Fundamental Journal of Modern Physics
ISSN: 2249-9768
Vol. 11, Issue 2, 2018, Pages 181-189
This paper is available online at http://www.frdint.com/
Published online December 12, 2018

SCALING CHARACTERISTICS OF NON-STATIONARY MARKOVIAN REPLICATION PROCESSES WITH HARMONIC REPLICATION PROBABILITY

HYUN-JOO KIM

Department of Physics Education
Korea National University of Education
Chungbuk 363-791
Korea
e-mail: hjkim21@knue.ac.kr

Abstract

We have studied the characteristics of diffusions generated by the non-stationary Markovian replication process with harmonic replication probability. We have found that the mean-squared displacement depends on the time as well as the period of the harmonic replication probability, that is, at the early stage, the MSD grows with time while the MSD is saturated after a crossover time which increases as a power of the period, and it has been described by a scaling function. Also, we have measured the distribution of persistent length for various periods of replication probability. For small size of persistent length, the distribution is constant while for large size, it decays as power law. The crossover size increases as a power of the period and the rescaled distributions collapse into a single curve by scaling function.

Keywords and phrases: random walk, anomalous diffusion, stochastic process.

Received September 26, 2018; Accepted October 11, 2018

1. Introduction

The Brownian motion is a prototype of the diffusive phenomena. It has been modelized by a simple stochastic model, random walk which is characterized by the mean-squared displacement (MSD) growing with time linearly [1]. In recent, anomalous diffusions deviating from the normal diffusion has been received much attention in statistical physics as well as various movements of humans and animals and even bacteria [1-5]. The anomalous diffusions is characterized by the MSD $M(t)$ that is not linearly dependent on time unlike the normal diffusion and follows power-law behavior [6, 7]

$$M(t) = \langle x(t)^2 \rangle \sim t^{2H}, \tag{1}$$

where H is the Hurst exponent. For $H > 1/2,$ persistence in a diffusion exists and is called superdiffusion, while for $H < 1/2,$ anti-persistence is exhibited and is called subdiffusion.

Recently, the non-stationary Markovian replication process (NMRP) was been introduced to describe the non-stationary diffusions affected by complex temporal stimuli coming from natural environments [8]. In the NMRP, a step just replicates or anti-replicates the latest step, at each time t with a replication probability which step process is Markovian, that is, the next step only is affected by the current step. In the meanwhile, allowing a replication probability varying with time by which the degree of replication or anti-replication is controlled results in long-term correlations between steps and describe a variety of diffusions with different replication probabilities [8, 9].

Given the oscillatory replication probability, the MSD has been shown the step-like behavior [8], which is different from the anomalous diffusion with power-law behavior of the MSD. When the period of the replication probability is a half of the total number of steps, the MSD is changed from the ballistic diffusion to the saturated phase, that is, the phase of diffusion is changed [10]. The phase change of movement is important to the intermittent search strategy [11] and run-tumble motions of bacteria [5]. Therefore it is meaningful to study the characteristics of the phase changing diffusions using the NMRP with oscillatory replication probability in detail. In this paper, we have considered the harmonic replication probability with various periods. We have measured the MSDs and the persistent length distribution for various period and found that they show the scaling behavior with the periods.

2. Model and Results

In one dimension, the NMRP model is defined as a stochastic process of which dynamics is controlled by the time-varying probability $\alpha(t)$, we call it the replication probability as follows,

$$\sigma_t = \begin{cases} \sigma_{t-\tau}, & \text{with probability } \alpha(t), \\ -\sigma_{t-\tau}, & \text{with probability } 1-\alpha(t), \end{cases} \tag{2}$$

where l is the uniform spacing between neighboring sites in one dimensional lattice and τ is the time interval, a walker stochastically moves to one of the two neighboring sites. The relation between the walker's position at time t, x_t, and the step a walker takes, σ_t, is defined as $x(t) = x(t-\tau) + \sigma_t$. The first step (σ_τ) is randomly chosen between the two possibilities $\pm l$ with equal probabilities. Successive steps at $t \geq 2\tau$ are determined by the rule (2).

We have used the replication probability of $\alpha(t) = \cos^2(\pi t / TN)$, where N is the number of total steps and T is a parameter controlling the period that is given as TN. Figure 1(a) shows $\alpha(t)$ for the various parameters $T = 2, 4, 8, 16$, and 32 and the corresponding MSDs are shown in (b). The MSDs grow with time initially and then are saturated to constant values that become larger when T is larger. In the (c), we measured the saturated values of MSDs as a function of the parameter T and found that the saturated MSD scales as $M_{\text{sat}} \sim T^{\mu}$ with $\mu \approx 1.3$. Thus the MSD behaves as

$$M(t, T) = \begin{cases} t^{2H}, & t \ll t_c, \\ T^{\mu}, & t \gg t_c, \end{cases} \tag{3}$$

where t_c is a crossover time separating two phases and the dashed line represents that $H = 1$. As shown in the Figure 1(d) the crossover time t_c depends on the parameter T as

$$t_c \sim T^{\kappa}, \tag{4}$$

and the fitting line represents that $\kappa \approx 0.65$. If we rescale the time and the MSD with appropriate quantities, the various curves of the MSDs will collapse a curve and we can find a scaling function as follows,

$$M(t, T) = T^{\mu} f\left(\frac{t}{T^{\kappa}}\right), \tag{5}$$

where the scaling function $f(x)$ is given by

$$f(x) = \begin{cases} x^{2H}, & x \ll 1, \\ \text{const.}, & x \gg 1. \end{cases} \tag{6}$$

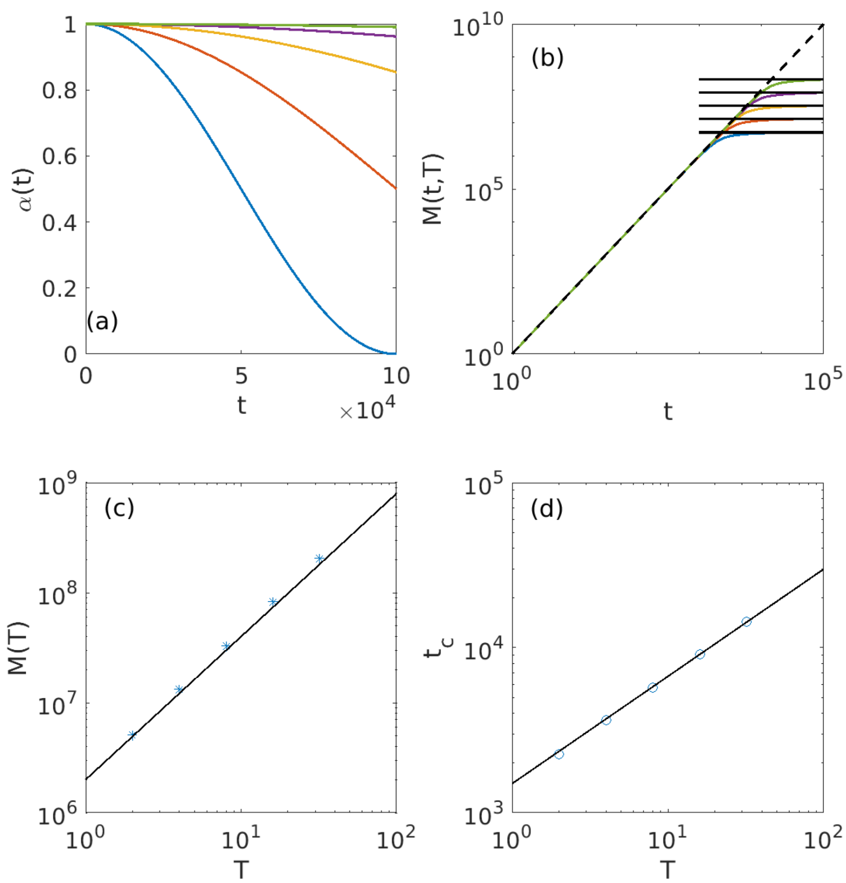

Figure 1. (a) Plot of the replication probability $\alpha(t)$ for various parameters T with values 2, 4, 8, 16, and 32 from the bottom to the top. (b) The MSDs $M(t, T)$ versus time t for $T = 2$, 4, 8, 16, and 32 from the bottom to the top. (c) The saturated MSD $M(T)$ versus the parameter T. The solid line represents that the MSD follows power-law behavior, $M(t) \sim T^{1.3}$. (d) The crossover time t_c versus the parameter T. t_c behaves as $t_c \sim T^{0.65}$.

The exponents μ and κ are not independent. As shown in the Figure 1(b) if we approach the crossover point $(t_c M(t_c))$ from the left, the MSD becomes $M(t_c) \sim t_c^2$ according to the Equation (3), while approaching the same point from the right we have $M(t_c) \sim T^{\mu}$. Therefore from the relation $t_c^2 \sim T^{2\kappa} \sim T^{\mu}$ at the crossover point, we obtain the relation between the two exponents,

$$\mu = 2\kappa, \tag{7}$$

which is excellent agreement in the relation between values obtained by this simulations. Figure 2 shows the collapse of the MSDs with the various parameter T into a single curve by the scaling function of Equation (6).

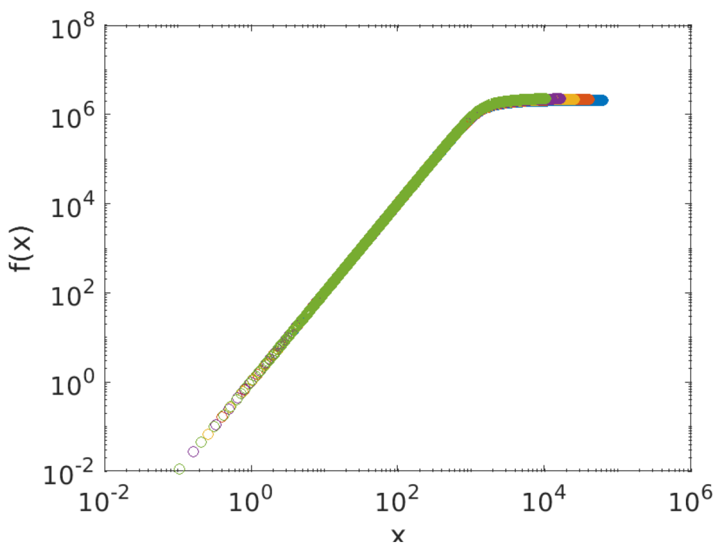

Figure 2. Plots of the scaling function $f(x)$ for various parameters. The rescaled MSDs collapse into a single curve.

The distribution $P(s)$ of the persistent length s that is the number of the successive steps into the same direction is another quantity being able to classify the characteristics of diffusions. We have also measured the $P(s)$ with the various values of T, and studied the scaling behavior of it. Figure 3(a) is the plot of the persistent length distribution $P(s)$ versus the persistent length s with $T = 2$, 4, 8, 16, and 32 from the top to the bottom. The slope of the dashed line is –2.5, which means that the persistent length distribution follows the power-law behavior $P(s) \sim s^{-\beta}$ with $\beta \approx 2.5$ for large s. While for small s, they have constant values. It indicates that persistent lengths of large size exist in the NMRP unlike the normal diffusions where there is no large persistent length [12]. Also, the number of small size of the persistent length decreases for large T. Figure 3(b) shows the plot of the constant persistent length distribution versus T and the slope of the solid line is 3.87.

Therefore the distribution of the persistent length $P(s)$ is given by

$$p(s,\ T) = \begin{cases} T^{\nu}, & s \ll s_c, \\ s^{\beta}, & s \gg s_c, \end{cases} \tag{8}$$

where the value of ν is $\nu \approx -3.87$ and s_c is the crossover length that follows power-law behavior $s_c = T^{\phi}$ with $\phi \approx 1.94$, which is shown in the Figure 3(c). The distribution can be also represented by a scaling function

$$p(s,\ T) = T^{\nu} g\left(\frac{s}{T^{\phi}}\right), \tag{9}$$

where the scaling function is given by

$$g(x) = \begin{cases} \text{const.}, & x \ll 1, \\ x^{\beta} & x \gg 1. \end{cases} \tag{10}$$

Figure 3(d) shows the collapse of the rescaled distribution for various T into a single curve with the scaling function of Equation (10) and thus confirm the scaling property of the diffusive process.

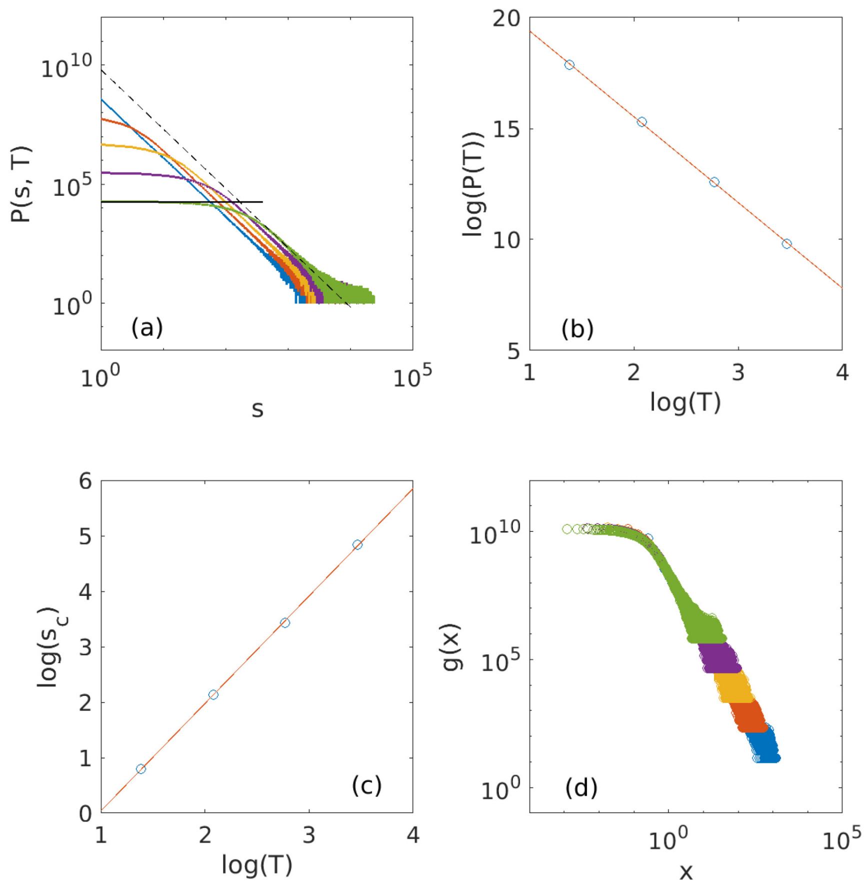

Figure 3. (a) Plot of the distribution of the persistent length $P(s, T)$ versus the persistent length s for parameters $T = 2, 4, 8,$ 16, and 32 from the top to the bottom. (b) The constant $P(T)$ versus T. The solid line represents that $P(T) \sim T^{-3.87}$. (c) The crossover size s_c versus T. s_c follows power-law behavior $s_c \sim T^{1.94}$. (d) The scaling function $g(x)$ as a function of $x = s / s_c$.

3. Summary

In summary, we have studied the scaling properties of diffusive behaviors changing phases from growing with time to the saturated phase using the NMRP with the harmonic replication probabilities having the different periods. We have found that the larger period, the larger saturated MSD and the saturated MSD follows power-law behavior with the period. Also the crossover time follows power-law behavior with the period. The distribution of persistent length depends on the period showing from initially constant behavior to the power-law behavior after the crossover size which follows the power-law behavior with the parameter.

References

[1] B. D. Hughes, Random Walks and Random Environments, Oxford Science, New York, 1995.

[2] D. Brockmann, L. Hufnagel and T. Geisel, Nature (2006), 439-462.

[3] M. C. Gonzalez, C. A. Hidalgo and A.-L. Barabási, Nature (2008), 453-779.

[4] L. Alessandretti, P. Sapiezynski, S. Lehmann and A. Baronchelli, PloS one 12 (2017), e0171686.

[5] H. C. Berg, E. coli in Motion ,Springer-Verlag, NewYork, 2004.

[6] R. Metzler and J. Klafter, Phys. Rep. 339 (2000), 1; R. Metzler and J. Klafter, J. Phys. A: Math. Gen. 37 (2004), R161.

[7] J. Klafter and I. M. Sokolov, First Steps in Random Walks, Oxford University Press, New York, 2011.

[8] Y. Choi and H.-J. Kim, Phys. Rev. E 96 (2017), 042144.

[9] H.-J. Kim, Phys. Rev. E 90 (2014), 012103.

[10] H.-J. Kim, Fundamental J. Modern Physics 9(2) (2016), 137.

[11] O. Bénichou, C. Loverdo, M. Moreau and R. Voituriez, Rev. Mod. Phys. 83 (2011), 137.

[12] D. Kim and H.-J. Kim, Fundamental J. Modern Physics 10(2) (2017), 143.