant that the latter clock is started.

(iii) C'_1 and C'_2 are started on receipt of the light signals from C'_0 .

(iv) C'_0 , C'_1 and C'_2 are now synchronised.

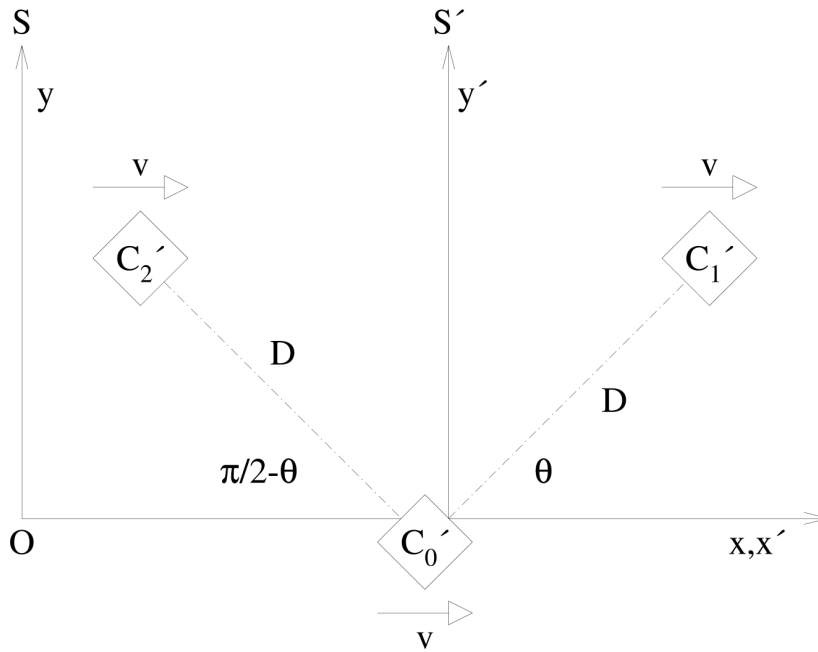


Figure 2. Geometrical configuration of three clocks C'_0 , C'_1 and C'_2 at rest in the frame S' , that moves with speed v relative to S . See text for discussion.

In the case that the angle θ in Figure 2 can be varied in a controlled manner a simpler strategy can be used to find, by measurements internal to the frame S' , the magnitude and direction of the speed of S' relative to S :

(i) The angle θ is varied until $\Delta t'_{010} = t'_{020}$.

(ii) Since inspection of (4.1) and (4.2) shows that now $\theta = -\pi/4$, the direction of the relative motion is the bisector of the angle $C'_1C'_0C'_2$.

(iii) By aligning the path $C'_0C'_1$ with the direction of the relative velocity (i.e., setting $\theta = 0$ in Figure 2) the speed of light is given (exactly³) by Eq. (4.2) as

$$c = \frac{2D}{t'_{020}(\theta = 0)} \tag{4.8}$$

and the (exact) magnitude of the relative velocity by (4.1) and (4.2) as:

$$v = c \frac{\sqrt{\gamma^2 - 1}}{\gamma}, \quad \gamma = \frac{t'_{010}(\theta = 0)}{t'_{020}(\theta = 0)}. \tag{4.9}$$

The formula (4.8) corresponds to the familiar ‘transverse photon clock’ geometry as discussed, for example, in the Feynman Lectures on Physics [13].

5. Synchronisation of Clocks without Light Signals

At the outset, two types of synchronisation procedures may be distinguished: *intra-frame synchronisation* in which the clocks-to-be-synchronised are at rest in the same reference frame and *inter-frame synchronisation* in which the clocks are in different inertial frames.

³That is, correctly to all orders in v/c .

A conceptually simple type of intra-frame synchronisation, applicable within any inertial frame, is one in which the light signal of the Einstein procedure is replaced by a ‘messenger object’ (MO) programmed to move in a known manner within the frame. In the ‘messenger-exchange’ procedure, strictly analogous to the Einstein light signal procedure, the MO moves away from clock A at A epoch t_1^A , being first accelerated, then decelerated so as to arrive at rest at clock B. The clock B which has previously been set to the epoch t_1^A is started by the arrival of MO. After an arbitrary time interval, T^B as recorded by B, the MO moves back in a symmetrical (time reversed) manner to clock A, arriving there at A epoch t_2^A . On advancing the epoch recorded by B by the time interval:

$$\Delta t^B = \frac{t_2^A - t_1^A - T^B}{2} \quad (5.1)$$

the clocks A and B are synchronised.

In the even simpler ‘dual-messenger’ intra-frame procedure two MOs are placed midway between (or, more generally, at any point on a line perpendicular to the line joining the two clocks, and passing through the mid-point of the latter line) the two clocks-to-be-synchronised, A and B. The clocks are stopped and set to any desired epoch: $t^A = t^B = t$. The two adjacent MOs simultaneously initiate identical acceleration/deceleration programs one towards A, the other towards B. The clocks A and B, started by the arrivals of the respective MOs are then synchronised. Worldline diagrams of the ‘messenger-exchange’ and ‘dual-messenger’ procedures can be found in Figures 1 and 2, respectively, of Ref. [14].

Synchronised clocks at rest in the frame S' , as considered in Section 2 can be alternatively obtained by symmetrical transport of clocks initially at rest in the frame S and synchronised there by any convenient

method (for example, Einstein light signal synchronisation). This method exploits the spatial-position-independence of the time dilation relation (2.9). In the simplest application of this method two clocks A and B are placed together at some point in the frame S and synchronised so that $t^A = t^B = t$. At some arbitrary later instant both clocks are accelerated and decelerated in an arbitrary, though identical, manner in any direction, so they are both finally at rest in the frame S at points equidistant from their original position. Since the proper time evolution of the clocks, given by integration of the time dilation relation (2.7) is the same, they remain synchronised throughout this clock transport. This synchronisation is maintained when they are finally accelerated in an arbitrary, but identical, manner, into the frame S'. Notice that the clock transport method is valid for an arbitrary acceleration/deceleration program provided that it is applied in an identical manner to both clocks. This is to be contrasted with the 'slow clock transport' method considered by Bridgman [15] and Mansouri and Sexl [16].

A method to synchronise four clocks, two in each of two inertial frames by 'length transport' has been described in Ref. [17]. This method therefore combines intra-frame and inter-frame synchronisations in a single procedure. Its application to two clocks at rest in the frame S: A and B, and two clocks at rest in the frame S': A' and B', can be understood by reference to Figure 1. Initially, all four clocks are stopped and set to the same epoch. The clocks A and B are then placed in the position of C₀' and C₁' in Figure 1; the clocks A' and B' just above them, but with the same x-coordinates. A' and B' are then moved the same distance in the negative x-direction till they are adjacent to two other clocks A₀ and B₀, at rest in S, that have previously been synchronised by any convenient procedure. At a pre-determined instant, controlled by the epoch of A₀ and B₀, A' and B' are accelerated, in an identical manner, in the positive x-direction up to speed v in the frame S. When A'

is later aligned with A and, simultaneously, B' is aligned with B, all four clocks are started. At this instant all four clocks register the same epoch. At later times A and B as well as A' and B' remain synchronised, but because of the time dilation effect the clocks at rest in S (A and B) are no longer synchronised with those at rest in S' (A' and B').

A variation of this method where four synchronised clocks at known positions in the frame S are used to measure the essential S-frame parameters v , D and θ of the space-time experiments discussed in Section 2 above, is described in the Appendix.

6. Internal Detection of Uniform Translational Motion by Clock Transport

The time dilation relation (2.9) which is an immediate consequence of postulate (iii) (The Lorentz transformation of space and time intervals between the frames S and S') gives the possibility, actually realised in the Hafele-Keating (HK) experiment [18, 19] performed in 1971, to detect the uniform motion of an inertial frame by observation of time intervals recorded by a clock moving, in a known manner, relative to the inertial frame. Since only time intervals recorded by a single clock are considered in this analysis, all considerations of clock synchronisation are irrelevant.

Referring to Figure 2, a single clock, C'' , is moved with constant speed v' in the frame S' between C'_0 and C'_1 or C'_0 and C'_2 . Only the clock C'' is observed, the others are used only as spatial markers. Denoting by (+) the outward displacement from C'_0 and by (-) the return displacement, in the analysis at order $(v/c)^2$, considered here, it is sufficient to use Galilean transformations to determine the velocity of the clock C'' in the frame S. In an obvious notation, it is found that:

$$v_1(+) = [v^2 + (v')^2 + 2vv' \cos \theta]^{\frac{1}{2}}, \quad (6.1)$$

$$v_1(-) = [v^2 + (v')^2 - 2vv' \cos \theta]^{\frac{1}{2}}, \quad (6.2)$$

$$v_2(+) = [v^2 + (v')^2 - 2vv' \sin \theta]^{\frac{1}{2}}, \quad (6.3)$$

$$v_2(-) = [v^2 + (v')^2 + 2vv' \sin \theta]^{\frac{1}{2}}. \quad (6.4)$$

Denoting the transit times in the frame S, S' of each passage of the clock C'' by Δt , $\Delta t'$, respectively, and the time intervals recorded by C'' during the passages as: $\Delta t_1''(+)$, $\Delta t_1''(-)$, $\Delta t_2''(+)$ and $\Delta t_2''(-)$, the time dilation relation (2.9) gives the relations:

$$\begin{aligned} \Delta t &= \gamma[v]\Delta t' = \gamma[v_1(+)]\Delta t_1''(+)= \gamma[v_1(-)]\Delta t_1''(-), \\ &= \gamma[v_2(+)]\Delta t_2''(+)= \gamma[v_2(-)]\Delta t_2''(-), \end{aligned} \quad (6.5)$$

where $\gamma[v] \equiv 1/\sqrt{1 - (v/c)^2}$. On retaining only order $(v/c)^2$ terms it follows from (6.1) to (6.5) that

$$\Delta t_1'' \equiv \Delta t_1''(+)- \Delta t_1''(-) = -\frac{2\Delta t}{c^2} vv' \cos \theta + O(\beta^4) \quad (6.6)$$

$$\Delta t_2'' \equiv \Delta t_2''(+)- \Delta t_2''(-) = \frac{2\Delta t}{c^2} vv' \sin \theta + O(\beta^4). \quad (6.7)$$

Since $\Delta t = \gamma[v]\Delta t' \simeq \Delta t'[1 + v^2 / (2c^2)]$, (6.6) and (6.7) simplify, further, at order $(v/c)^2$, to

$$\begin{aligned} \Delta t_1'' &\simeq -\frac{2\Delta t'}{c^2} vv' \cos \theta = -\frac{2}{c^2} \left(\frac{D}{v'}\right) vv' \cos \theta \\ &= -\frac{2Dv \cos \theta}{c^2} + O(\beta^4), \end{aligned} \quad (6.8)$$

$$\Delta t_2'' = \frac{2Dv \sin \theta}{c^2} + O(\beta^4). \quad (6.9)$$

It is interesting to note that at order $(v/c)^2$, $\Delta t_1''$ and $\Delta t_2''$, although sensitive to v and θ (and so to the motion of S' relative to S) are independent of the value of the velocity v' .

Solving (6.8) and (6.9) for θ and v gives:

$$\theta = -\arctan\left[\frac{\Delta t_2''}{\Delta t_1''}\right] = \arctan\left[\frac{\Delta t_2''(+)-\Delta t_2''(-)}{\Delta t_1''(-)-\Delta t_2''(+)}\right], \quad (6.10)$$

$$v = \frac{c^2}{2D} [(\Delta t_1'')^2 + (\Delta t_2'')^2]^{\frac{1}{2}} \quad (6.11)$$

formulas with a similar structure to (3.7) and (3.6), respectively, in a space-time experiment with a similar geometry involving the exchange of light signals.

In the case that the values of v , v' and θ are known, the relation (6.5) can be used to synchronise, for example, the clock C'_0 and the moved clock C'' . In order to do this C'_0 and C'' are set to the same epoch at the beginning of the outward displacement of C'' between C'_0 and C'_1 . The clock C'' is synchronised with C'_0 , by adding to its epoch, at any instant after it has been displaced to the position of C'_1 , where it is brought to rest in the frame S' , the quantity $\delta t'$ where:

$$\begin{aligned} \delta t' &\equiv \Delta t' - \Delta t_1''(+) = \Delta t' \left[1 - \frac{\Delta t_1''(+)}{\Delta t'} \right] \\ &= \frac{D}{v'} \left[1 - \frac{\gamma[v]}{\gamma[v_1(+)]} \right] \\ &= \frac{Dv'}{2} [v' + 2v \cos \theta] + O(\beta^4). \quad (6.12) \end{aligned}$$

This method of synchronising clocks in an inertial frame with known motion was suggested by Ives [20] with a view to performing a measurement of the one-way speed of light.

7. Physical Preferred Frames

The simplest example of a preferred frame for isotropic light propagation with a fixed velocity is provided by the local inertial frame, at any point in the universe remote from the gravitational fields of discrete objects, within which the frequency distribution of the locally measured Cosmic Microwave Background (CMB) is observed to be isotropic. This frame is experimentally determined by use of a direction-sensitive microwave detector to measure the spectrum of the CMB. The measured direction-dependent Doppler shift ('Dipole Term') defines a boost into a definite frame where the frequency anisotropy vanishes. According to the COBE experiment [22] the Solar system is moving with a velocity of 369 km/s relative to the frame in which the CMB is isotropic. If a transmitter of electromagnetic signals of known frequency is placed at rest in this inertial frame, measurement of the observed Doppler shift of this signal determines the velocity of any receiver relative to the preferred frame tagged by the transmitter.

Of more practical importance are the preferred frames associated with the gravitational fields around massive astronomical bodies such as the Earth or the Sun. As remarked by Su [23, 24] these gravitational fields constitute effective 'local aethers' where the speed of light is less than, but close to, its speed in free space. For the case of the Earth it is a prediction of the Schwarzschild metric equation [25, 26] of general relativity that the ECI (Earth-Centered Inertial) frame [27] is a preferred one of this type. The ECI frame is an inertial frame instantaneously co-moving with the centroid of the Earth, with coordinate axes pointing in fixed directions relative to the Celestial Sphere. Since it is the 'fixed stars'

that serve as the reference for rotational motion, this can be considered as a practical application of Mach's Principle. The SCI (Sun-Centered Inertial) frame is defined in a similar manner. In the operation of the Global Positioning System (GPS) microwave signals transmitted by Earth-satellites are assumed to have a uniform speed c in the ECI frame [27].

For a light signal moving parallel to the surface of the Earth at low altitude, h , ($h \ll R_E$ where R_E is the radius of the Earth, assumed spherical) the velocity is given by the Schwarzschild metric equation:

$$0 = (d\tau)^2 = \left(1 + \frac{2\phi_E}{c^2}\right)(dt)^2 - \frac{R_E^2(d\phi)^2}{c^2}, \quad (7.1)$$

where $\phi_E = -GM_E / R_E$ is the gravitational potential at the surface of the Earth. To first order in ϕ_E , the light signal speed in the ECI frame is:

$$c_E \equiv R_E \frac{d\phi}{dt} = \left(1 + \frac{\phi_E}{c^2}\right)c. \quad (7.2)$$

The known values of the mass, M_E , and radius of the Earth give; $\phi_E / c^2 = -6.94 \times 10^{-10}$ so that

$$\frac{\Delta c_E}{c} = \frac{c_E - c}{c} = -6.94 \times 10^{-10}.$$

For practical applications of the GPS [27] any reduction of the speed of light signals in the vicinity of the Earth due to the effect of the gravitational field of the latter is negligible. On the surface of the Earth then, the preferred frame S considered in the calculations of Sections 2-6 above can, to a very good approximation, be identified with the ECI frame, while the instantaneous co-moving inertial frame of a point at rest on the surface of the rotating Earth may be identified with the frame S'.

It is interesting to note that the isotropic propagation of light, at fixed

speed c , assumed to exist in the frame S in postulate (i) is actually a *necessary consequence* of relativistic kinematics and the massless (or almost massless) nature of photons [28]. Consider any process in the frame S (i.e., for experiments performed on the surface of the Earth, the ECI frame) in which particles are produced. Independently of the details of the production mechanism, the velocity, v , of any particle is related to its Newtonian mass, m , relativistic energy, E , and relativistic momentum \vec{p} by the formula:

$$v = \frac{pc^2}{E} = \frac{pc^2}{[m^2c^4 + p^2c^2]^{\frac{1}{2}}}, \quad (7.3)$$

where $p \equiv |\vec{p}|$. This predicts that for *any particle* respecting the condition $p/c \gg m$ then $v \simeq c$ and for a strictly massless particle $v = c$. Since (7.3) depends only on p and not \vec{p} there can be no ‘intrinsic’ anisotropy in the velocity distribution of the produced particle. Indeed, the angular distribution of the velocity is controlled by the physics of the production process, not by any anisotropy of the space-time metric.

For example, to correctly describe the propagation of neutrinos produced in laboratory experiments on the surface of the Earth, the kinematics of the production process should be calculated, not in the Earth-fixed laboratory frame, but in the ECI frame. The time of flight of neutrinos, produced with speed $\simeq c$ according to Eq. (7.3) in the ECI frame, between Earth-fixed sources and detectors must then take into account the motion of these due to the Earth’s rotation [30]. In a similar manner the microwave signals of the GPS have a speed close to c only in the ECI frame, not in the proper frame of a receiver at a fixed position on the surface of the Earth. This ‘Sagnac effect’ is taken into account in the GPS software [27]. After correcting for the Sagnac effect a limit on the speed anisotropy of microwave signals of $\delta c/c < 5 \times 10^{-9}$ has been

obtained [31].

Proper time intervals $d\tau$ of a clock in the vicinity of the Earth, where space-time curvature is dominated by the Earth's gravitational field, are described by the Schwarzschild metric equation, which is the solution of Einstein's field equations of general relativity for a non-rotating, spherically symmetrical source [25, 26]:

$$d\tau = \left[1 + \frac{2\phi_E(r)}{c^2} - \frac{1}{c^2} \left(\frac{v_r^2}{1 + \frac{2\phi_E(r)}{c^2}} + v_\theta^2 + v_\phi^2 \right) \right]^{\frac{1}{2}} dT, \quad (7.4)$$

where $\phi_E(r) \equiv -GM_E/r$ is the gravitational potential at distance r from the center of the Earth (assumed to be spherical) and M_E is the mass of the Earth. The origin of the spherical polar coordinates (r, θ, ϕ) is at the centre of the Earth and the 'coordinate time interval', dT , is that which would be recorded by a hypothetical clock, at rest in the ECI frame, sufficiently far from the Earth that $\phi_E \simeq 0$. For a clock moving parallel to the surface of the Earth with velocity $v = \sqrt{v_\theta^2 + v_\phi^2}$, then $v_r = 0$ and (7.4) simplifies to:

$$d\tau = \left[1 + \frac{2\phi_E(R_E)}{c^2} - \beta^2 \right]^{\frac{1}{2}} dT, \quad (7.5)$$

where $\beta \equiv v/c$ and R_E is the radius of the Earth. Denoting the proper frame of the moving clock by S' and the ECI frame, where $\beta = 0$, by S gives the metric interval equations:

$$\text{In the frame } S': \quad d\tau = dt' = \left[1 + \frac{2\phi_E(R_E)}{c^2} - \beta^2 \right]^{\frac{1}{2}} dT, \quad (7.6)$$

$$\text{In the frame S: } d\tau = dt = \left[1 + \frac{2\phi_E(R_E)}{c^2} \right]^{\frac{1}{2}} dT. \quad (7.7)$$

Retaining only order β^2 terms on taking the ratio of (7.7) to (7.6) it is found that

$$\begin{aligned} dt &= \left[1 + \frac{\beta^2}{2 \left(1 + \frac{\phi_E(R_E)}{c^2} \right)} \right] dt' + O(\beta^2) \\ &= \left[1 + \frac{\beta^2}{2} \right] dt' + O\left(\frac{\beta^2 \phi_E(R_E)}{c^2} \right). \end{aligned} \quad (7.8)$$

Since (7.8) gives the same order β^2 approximation as the time dilation relation (2.7), it is clear that, at this level of approximation, and for experiments performed on the surface of the Earth, the frame S introduced in Section 2 may be identified with the preferred ECI frame.

That the ECI frame is then a preferred one, not only for the isotropic propagation of light at speed close to c , but also for the calculation of time dilation, becomes evident on considering different values of the velocity v in the interval Lorentz transformation equations (2.1) and (2.2). For example, $v = v_+ > 0$ (frame S'_+) and $v = -v_- < 0$ (frame S'_-) give, from (2.9), the time dilation relations:

$$\Delta t = \gamma(v_+) \Delta t'_+ = \gamma(v_-) \Delta t'_-. \quad (7.9)$$

If $v_+ = -v_- = v$, it follows from (7.8) that $\Delta t'_+ = \Delta t'_-$ so that although the clocks at rest in S'_+ and S'_- have a relative velocity of $2v$ in the frame S, there is no time dilation effect for these two clocks [21]. The time dilation effect therefore does not depend only on the relative velocity of the clocks, as might be concluded from a naive inspection of Eq. (2.9). The second

member of Eq. (7.9) can be considered as the basis for the calculation of special relativistic contributions to the time intervals recorded by the clocks of the Hafele-Keating experiment [32, 33, 18, 19, 12] and is verified by the good agreement between prediction and observation found in this experiment [19].

8. Discussion

The much discussed concept of ‘Conventionality of Clock Synchronisation’ [34], originally developed from a philosophical standpoint by Reichenbach [7] and Grünbaum [35] was based on three independent arguments:

- (1) A misinterpretation of the result of the Michelson-Morley experiment (MME) [36].
- (2) Application of ‘Lorentz-Fitzgerald Contraction’ or ‘relativistic length contraction’ to the analysis of the MME.
- (3) The assumption that clock synchronisation is possible only by exchange of light signals.

It was assumed, in the original analysis of the MME, that the putative ‘aether frame’ is identified with the SCI, instead of the ECI, as predicted by general relativity, and discussed in the previous section. In this case, the speed of the ‘aether wind’, to which it was assumed the MME would be sensitive, is identified with the speed of rotation of the Earth in its orbit around the Sun: $\approx 30\text{km/s}$ instead of the speed of rotation of the surface of the Earth about its polar axis: $\approx 300\text{ m/s}$ - an ‘aether wind’ a factor of 10^{-2} weaker and a phase shift in the MME a factor 10^{-4} times smaller. As pointed out by Su [23, 24] neither the original MME, nor any of its successors, was sufficiently sensitive to observe the rotation of the Earth in these order β^2 experiments. In

contrast, using the order β Sagnac effect, rotational motion of an interferometer relative to the ECI frame was measured in 1913 [37] and the rotation of the Earth relative to the ECI frame was measured by Michelson and Gale in 1925 [38]. As discussed in the previous section, the gravitational field of the Earth constitutes an ‘effective aether’ which renders the ECI frame a preferred one for propagation of light at speeds close to c in the region of the Earth. The existence of a similar, but different, ‘effective aether’ constituted by the gravitational field of the Sun in the SCI frame, was demonstrated by the Shapiro radar-echo-delay experiments [39], where microwave signals were reflected back to the Earth from the inner planets Venus and Mercury.

In the analysis presented in Section 2 above, the distance D separating the to-be-synchronised clocks is defined in the frame S, not in the proper frame, S' , of the clocks.

The measurement of this separation, using synchronised clocks at known positions in the frame S is described in the Appendix. If instead, D is identified with the clock separation in the frame S' and ‘length contraction’ parallel to the x -axis is assumed to occur in the frame S, then the distance from C'_0 to N in Figure 1 becomes $D \cos \theta / \gamma$. Application of the Theorem of Pythagoras to the triangle OC'_0N then gives, instead of Eqs. (2.8) and (2.9):

$$\Delta t'_{\pm}(\beta, \theta) = \frac{D}{c} [1 \pm \beta \cos \theta] \quad [D \cos \theta \rightarrow (D \cos \theta / \gamma)], \quad (8.1)$$

$$\Delta t'_{010} = \Delta t'_{101} = \frac{2D}{c} \quad [D \cos \theta \rightarrow (D \cos \theta / \gamma)]. \quad (8.2)$$

The null result of the MME was incorrectly interpreted as evidence for the correctness of Eq. (8.2) and, consequently, of the existence of relativistic ‘length contraction’.

Another prediction of (8.2) (due to the independence of the right side

on v and θ) is the velocity-independence of the two-way speed of light. So, in agreement with Poincaré's statement of the special relativity principle, observation of $\Delta t'_{010}$ or $\Delta t'_{101}$ gives no information on the relative velocity of the frames S and S'. A corollary, which is the basis for the 'Conventionality of Clock Synchronisation' concept is that, if light signal exchange is the *only* way to synchronise spatially-separated clocks (i.e., the argument (3) above) it is impossible to measure the one way speed of light. This is because this measurement requires clocks to be synchronised, but clocks can only be synchronised if the one way speed of light is already known. However, regardless of whether length contraction exists or not, if the distance D is defined and measured in the frame S, which is the case for the calculation of Section 2 above, $\Delta t'_{010}$ and $\Delta t'_{101}$ are given by Eq. (2.12) *and are* sensitive to the relative velocity of the frames S and S', violating Poincaré's statement [9] of the special relativity principle. This is a necessary consequence solely of the postulates (i) and (ii) of Section 2 above and so holds both for Galilean relativity ($\Delta t = \Delta t'$) and special relativity ($\Delta t = \gamma \Delta t'$). Also, as described in Sections 5 and 6, many methods of clock synchronisation not requiring exchange of light signals exist. If any of these methods is used, there is no logical difficulty preventing the measurement of the one-way speed of light.

Since all space-time geometrical calculations in the present paper have been performed in the frame S where, by hypothesis, light is propagated isotropically with speed c , no consideration of the velocity of light in the proper frame S' of the moving clocks has been made. Einstein's formulation of special relativity theory is based, as well as on the special relativity postulate - that the laws of physics are the same in all inertial frames - on the second postulate that the speed of light is the same in all inertial frames. However, as will now be demonstrated, Einstein's second postulate cannot be true if the postulates (i), (ii) and (iii)

introduced in Section 2 hold. Indeed, correct calculations presented in the original special relativity paper [5] show, when combined, that the postulate of the frame-independence of the speed of light is untenable.

In Section 2 of Ref. [5] with the title ‘On the Relation of Lengths and Times’ Einstein performed the calculation of Section 2 above for the special case $\theta = 0$. Einstein also gave explicitly the postulates (i) and (ii) above as the basis for the calculation. The result obtained (in the notation of the present paper) was:

$$\Delta t_{\pm} = \frac{D}{c \mp v} \quad (8.3)$$

as obtained by setting $\theta = 0$ in Eqs. (2.5) and (2.6).

In Section 3 of Ref. [5], the Lorentz transformation equations, equivalent to postulate (iii) above, were derived and in the following section, the time dilation relation, Eq. (2.9) in the notation of the present paper, was derived. If D' is the separation of the clocks in the frame S' , then the postulate (ii), if also applied in the frame S' , together with the time dilation relation (2.9), give for the velocity of light in this frame:

$$\begin{aligned} c'_{\pm} &= \frac{D'}{\Delta t'_{\pm}} = \frac{\gamma D'}{D} \frac{D}{\Delta t_{\pm}} = \frac{\gamma D'}{D} (c \mp v) \neq c \\ &= c \mp v + O(\beta^2). \end{aligned} \quad (8.4)$$

So regardless of whether ‘length contraction’ exists ($D' = \gamma D$) or not ($D' = D$), it is impossible that the speed of light, defined according to the postulate (ii) in the frame S' , can be c .

Just after the derivation of the Lorentz transformation in Ref. [5] an argument based on a thought experiment involving light waves was given that claimed to show that the Lorentz transformation predicted the frame-independence of the speed of light. It was concluded that: ‘This

shows that our two fundamental principles are compatible'. It is also stated in a footnote to the English translation of Ref. [5] that assuming the speed of light is the same in all inertial frames is sufficient to derive the LT - a simple calculation to be found in many text books and pedagogical papers.

Actually Einstein's 'light wave' calculation is marred by a trivial mathematical error - use of the same mathematical symbol to represent quantities that have completely different physical meanings. The parameters β and γ may be eliminated for the interval LT (2.1)-(2.4) to obtain the invariant interval relation:

$$(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2(\Delta t)^2 = (\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 - c^2(\Delta t')^2. \quad (8.5)$$

In the notation of Ref [5], this is written:

$$x^2 + y^2 + z^2 - c^2t^2 = \xi^2 + \eta^2 + \zeta^2 - c^2\tau^2. \quad (8.6)$$

In Ref. [5], the right and left members of (8.6) were spuriously identified with spherical 'light waves' in the frames S and S', respectively:

$$x^2 + y^2 + z^2 - c^2t^2 = 0, \quad (8.7)$$

$$\xi^2 + \eta^2 + \zeta^2 - c^2\tau^2 = 0. \quad (8.8)$$

With this identification, then assuming (8.7) holds (i.e., a spherical light wave moving with speed c in the frame S) and applying the relation (8.6), derived from the LT, then (8.8) is derived, showing that a spherical light wave moving with speed c also exists in the frame S' - consistent with the second postulate. However, this is a physically meaningless calculation since the coordinate intervals in (2.1)-(2.4) and in Einstein's equation (8.6) are those on the world line of *an object at rest in the frame S'*, *not of a light signal or a photon*, as is assumed to be the case in Eq. (8.7). Indeed, since the world line equation in S: $x = vt$ was assumed in Ref.

[5] in order to derive the time dilation relation it follows from the space LT (2.1) that $\xi = 0$. Since also (see (2.2) and (2.3)) $y^2 + z^2 = \eta^2 + \zeta^2 = 0$, (8.6) simplifies to

$$x^2 - c^2 t^2 = -c^2 \tau^2 \quad (8.9)$$

equivalent to the time dilation relation: $t = \gamma\tau$. Clearly the two members of (8.6) can never vanish if $\tau > 0$ as is assumed to be the case in the 'light wave' equations (8.7) and (8.8). Einstein's claimed derivation of the second postulate from the LT - or the possibility to derive the LT by assuming that the second postulate holds - is then invalidated by a trivial mathematical error. This is the use of the same symbols to denote different physical quantities, in one case intervals on the world line of a ponderable object at rest in the frame S' , as in Eq. (8.6), in the other, intervals on the world line of a light signal propagating with speed c in frame S , as in Eq (8.7).

The equality of the speed of light in the frames S and S' is also a consequence of Einstein's velocity composition formula derived in Section 5 of Ref. [5]. However, the derivation of this formula is also invalidated by a similar mathematical error to the one concerning 'light waves' in the frames S and S' just discussed. Retaining the notation of the present paper, Einstein's velocity transformation formula is derived by taking the ratio of the interval LT equations (2.1) and (2.4) to give:

$$\frac{\Delta x'}{\Delta t'} = \frac{\Delta x - v\Delta t}{\Delta t - \frac{v\Delta x}{c^2}} = \frac{\frac{\Delta x}{\Delta t} - v}{1 - \frac{v}{c^2} \frac{\Delta x}{\Delta t}}. \quad (8.10)$$

Einstein then defines two velocities $w' \equiv \Delta x'/\Delta t'$ and $w \equiv \Delta x/\Delta t \neq v$ in order to obtain from (8.10) the velocity transformation formula:

$$w' = \frac{w - v}{1 - \frac{vw}{c^2}}. \quad (8.11)$$

Setting $w = c$ in this equation gives $w' = c'_+ = c$ in accordance with the second postulate. However, since, in fact, $\Delta x/\Delta t = v$, as correctly assumed by Einstein in order to derive the TD relation (2.9) above, the right side of (8.10) is actually

$$w' = \frac{v - v}{1 - \frac{v^2}{c^2}} = 0. \quad (8.12)$$

Then $\Delta x'/\Delta t' \equiv w' = 0 \neq c$.

The relation

$$\Delta t' = \gamma \left(1 - \frac{vw}{c^2} \right) \Delta t \quad (8.13)$$

which is assumed to hold in order to derive (8.11) is in contradiction with the time dilation relation $\Delta t' = \Delta t/\gamma$ if $w \neq v$. Indeed Eq. (8.13) is logically absurd since it predicts that the time interval, $\Delta t'$, recorded by a clock at rest in S' depends, for a given value of Δt , on the speed of motion w of an *arbitrary object* in the frame S ! When $w = \Delta x/\Delta t = v$ (the interval world line, in the frame S , of an arbitrary object at rest in the frame S'), then (8.13) correctly yields the time dilation relation (2.9).

The correct value of c'_+ is given by (8.4) with $D = D'$ [10, 11, 12]. The interval LT equations (2.1) and (2.4) are only valid for intervals on the world line of a ponderable object *at rest in the frame S'* , not as assumed by Einstein, for an object with *an arbitrary velocity w' in the frame S'* . Again, the same mathematical symbol is used to denote two physically distinct quantities.

The space-time LT were originally derived as the transformations that render the form of Maxwell's equations of classical electromagnetism the same in every inertial frame. Maxwell's application [40] of these equations in 'free space' to demonstrate the existence of 'electromagnetic waves' with a certain speed c then necessarily implies that such waves must have this speed in all inertial frames. On the other hand if light does propagate as a 'signal' (of unspecified nature) with fixed speed c in any particular frame of reference, the space-time geometrical calculation of Section 2 above, performed in this frame, shows that it is impossible that the signal has the same speed in any other inertial frame. This was clearly shown (but not remarked upon) by Einstein in Ref. [5] where Eq. (8.3) was given. This equation together with the time dilation relation (2.9) (also derived in Ref. [5]) necessarily leads, with the Galilean definition of velocity of postulate (ii) (as assumed in Ref. [5]) also in the frame S' , to Eq. (8.4), which negates the second postulate of special relativity.

If light is identified with 'electromagnetic waves in free space' there is then a clear antinomy between the predictions of, on the one hand, space-time geometry and the interval LT and, on the other, the second postulate. A possible way to resolve this antinomy at the time of this writing (the beginning of the 21st Century) may be to finally recognize that light actually consists not of the 'electromagnetic waves' predicted by Maxwell in 1865 but more likely of massless particles: 'light quanta' or photons for the discovery of which in 1905 Einstein was awarded the 1921 Nobel Prize for Physics. The word 'consists' above is used in the ontological sense - the answer to the question: 'What is light?' - not as the specification of some attribute. This is not to say that the 'electromagnetic wave' concept is without any physical significance whatever. Indeed phenomena involving very large numbers of real photons interacting with very large numbers of electrons are very conveniently described as the effect of phenomenological 'electromagnetic waves', obtained as the

solution of Maxwell's equations with sources, according to certain boundary conditions. Examples are radio antennas, wave guides or the accelerating cavities of particle accelerators. However, the Feynman diagrams of quantum electrodynamics do not allow the production of photons without sources, completely forbidding any identification of Maxwell's 'free space' electromagnetic waves with photons. Indeed retaining them and their associated electromagnetic fields after abandoning the aether, as suggested by Einstein in Ref. [5], was tantamount to banishing the ocean but still retaining the waves on the shore.

An analogy can be made between phenomenological electromagnetic fields and photons, and an army and the individual soldiers of which it is composed. It is convenient to describe the movements of different regiments making up the army: infantry, cavalry, artillery, etc. (c.f., electric fields, magnetic fields) but the army always is a certain number of soldiers - actual members of the species *homo sapiens* - and their equipment (c.f., numbers of real or virtual photons).

As described in detail elsewhere [41], the physical significance of electromagnetic fields in the limit of low photon density becomes identical to that of the quantum wave function, or probability amplitude, for experiments in which single photons are observed. Further critical discussion of the concept of 'wave particle duality' and the connection between quantum mechanics and classical wave theories of light or massive particles is found in Ref. [42].

Pais has discussed [43] the extreme reluctance of the physics community to accept the light quantum concept - that light actually *consists* of particles - between its discovery by Einstein in 1905 [44] and its confirmation by Compton in 1923 [45]. Even stranger, perhaps, is Einstein's own reluctance to see the connection between his own light quantum concept, relativistic mechanics, and the speed of light. Indeed

the remark in Ref. [5] that:

‘It is remarkable that the energy and the frequency of a light complex vary with the state of motion of the observer in the same manner.’

is a necessary consequence of the Planck-Einstein relation $E = h\nu$ for an individual light quantum given earlier by Einstein in 1905 in Ref. [44]. Also the constancy of the speed of light in *some* inertial frame of reference⁴ necessarily follows from relativistic kinematics - Eq. (7.3) above - in the case that light consists of massless particles. This assertion [28] could have been made at any time after Planck wrote down the formulas for relativistic energy ($E = \gamma mc^2$) and momentum ($p = \gamma mv$) in 1906 [46, 48].

As discussed in Section 7 above, the ‘free space’ within which special relativity is supposed to be valid is not an obvious feature of the known universe. Far from gravitating matter, in interstellar or intergalactic space, a natural reference frame is provided by the isotropy frame of the CMB. Near to gravitating matter, e.g., in the Solar system near to the Sun, or near to the surface of a planet, a preferred frame for light propagation with a speed close to c , is provided by general relativity. For a spherical gravitating body this is the frame in which space-time curvature is described by the Schwarzschild metric - the ECI frame for the Earth and the SCI frame for the Sun. As demonstrated by the Hafele-Keating experiment [47], the same preferred frame controls the relative rates of clocks in motion near to the gravitating body. These rates are correctly given at order $(v/c)^2$ by the special relativistic time dilation effect using the preferred inertial frame to specify the clock velocities (Eq. (7.8) above).

The above considerations show that it was not the arguments given in

⁴The frame in which it is chosen to calculate the energy and momentum of the created photon according to the laws of relativistic kinematics.

Einstein's 1905 light quantum paper that were 'heuristic' but rather those concerning the LT in the special relativity paper. Maxwell's prediction of 'electromagnetic waves' in free space can similarly be considered as the heuristic discovery that light consists of massless particles. The existence of light quanta (photons) of energy $h\nu$ and momentum $h\nu/c$ is verified by their essential role in the most successful and precise theory in the history of physics - quantum electrodynamics. Einstein gave a mathematically flawed derivation of the LT by assuming that the speed of light is the same in all inertial frames but then proceeded to derive from it both the experimentally confirmed time dilation effect and the equivalence of mass and energy, a completely novel concept with enormous practical consequences [48].

Maxwell used fields in 'free space' (sourceless fields that do not exist in quantum electrodynamics) to predict 'electromagnetic waves' - identified as light - as a disturbance of a putative luminiferous aether (now known to effectively exist, with predicted properties, as a consequence of general relativity) with fixed speed c . The almost constant value of the speed of light, in certain preferred frames of reference, is now understood most simply as a consequence of relativistic kinematics (Eq. (7.8) above) and the fact that light consists of massless (or very light) particles.

In summary, Einstein's light quantum paper was, to the best of our current knowledge not at all 'heuristic' but a major experimentally verified discovery about the nature of the real world. It contained the seed from which grew the splendid plant of quantum electrodynamics [49, 50] and other analogous modern particle physics theories. Einstein's 'special relativity theory' and Maxwell's prediction of 'electromagnetic waves' in free space were mathematically and/or conceptually flawed but nevertheless gave some important predictions that were in accord with experiment - predictions that were therefore obtained in a heuristic

manner. The enormous practical ramifications of Maxwell's classical electromagnetism and Einstein's special relativity were not in any way affected by the flawed manner in which these theories were discovered.

9. Summary and Conclusions

Assuming that a reference frame, S , exists in which light propagates isotropically with uniform speed, as well as the validity of the time dilation relation (2.9) (which is an immediate consequence of the interval LT (2.1)-(2.4)), measurements of time intervals between light signals recorded by the single clock C'_0 at rest in an arbitrary inertial frame S' are sufficient to determine the parameters $\beta = v/c$ and θ specifying the relative motion of the frames S and S' . This is done up to corrections of order v/c by use of Eqs. (3.6) and (3.7) or exactly (to all orders in v/c) by use of Eqs. (4.6) and (4.7). This knowledge of the motion of the frame S' relative to S enables two (or three) clocks at rest in S' to be synchronised by exchange of light signals using the procedures described in Section 3 (or Section 4).

These determinations of the relative motion of two inertial frames are counter examples to Poincaré's statement of the special relativity principle as the impossibility, by internal measurements, to detect uniform translational motion. Another counter example is provided by observation of time intervals recorded by clocks moving with a uniform velocity relative to the frame S' , as described in Section 6. This method of internal detection of relative motion has been verified by the Hafele-Keating experiment.

Actual examples of reference frames where light propagates, to a very good approximation, uniformly and isotropically are the local reference frame, at any point in the Universe, distant from discrete gravitating objects, where the observed frequency of the CMB is isotropic, or the ECI

and SCI frames in the proximity of the Earth and Sun respectively, as discussed in Section 7. The approximate isotropy and uniformity of light propagation in these preferred frames is a consequence of general relativity and of relativistic kinematics (Eq. (7.3)) if light is assumed to consist of massless (or very light) particles.

The concept of ‘Conventionality of Clock Synchronisation’ and its corollary, Poincaré’s formulation of the special relativity principle as a statement of the impossibility of internal detection of uniform translational motion, stems from misinterpretation of the results of the MME and its successors. It was assumed that the aether frame in the vicinity of the Earth is the SCI frame, not as predicted by general relativity, and confirmed by observation of the Sagnac effect, the ECI frame. Because the magnitude of the ‘aether wind’ associated with the ECI frame is a factor 10^{-2} weaker than for the SCI frame no MM-type experiment was sufficiently sensitive to observe it. Wrongly interpreting the negative result of the MME as evidence for length contraction, it was concluded (from Eq. ((8.2)) that the two-way speed of light is the same in all inertial frames. On further assuming that clock synchronisation is possible *only* by exchange of light signals, as suggested by Einstein, it follows that no measurement of the one-way speed of light is possible. Since then no physical significance could be assigned to one-way light speed, the concept of ‘Conventionality of Clock Synchronisation’, analogous to ‘gauge freedom’ in classical electromagnetism was introduced.

Actually the MME does not give evidence for the existence of length contraction and, as described in Sections 5 and 6, many methods of clock synchronisation, not relying on light signal exchange, exist. If any one of these is used there is no problem to measure the one-way speed of light. Furthermore, if the clock separation is defined and measured in the frame S, the two-way speed of light (see Eqs. (2.7) and (2.12)) is not the

same in the frames S and S'. In summary the 'Conventionality of Clock Synchronisation' concept is invalid, being based on misinterpreted experimental data and the false theoretical premise of frame independence of the two-way speed of light.

In the calculations presented in Sections 2, 3 and 4, only space-time geometry in the frame S was considered, i.e., the Galilean definition of velocity of postulate (ii). There was no mention of the speed of light in the frame S'. However, using the postulate (ii) also to define the velocity of a light signal in the frame S', it follows that, at order v/c , for a light signal moving parallel to the x -axis, the speed of light in the frame S' is $c \pm v$ (Eq. (8.4)), not c , as tacitly postulated by Einstein in Ref. [5], in spite of giving explicitly, in the same paper, Eq. (8.3) which contradicts this postulate. Indeed as pointed out elsewhere [51], light speed c in the frame S' is also incompatible with the existence of the Sagnac effect. Thus if Galilean space-time geometry⁵ holds for velocity measurements in both S and S' and the time dilation effect of special relativity occurs (as is confirmed by experiment [52]), the special relativistic concept most at odds with common sense - the frame independence of the speed of light - is untenable. How mathematical errors in Einstein's original special relativity paper [5] obscured this antinomy between the second postulate and Einstein's Eq. (8.3) above is explained in Section 8.

The motivation for Einstein's second postulate was most likely the invariance of Maxwell's equations under the LT and Maxwell's identification of light with 'electro-magnetic waves', derived from free space Maxwell's equations, which in virtue of the frame-invariance of these equations, must have the same speed in all inertial frames.

However, according to the argument just given, this is impossible. Such 'free space' electromagnetic waves also cannot be identified with the

⁵i.e., that time and space intervals are related as in postulate (ii) of Section 2.

photons of quantum electrodynamics which of necessity have a source in order to exist and, if they participate in an observed physical process, must also have a sink, i.e., must be both created and destroyed.

These antinomies are simply resolved if it is finally recognised that light really does *consist* of particles - the light quanta, for the discovery of which, Einstein was awarded the Nobel Prize - not (Maxwell's fame notwithstanding) electromagnetic waves in free space.

In Maxwell's theory such waves were considered to be disturbances of, i.e., attributes of, some luminiferous aether in just the way that ocean waves are attributes of the ocean and sound waves attributes of the air. The essential particulate ontology of any valid description of light is then completely missing.

From this modern perspective, Maxwell's prediction of 'electromagnetic waves' with the same speed as light, would be considered as a valid heuristic motivation for Hertz' experiments in which low energy photons were first discovered⁶.

Similarly in Einstein's first special relativity paper, an incorrect derivation of the LT, based on the second postulate, was given, but the LT was then used to correctly derive time dilation and the equivalence of mass and energy [48]. The role of the second postulate, like Maxwell's 'electromagnetic waves', was again a heuristic one. In contrast, in spite of its title, Einstein's recognition that light does consist of particles [44] is, by itself, a major advance in the understanding of nature, not a heuristic one: a false argument or postulate that still leads to a prediction that does correctly describe some aspect of the real world. Maxwell's derivation of the existence of electromagnetic waves as a consequence of his electromagnetic field equations and Einstein's postulate concerning the

⁶Hertz actually showed that the *one-way* speed of 'electromagnetic waves' created by an antenna was, at large distances from the source, equal to the speed of light [53, 54].

constancy of the speed of light, were both heuristic in this sense.

Appendix

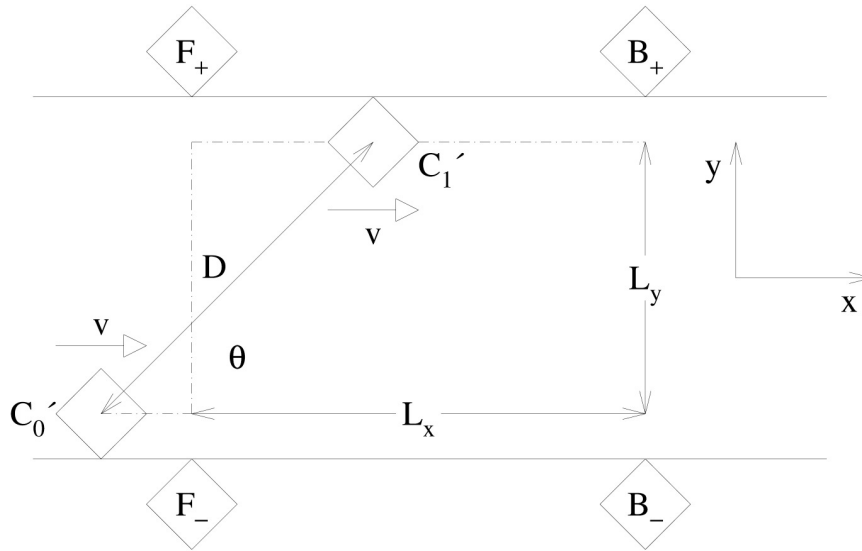


Figure 3. Scheme of an experiment to determine the parameters v , θ and D , by measuring the epochs of spatial coincidences of the clock C_1' with the clocks F_+ and B_+ , and of the clock C_0' with the clocks F_- and B_- . The synchronised clocks F_+ , F_- , B_+ and B_- are at rest in the frame S , C_0' and C_1' at rest in the frame S' . See text for discussion.

An experimental set-up to measure the geometrical parameters D , θ and the common velocity, v , in the frame S of the clocks C_0' and C_1' discussed in Section 2 is shown schematically in Figure 3. Synchronised clocks F_+ , F_- , B_+ and B_- ⁷ are at rest in S at the corners of a rectangle of known dimensions. Lines joining F_+ to B_+ and F_- to B_- are parallel to

⁷F stands for 'front' and B for 'back' as viewed from the moving clocks C_0' and C_1' .

the x -axis and the direction of motion of C'_0 and C'_1 . The epochs of the clocks at rest in S are recorded when the x -coordinates of C'_1 and F_+ or B_+ , or C'_0 and F_- or B_- are the same. Denoting the epochs of these spatial coincidences as: $t(F_+)$, $t(B_+)$, $t(F_-)$ and $t(B_-)$ the space-time geometry of Figure 3 gives the relations:

$$t(B_+) - t(F_+) = \frac{L_x}{v}, \quad (\text{A.1})$$

$$t(B_+) - t(F_-) = \frac{L_x - D \cos \theta}{v}, \quad (\text{A.2})$$

$$t(B_-) - t(F_-) = \frac{L_x}{v}. \quad (\text{A.3})$$

The velocity v is determined by (A.1) and (A.2) as:

$$v = \frac{L_x}{2} \left[\frac{1}{t(B_+) - t(F_+)} + \frac{1}{t(B_-) - t(F_-)} \right]. \quad (\text{A.4})$$

The geometry of Figure 3 gives:

$$D \sin \theta = L_y \quad (\text{A.5})$$

while transposing (A.2) gives:

$$D \cos \theta = L_x - v[t(B_+) - t(F_-)]. \quad (\text{A.6})$$

The parameters θ and D are then determined by (A.5) and (A.6) to be:

$$\theta = \arctan \left[\frac{L_y}{L_x - v[t(B_+) - t(F_-)]} \right], \quad (\text{A.7})$$

$$D = [L_y^2 + (L_x - v[t(B_+) - t(F_-)])^2]^{\frac{1}{2}} \quad (\text{A.8})$$

where v is given by Eq. (A.4).

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