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SLANT HELICES GENERATED BY PLANE CURVES IN E_1^3

MESUT ALTINOK* and LEVENT KULA

Department of Mathematics
Faculty of Sciences and Arts
Ahi Evran University
Kirsehir, Turkey
e-mail: altnokmesut@gmail.com
lkula@ahievran.edu.tr

Abstract

In this paper, we investigate the relationship between the plane curves and slant helices in E_1^3. Moreover, we show how could be obtained a slant helix from a plane curve. Finally, we give some slant helix examples generated by plane curves in E_1^3.

1. Introduction

In classical differential geometry, a general helix in the Euclidean 3-space, $\mathbb{E}^3$, is a curve with constant slope which means that it makes a constant angle with some fixed direction (the axis of the helix). A necessary and sufficient condition that a curve be a general helix is that the ratio of curvature to torsion be constant. In particular, circular helices where both curvatures and torsion are constant so as plane curve where the torsion vanishes identically provide two subclasses of general

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*Corresponding author

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helices.

Recently, Izumiya and Takeuchi, in [7], have introduced the concept of slant helix in Euclidean 3-space. A slant helix in Euclidean space $\mathbb{E}^3$ was defined by the property that the principal normal makes a constant angle with a fixed direction. Moreover, Izumiya and Takeuchi showed that γ is a slant helix in Euclidean 3-space if and only if the geodesic curvature of the principal normal of a space curve γ is a constant function.

In [8], L. Kula and Y. Yayli studied the spherical images under both tangent and binormal indicatrices of slant helices and obtained that the spherical images of a slant helix are spherical helix. In [9], the authors characterize slant helices by certain differential equations verified for each one of obtained spherical indicatrix in Euclidean 3-space. Recently, Ali and Lopez, in [1], have studied slant helix in Minkowski 3-space. They showed that the spherical indicatrix of a slant helix in $\mathbb{E}^3$ are helices.

In [3], M. Altinok and L. Kula studied the relationship between the plane curves and slant helices in $\mathbb{R}^3$. Moreover, they get slant helix from plane curve.

In this paper, we consider the relationship between the plane curves and slant helices in $\mathbb{E}^3$. Moreover, we get slant helix from plane curve. Also, we give some slant helix examples in $\mathbb{E}^3$.

2. Preliminaries

The Minkowski 3-space $\mathbb{E}_1^3$ is the Euclidean 3-space $\mathbb{E}^3$ equipped with indefinite at metric given by

$$g = -dx_1^2 + dx_2^2 + dx_3^2,$$

where (x_1, x_2, x_3) is a rectangular coordinate system of $\mathbb{E}_1^3$. Recall that a vector $v \in \mathbb{E}_1^3$ is called *spacelike* if $g(v, v) > 0$ or $v = 0$, *timelike* if $g(v, v) < 0$ and *null* (*lightlike*) if $g(v, v) = 0$ and $v \neq 0$. The norm of a vector v is given by $\|v\| = \sqrt{|g(v, v)|}$ and two vectors v and w are said to be orthogonal if $g(v, w) = 0$. An arbitrary curve $\alpha(s)$ in $\mathbb{E}_1^3$ can locally be *spacelike*, *timelike* or *null* (*lightlike*), if all its velocity vectors $\alpha'(s)$ are spacelike, timelike or null, respectively. Spacelike or a timelike curve α has unit speed, if $g(\alpha'(s), \alpha'(s)) = \pm 1$. A null curve α is parameterized by pseudo-arc s, if $g(\alpha''(s), \alpha''(s)) = 1$ ([10]). For a non-null unit speed space curve $\alpha(s)$ in the space $\mathbb{E}_1^3$ with non-null normals.

The Frenet frame $\{T, N, B\}$ of α and the curvatures κ, τ of α is given by

$$T(s) = \alpha'(s), \quad N(s) = \frac{\alpha''(s)}{\|\alpha''(s)\|}, \quad B(s) = T(s) \times N(s),$$

$$\kappa(s) = \varepsilon_1 g(T'(s)\ N(s)), \quad \tau(s) = \varepsilon_2 g(N'(s)\ B(s)).$$

The following Frenet formulae are given in [4, 5]

$$T'(s) = \kappa(s)N(s),$$

$$N'(s) = -\varepsilon_0\varepsilon_1\kappa(s)T(s) + \tau(s)B(s),$$

$$B'(s) = \varepsilon_1\varepsilon_2\tau(s)N(s) \tag{2.1}$$

and the Minkowski vector products of Frenet vectors are given as

$$T(s) \times N(s) = B(s),$$

$$N(s) \times B(s) = -\varepsilon_1 T(s),$$

$$B(s) \times T(s) = -\varepsilon_0 N(s),$$

where $g(T(s), T(s)) = \varepsilon_0 = \pm 1$, $g(N(s), N(s)) = \varepsilon_1 = \pm 1$ and $g(B(s), B(s)) = \varepsilon_2 = \pm 1$ and two ε_i's are equal to 1 the other ε_i is -1. It is well known that, the pseudo-Riemannian sphere with radius $r = 1$ and centered at origin is defined by

$$S_1^2 = \{p \in \mathbb{E}_1^3 : g(p, p) = 1\}, \tag{2.2}$$

the pseudohyperbolic space of radius $r = 1$ and centered at origin is defined by

$$H_0^2 = \{p \in \mathbb{E}_1^3 : g(p, p) = -1\}, \tag{2.3}$$

are the hyperquadrics with dimension 2 and index 1 and with dimension 2 and index 0, respectively, ([10]).

Definition 2.1. A unit speed curve $\tilde{\gamma}$ is called a *slant helix* if there exists a non-zero constant vector field U in E_1^3 such that the function $g(N(s), U)$ is constant [1].

It is important to point out that, in contrast to what happens in Euclidean space, in Minkowski ambient space we can not define the angle between two vectors (except that both vectors are of timelike type). For this reason, we avoid to say about the angle between the vector fields $N(s)$ and U [1].

3. Plane Curves and Slant Helix

In this section, we investigate the relationship between the plane curves and slant helices in E_1^3.

Theorem 3.1. *Let* γ *be a timelike curve on spacelike plane with unit spacelike normal vector. If* $\theta = -\frac{\|\gamma''\|\sqrt{\|\gamma'\|^2 - 1}}{g(\gamma', \gamma'')}$ *is constant, then*

$$\tilde{\gamma} = \gamma - \left(\tan\theta \int \frac{g(\gamma', \gamma'')}{\|\gamma''\|} dt\right)\vec{a} + \vec{c} \tag{3.1}$$

$\tilde{\gamma}$ *is a timelike slant helix, where* θ *is a constant and* $\vec{a}, \vec{c}$ *are constant vectors.*

Proof. Suppose that γ is a plane curve with Frenet frame $\{\vec{t}, \vec{n}\}$ and with curvature κ_p.

Differentiating equation (3.1), we get

$$\tilde{\gamma}' = \|\gamma'\|\vec{t} - \tan\theta\left(\frac{g(\gamma', \gamma'')}{\|\gamma''\|}\right)\vec{a},$$

$$\tilde{\gamma}'' = \|\gamma'\|'\vec{t} + \|\gamma'\|^2 \kappa_p \vec{n} - \frac{\|\gamma'\|\|\gamma'\|'}{\sqrt{\|\gamma'\|^2 - 1}}\vec{a}. \tag{3.2}$$

Since

$$\tan\theta = -\frac{\|\gamma''\|\sqrt{\|\gamma'\|^2 - 1}}{g(\gamma', \gamma'')}, \tag{3.3}$$

we can write

$$\sqrt{\|\gamma'\|^2 - 1} = -\frac{\tan\theta g(\gamma', \gamma'')}{\|\gamma''\|}. \tag{3.4}$$

By using equation (3.4) in equation (3.2), we obtain

$$\tilde{\gamma}'' = \|\gamma'\|' \vec{t} + \|\gamma'\|^2 \kappa_p \vec{n} + \cot\theta \|\gamma''\| \vec{a}.$$

Since γ is a timelike curve, $g(\vec{t}, \vec{t}) = -1$, $g(\vec{n}, \vec{n}) = 1$ and

$$g(\tilde{\gamma}', \tilde{\gamma}') = \|\gamma'\|^2 g(\vec{t}, \vec{t}) + (\|\gamma'\|^2 - 1) g(\vec{a}, \vec{a})$$
$$= -1,$$

i.e., $\tilde{\gamma}$ is a timelike curve. Then

$$T = \|\gamma'\| \vec{t} + (\|\gamma'\|^2 - 1)^{\frac{1}{2}} \vec{a}. \tag{3.5}$$

Also

$$g(\tilde{\gamma}'', \tilde{\gamma}'') = (\|\gamma'\|')^2 g(\vec{t}, \vec{t}) + \|\gamma'\|^4 \kappa_p^2 g(\vec{n}, \vec{n}) + \cot^2\theta \|\gamma''\|^2 g(\vec{a}, \vec{a})$$
$$= \frac{\|\gamma'\|^2}{\sin^2\theta}.$$

So

$$N = \frac{\sin\theta}{\|\gamma''\|} (\|\gamma'\|' \vec{t}' + \|\gamma'\|^2 \kappa_p \vec{n} + \cot\theta \|\gamma''\| \vec{a}) = \cos\theta = \text{constant} \tag{3.6}$$

and

$$B = \frac{\sin\theta}{\|\gamma''\|} \left(-\|\gamma'\|^2 (\|\gamma'\|^2 - 1)^{\frac{1}{2}} \kappa_p \vec{t} - \frac{\|\gamma''\| \cot\theta}{\|\gamma'\|} \vec{n} + \kappa_p \|\gamma'\|^3 \vec{a} \right). \tag{3.7}$$

We can easily see that $g(T, T) = -1$, $g(N, N) = 1$ and $g(B, B) = 1$, that is, T is a timelike vector, N is a spacelike vector and B is a spacelike vector. Moreover

$$\kappa = \frac{\|\gamma''\|}{\sin\theta},$$

$$\tau = \frac{\sqrt{\|\gamma'\|^2 - 1}}{\cos\theta}$$

and

$$\kappa_g = \cot\theta. \tag{3.8}$$

Consequently

$$\langle N, \vec{a} \rangle = \frac{\sin\theta}{\|\gamma'\|}\cot\theta\|\gamma'\| = \cos\theta = \text{constant}, \tag{3.9}$$

which means that $\tilde{\gamma}$ is a timelike slant helix.

Theorem 3.2. *Let* $\tilde{\gamma}$ *be a unit speed timelike curve. If* $\tilde{\gamma}$ *is a timelike slant helix in* E_1^3, *then the spherical image of the tangent indicatrix* (T) *of* $\tilde{\gamma}$ *is a spherical helix* [8].

Corollary 3.1. *We denote the curvatures of tangent indicatrix* (T) *of* $\tilde{\gamma}$ *generated by plane curve* γ *by* κ_1, τ_1.

$$\kappa_1 = \frac{\sin\theta}{\sqrt{\|\gamma'\|^2 - \cos^2\theta}} \tag{3.10}$$

and

$$\tau_1 = \frac{\cos\theta}{\sqrt{\|\gamma'\|^2 - \cos^2\theta}}. \tag{3.11}$$

Then

$$\frac{\tau_1}{\kappa_1} = \cot\theta.$$

Theorem 3.3. *Let* $\tilde{\gamma}$ *be a unit speed timelike curve. If* $\tilde{\gamma}$ *is a timelike slant helix in* E_1^3, *then the spherical image of the binormal indicatrix* (B) *of* $\tilde{\gamma}$ *is a spherical helix* [8].

Corollary 3.2. *We denote the curvatures of binormal indicatrix* (B) *of* $\tilde{\gamma}$ *generated by plane curve* γ *by* κ_2, τ_2. *Therefore*

$$\kappa_2 = \frac{\sin\theta}{\sqrt{\|\gamma'\|^2 - 1}} \tag{3.12}$$

and

$$\tau_2 = \frac{\cos\theta}{\sqrt{\|\gamma'\|^2 - 1}}. \tag{3.13}$$

Then

$$\frac{\tau_2}{\kappa_2} = \cot\theta.$$

Theorem 3.4. *Let* γ *be a spacelike curve on spacelike plane with unit timelike normal vector. If* $\tanh\theta = -\frac{\|\gamma''\|\sqrt{\|\gamma'\|^2 - 1}}{g(\gamma', \gamma'')}$ *is constant, then*

$$\tilde{\gamma} = \gamma - \left(\tanh\theta \int \frac{g(\gamma', \gamma'')}{\|\gamma''\|}\, dt\right)\vec{a} + \vec{c} \tag{3.14}$$

$\tilde{\gamma}$ *is a spacelike slant helix, where* θ *is a constant and* $\vec{a}$, $\vec{c}$ *are constant vectors.*

Proof. By using the method in Theorem 3.1, the proof of Theorem 3.4 is obvious.

Theorem 3.5. *Let* γ *be a timelike curve on timelike plane with unit spacelike normal vector.*

(a) *If* $\tan\theta = -\frac{\|\gamma''\|\sqrt{\|\gamma'\|^2 + 1}}{g(\gamma', \gamma'')}$ *is constant, then*

$$\tilde{\gamma} = \gamma - \left(\tan\theta \int \frac{g(\gamma', \gamma'')}{\|\gamma''\|}\, dt\right)\vec{a} + \vec{c} \tag{3.15}$$

$\tilde{\gamma}$ *is a unit speed spacelike curve with spacelike principal normal vector.*

(b) *If* $\coth\theta = -\frac{\|\gamma''\|\sqrt{\|\gamma'\|^2 + 1}}{g(\gamma', \gamma'')}$ *is constant, then*

$$\tilde{\gamma} = \gamma - \left(\coth\theta \int \frac{g(\gamma', \gamma'')}{\|\gamma''\|}\, dt\right)\vec{a} + \vec{c}. \tag{3.16}$$

$\tilde{\gamma}$ *is a unit speed spacelike curve with timelike principal normal vector*, *where* θ *is a constant and* $\vec{a}, \vec{c}$ *are constant vectors*.

Proof. By using the method in Theorem 3.1, the proof of Theorem 3.5 is obvious.

Theorem 3.6. *Let* γ *be a timelike curve on timelike plane with unit spacelike normal vector*.

(a) *If* $\tan\theta = -\dfrac{\|\gamma''\|\sqrt{1-\|\gamma'\|^2}}{g(\gamma', \gamma'')}$ *is constant*, *then*

$$\tilde{\gamma} = \gamma - \left(\tan\theta \int \frac{g(\gamma', \gamma'')}{\|\gamma''\|}\, dt\right)\vec{a} + \vec{c} \tag{3.17}$$

$\tilde{\gamma}$ *is a unit speed spacelike curve with spacelike principal normal vector*.

(b) *If* $\coth\theta = -\dfrac{\|\gamma''\|\sqrt{1-\|\gamma'\|^2}}{g(\gamma', \gamma'')}$ *is constant*, *then*

$$\tilde{\gamma} = \gamma - \left(\coth\theta \int \frac{g(\gamma', \gamma'')}{\|\gamma''\|}\, dt\right)\vec{a} + \vec{c} \tag{3.18}$$

$\tilde{\gamma}$ *is a unit speed spacelike curve with timelike principal normal vector*, *where* θ *is a constant and* $\vec{a}, \vec{c}$ *are constant vectors*.

Proof. By using the method in Theorem 3.1, the proof of Theorem 3.6 is obvious.

Theorem 3.7. *Let* $\tilde{\gamma}$ *be a unit speed spacelike curve*. *If* $\tilde{\gamma}$ *is a spacelike slant helix in* E_1^3, *then the spherical image of the tangent indicatrix* (T) *of* $\tilde{\gamma}$ *is a spherical helix* [1].

Theorem 3.8. *Let* $\tilde{\gamma}$ *be a unit speed spacelike curve*. *If* $\tilde{\gamma}$ *is a spacelike slant helix in* E_1^3, *then the spherical image of the binormal indicatrix* (B) *of* $\tilde{\gamma}$ *is a spherical helix* [1].

4. Examples

In this section, we give an example of timelike and spacelike slant helix in Minkowski 3-space and draw its pictures and its tangent indicatrix, and binormal indicatrix by using Mathematica.

Let γ be a timelike curve on timelike plane with unit spacelike normal vector. γ is defined by

$$\gamma(t) = \left(2\sinh t + \frac{1}{2}\sinh(2t), -2\cosh t + \frac{1}{2}\cosh(2t), 0\right) \tag{4.1}$$

and is rendered in Figure 1. Curves $\tilde{\gamma}$ generated by timelike curve γ on timelike plane with unit spacelike normal vector is a timelike slant helix, which is rendered in Figure 1. Timelike slant helix is obtained as

$$\tilde{\gamma}(t) = \left(2\sinh t + \frac{1}{2}\sinh(2t), -2\cosh t + \frac{1}{2}\cosh(2t), -\frac{4\sqrt{2}}{3}\sinh\left(\frac{3t}{2}\right)\right)$$

and its tangent indicatrix, principal normal indicatrix and binormal indicatrix, respectively, are

$$T(t) = \left(2\cosh t + \cosh(2t), -2\sinh t + \sinh(2t), -2\sqrt{2}\cosh\left(\frac{3t}{2}\right)\right),$$

$$N(t) = \left(2\sqrt{2}\cosh\left(\frac{t}{2}\right), 2\sqrt{2}\sinh\left(\frac{t}{2}\right), -3\right),$$

$$B(t) = \left(2\sinh t + \sinh(2t), 2\cosh t - \cosh(2t), 2\sqrt{2}\sinh\left(\frac{3t}{2}\right)\right)$$

and are rendered in Figure 2.

Let γ be a spacelike curve on spacelike plane with unit timelike normal vector. γ is defined by

$$\gamma(t) = \left(0, \frac{4}{3}\sin t + \frac{1}{12}\sin(4t), -\frac{4}{3}\cos t + \frac{1}{12}\cos(4t)\right) \tag{4.2}$$

and is rendered in Figure 3. Curves $\tilde{\gamma}$ generated by spacelike curve γ on spacelike plane with unit timelike normal vector is spacelike slant helix, which is rendered in Figure 3. The spacelike slant helix is obtained as

$$\tilde{\gamma}(t) = \left(-\frac{8}{15}\sin\left(\frac{5t}{2}\right), \frac{4}{3}\sin t + \frac{1}{12}\sin(4t), -\frac{4}{3}\cos t + \frac{1}{12}\cos(4t) \right)$$

and its tangent indicatrix, principal normal indicatrix and binormal indicatrix, respectively, are

$$T(t) = \left(-\frac{4}{3}\cos\left(\frac{5t}{2}\right), \frac{4}{3}\sin t - \frac{1}{3}\sin(4t), \frac{4}{3}\cos t + \frac{1}{3}\cos(4t) \right),$$

$$N(t) = \left(\frac{5}{3}, -\frac{4}{3}\cos\left(\frac{3t}{2}\right), \frac{4}{3}\sin\left(\frac{3t}{2}\right) \right),$$

$$B(t) = \left(\frac{4}{3}\sin\left(\frac{5t}{2}\right), \frac{1}{3}(4\sin t + \sin(4t)), \frac{1}{3}(-4\cos t + \cos(4t)), \right)$$

and are rendered in Figure 4.

Let γ be a timelike curve on timelike plane with unit spacelike normal vector. γ is defined by

$$\gamma(t) = \left(2\cosh t + \frac{1}{2}\cosh(2t), 2\sinh t - \frac{1}{2}\sinh(2t), 0 \right) \tag{4.3}$$

and is rendered in Figure 5. If $5 > 4\cosh[3t]$, curves $\tilde{\gamma}$ generated by timelike curve γ on timelike plane with unit spacelike normal vector is spacelike slant helix with spacelike principal normal vector, which is rendered in Figure 5. The spacelike slant helix is obtained as

$$\tilde{\gamma}(t) = \left(2\cosh t + \frac{1}{2}\cosh(2t), 2\sinh t - \frac{1}{2}\sinh(2t), -\frac{4\sqrt{2}}{3}\cosh\left(\frac{3t}{2}\right) \right)$$

and its tangent indicatrix, principal normal indicatrix and binormal indicatrix, respectively, are

$$T(t) = \left(2\sinh t + \sinh(2t), 2\cosh t - \cosh(2t), -2\sqrt{2}\sinh\left(\frac{3t}{2}\right) \right),$$

$$N(t) = \left(2\sqrt{2}\cosh\left(\frac{t}{2}\right), -2\sqrt{2}\sinh\left(\frac{t}{2}\right), -3 \right),$$

$$B(t) = \left(-2\cosh t - \cosh(2t), 2\sinh t - \sinh(2t), -2\sqrt{2}\cosh\left(\frac{3t}{2}\right) \right)$$

and are rendered in Figure 6.

Let γ be a spacelike curve on timelike plane with unit spacelike normal vector. γ is defined by

$$\gamma(t)\left(-2\cosh t - \frac{1}{2}\cosh(2t),\ 2\sinh t - \frac{1}{2}\sinh(2t),\ 0\right) \tag{4.4}$$

and is rendered in Figure 7. If $5 < 4\cosh[3t]$, curves $\tilde{\gamma}$ generated by spacelike curve γ on timelike plane with unit spacelike normal vector is spacelike slant helix with spacelike principal normal vector, which is rendered in Figure 7. The spacelike slant helix is obtained as

$$\tilde{\gamma}(t) = \left(2\cosh t + \frac{1}{2}\cosh(2t),\ 2\sinh t - \frac{1}{2}\sinh(2t),\ -\frac{4\sqrt{2}}{3}\cosh\left(\frac{3t}{2}\right)\right)$$

and its tangent indicatrix, principal normal indicatrix and binormal indicatrix, respectively, are

$$T(t) = \left(2\sinh t + \sinh(2t),\ 2\cosh t - \cosh(2t),\ -2\sqrt{2}\sinh\left(\frac{3t}{2}\right)\right),$$

$$N(t) = \left(2\sqrt{2}\cosh\left(\frac{t}{2}\right),\ -2\sqrt{2}\sinh\left(\frac{t}{2}\right),\ -3\right),$$

$$B(t) = \left(-2\cosh t - \cosh(2t),\ 2\sin t - \sinh(2t),\ -2\sqrt{2}\cosh\left(\frac{3t}{2}\right)\right)$$

and are rendered in Figure 8.

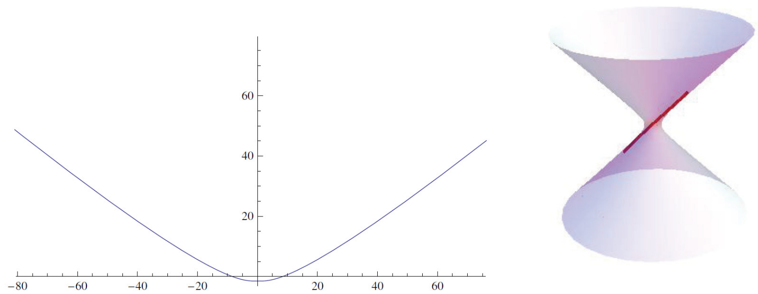

Figure 1. Timelike curve γ on timelike plane and $\tilde{\gamma}$ generated by the timelike curve.

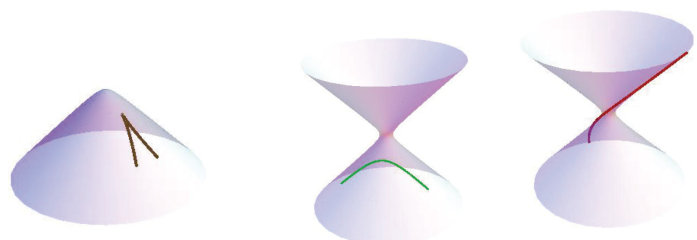

Figure 2. Tangent indicatricix T, principal normal indicatrix N of the timelike slant helix lie on S_1^2 and binormal indicatrix B of the timelike slant helix lies on H_0^2.

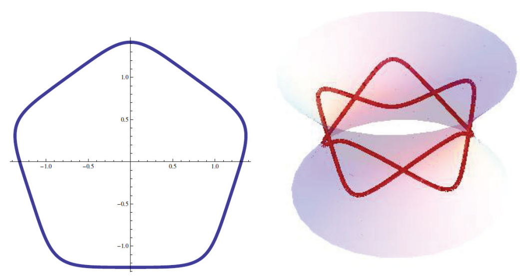

Figure 3. Spacelike curve γ on spacelike plane and $\tilde{\gamma}$ generated by the spacelike curve.

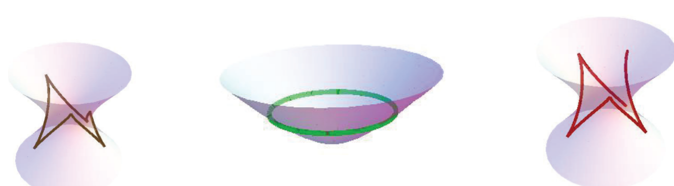

Figure 4. Tangent indicatrix T, binormal indicatrix B of the timelike slant helix lie on S_1^2 and principal normal indicatrix N of the timelike slant helix lies on H_0^2.

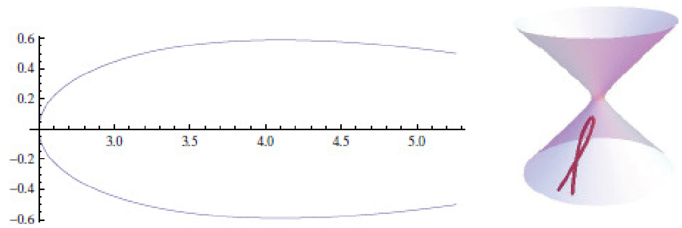

Figure 5. Timelike curve γ on timelike plane and $\tilde{\gamma}$ generated by the spacelike curve.

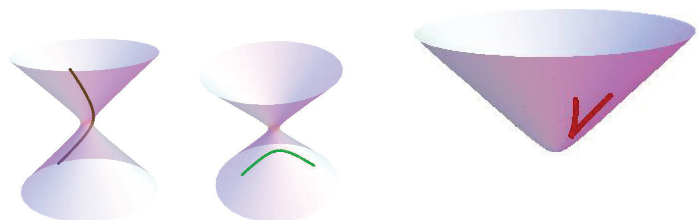

Figure 6. Tangent indicatrix T, principal normal indicatrix N of the timelike slant helix lie on S_1^2 and binormal indicatrix B of the timelike slant helix lies on H_0^2.

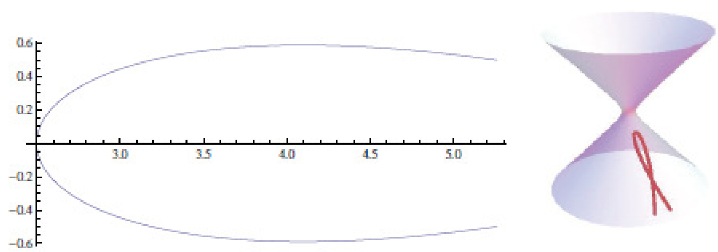

Figure 7. Timelike curve γ on timelike plane and $\tilde{\gamma}$ generated by the spacelike curve.

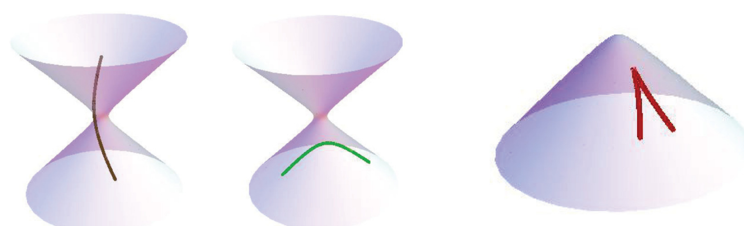

Figure 8. Tangent indicatrix T, principal normal indicatrix N of the timelike slant helix lie on S_1^2 and binormal indicatrix B of the timelike slant helix lies on H_0^2.

References

[1] T. Ali Ahmad and R. López, Slant helices in Minkowski space $\mathbb{E}_1^3$, J. Korean Math. Soc. 48(1) (2011), 159-167.

[2] M. Altinok, The characterization of some special space curves with plane curves, Master thesis, Ahi Evran University, Graduate School of Natural and Applied Sciences, 2011.

[3] M. Altinok and L. Kula, Slant helix generated by plane curve, to appear.

[4] N. Ekmekci and K. İlarslan, Null general helices and submanifolds, Bol. Soc. Mat. Mexicana 9(2) (2003), 279-286.

[5] K. İlarslan, Some special curves on non-Euclidean manifolds, Doctoral thesis, Ankara University, Graduate School of Natural and Applied Sciences, 2002.

[6] S. Izumiya and N. Takeuchi, Generic properties of helices and Bertrand curves, J. Geometry 74 (2002), 97-109.

[7] S. Izumiya and N. Takeuchi, New special curves and developable surfaces, Turkish J. Math. 28 (2004), 153-163.

[8] L. Kula and Y. Yayli, On slant helix and its spherical indicatrix, Appl. Math. Comput. 169 (2005), 600-607.

[9] L. Kula, N. Ekmekci, Y. Yayli and K. Ilarslan, Characterizations of slant helices in Euclidean 3-space, Turkish J. Math. 34 (2010), 261-274.

[10] B. O'Neill, Semi-Riemannian Geometry with Applications to Relativity, Academic Press, New York, 1983.