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REPULSIVE GRAVITATION, GENERAL RELATIVITY, AND ERRORS OF EINSTEIN

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Abstract

Galileo and Newton considered gravity to be independent of temperature, while Einstein claimed that gravity will increase as temperature increases. Further, Maxwell maintained that charge is unrelated to gravity. Experiments show, however, that the weight of a metal piece is reduced as its temperature increases. Thus, charge-initiated repulsive gravitation exists, and Galileo, Newton, Einstein and Maxwell are incorrect. Einstein's thought experiment for this case is misleading because of implicit assumptions. In fact, repulsive gravity has been demonstrated by the use of a charged capacitor hovering over Earth. Further, it is expected that a piece of heated metal would fall more slowly than a feather in a vacuum. Einstein developed an invalid notion of gravitational mass, and since he overlooked repulsive gravitation, he failed to establish the unification of gravitation and electromagnetism. Moreover, general relativity must be extended to a five-dimensional theory, where photons are a combination of the gravitational wave and the electromagnetic wave. Current space-time

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singularity theorems are based on an invalid physical assumption that all the couplings have a unique sign. For electromagnetic energy E, $E = mc^2$ is invalid, and is inconsistent with the Einstein equation. The non-linear Einstein equation has no bounded dynamic solution. The positive mass theorem of Schoen and Yau is misleading since their requirements cannot be satisfied by any dynamic problem, even a two-body problem. Thus, Yau was awarded a Fields Medal largely because the Fields Medal mathematicians and physicists did not sufficiently understand general relativity. Similarly, Penrose was awarded the 2020 Nobel Prize in Physics because the Nobel Committee did not sufficiently understand the physics of general relativity. A distinct characteristic of his work is that it is not verifiable.

1. Introduction

Einstein is considered a genius of 20th century physics. His credits are undeniably impressive. His paper, "*A New Determination of Molecular Dimensions*", indeed gives a correct method, and the paper "*On the Motion of Small Particles Suspended in Liquids at Best Required by the Molecular-Kinetic Theory of Heat*" gives the mean free path of such particles [1].

Moreover, his theory of special relativity is almost perfect,(1) although his conclusion of $E = mc^2$ is not always valid for any energy E [2]. However, his notion of photons [3] is only partially correct because the energy of photons would include the energy of the related gravitational wave [4, 5]. Although his theory of general relativity changed our view of the physical world, it is also general relativity that exposed some of Einstein's limitations in the field of physics [4, 5]. Nevertheless, it is true that because of Einstein, physics advanced and would never be the same again [1]. While he is the great physicist who opened the door to modern physics for us, he did, nevertheless, make certain mistakes.

First, although he gained confidence in his equation from obtaining the remaining perihelion of Mercury [6], he failed to justify it with a necessary perturbation approach.(2) Thus, the belief that general relativity had three kinds of strong supporting evidence is not exactly correct. Since the perihelion of Mercury cannot be counted as such, only two kinds, the bending of light rays and the gravitational redshifts, are valid.

In 1916, when general relativity was published, very few people had a good understanding of the theory. In fact, when Eddington was informed that there were three scientists who understood general relativity, his response [6] was to ask who the third person was. Now, even after more than 100 years have passed, there is still no-one, including Einstein, who has fully understood general relativity. Interestingly, it was an inaccurate comment made by Einstein himself [7] that led to this observation.

In 1946, Einstein [7] explained $E = mc^2$ to Science Illustrated by pointing out that a piece of metal would have an increased weight when its temperature increases, although it would be too small to measure because of the large factor c^2. Now, however, technology has improved such that we can measure the difference. To our surprise, the metal actually has a reduced weight when heated [8-10].(3) Thus, there is clearly an error in Einstein's theory. In this paper, we offer an analysis of Einstein's miscalculations.

It turns out that his central error is in overlooking the existence of repulsive gravitation [11]. This is also why he failed to achieve one of his principal goals, the unification of gravitation and electromagnetism [12]. His notion of gravitational mass was a source of his theoretical mistake, even though he was puzzled about why the inertial mass and the gravitational mass cannot be distinguished, since they are from very different origins in physics [13, 14].

While the errors related to the existence of repulsive gravitation are easier to identify, the errors related to the nonexistence of dynamic solutions are more difficult [15]. These errors are due to certain mathematical weaknesses in the work of Einstein and his followers [16].(4) Moreover, mathematicians such as S. T. Yau [17] and E. Witten [18], who are less schooled in physics,(5) have mistakenly suggested that the Einstein equation has dynamic solutions. Yet to date, they have not provided an explicit supporting example to support their claims [19].

Einstein [6] failed to see that one cannot produce the necessary perturbation approach to justify his calculation for the remaining perihelion of Mercury. This is perhaps the reason that D. Hilbert [6] gave all the credit for the field equation in general relativity to Einstein. It would seem that Hilbert, a distinguished mathematician, understood the errors that escaped the notice of other mathematicians, including those presiding over the Fields Medals [20].

In general relativity, it is often the case that what are considered important new results, actually stem from new, yet flawed conclusions.(6) In physics, the existence of a singularity in a solution is a clear sign of error. However, the space-time singularity theorems [21] were used by many to justify the development of the big bang theory of an expanding universe, and the existence of black holes. This was developed by assuming, incorrectly, that gravity is always attractive [22], and is based on an implicit, incorrect assumption that all the coupling constants have the same sign [23].

Moreover, the space-time singularity theorems are used by Hawking and Penrose to claim, incorrectly, that general relativity is unsuitable for explaining microscopic phenomena.(7) The Hubble law was misinterpreted, against Hubble's wishes [24], as evidence of the expanding universe by using an incorrect interpretation of the Doppler effects [25]. This was accomplished by using a measurement that leads to incorrect light speeds [25].

A great deal of confusion arose from the inability of physicists to see that the linearization of the Einstein equation is incompatible with physics [26].(8) In fact, for the dynamic cases, the linearized equation and the non-linear Einstein equation are independent equations [26]. It turns out that to have a dynamic solution for the non-linear Einstein equation with massive sources, one must add an energy momentum tensor with an anti-gravity coupling [15], as Lorentz [27] and Levi-Civita [28] have suggested. However, many simply accept Einstein's work without pursuing sufficient analysis.

Rectifying such errors is a difficult task, since many of the errors originated from top theorists in general relativity, including Einstein and theorists from Princeton, Harvard, and Cambridge.(9) Entrenched views that Einstein could not make errors in classical physics makes it difficult to find those who would accept theoretical challenges to Einstein. Thus, it is necessary to establish that Einstein has been wrong in classical physics, which we can do first by demonstrating the existence of the repulsive gravitation.

Moreover, Einstein's equivalence principle is valid only in the absence of repulsive gravitation. The covariance principle is invalid, as shown by explicit examples [29]. Einstein's justification for it was based on an invalid application of special relativity [30]. In short, for a dynamic situation, what has been derived from the linearized equation is correct, but what has been derived from the non-linear Einstein equation is questionable.

This paper seeks to provide a starting point to correct some of the errors of Einstein.

2. The Reduction of Gravitation as the Temperature Increases

Physics is based on experiments. Einstein's theory was accepted because the bending of light was experimentally confirmed. His theoretical shortcoming, however, has now been confirmed because the weight of a metal piece is actually reduced as temperature increases [8-10].

In physics, a theoretical conclusion might not be considered valid until supported by experiments because implicit assumptions might unintentionally have been used, as in, for example, Einstein's thought experiments. Thus, Einstein actually deviated from the teachings of Galileo on the importance of corroborative experiments. For example, an implicit assumption in the space-time singularity theorems is that all the coupling constants have the same sign. Such an assumption has recently been confirmed invalid for the photonic case [4, 5] in agreement with the case with a massive source that also has different coupling signs [15].

Einstein claimed that $E = mc^2$ means that a piece of heated-up metal would have an increment of weight [7]. He reasoned that if an increment of energy for matter implies an increment of mass, this will result in the increment of weight. Therefore, $E = mc^2$ is invalid if one can show that an increment of energy would reduce weight.

In 2003, Dmitriev, Nikushchenko and Snegov [8] established that a piece of heated-up brass has reduced weight. Their results can be shown in the following figures.

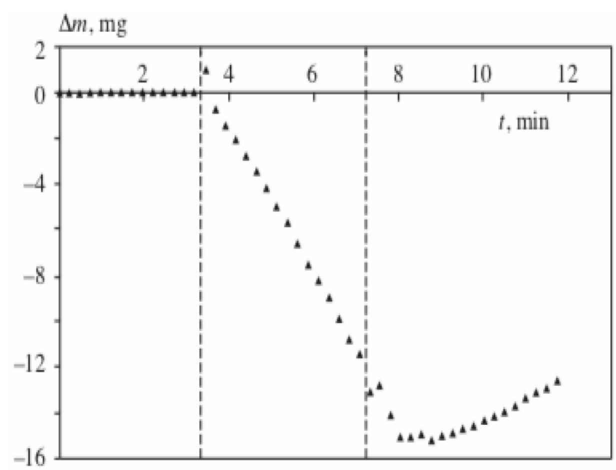

Figure 1. Change in mass of a brass rod mounted in an open holder. Ultrasound frequency 131.25 kHz. The dashed lines indicate the moments when the ultrasound was switched on and off.

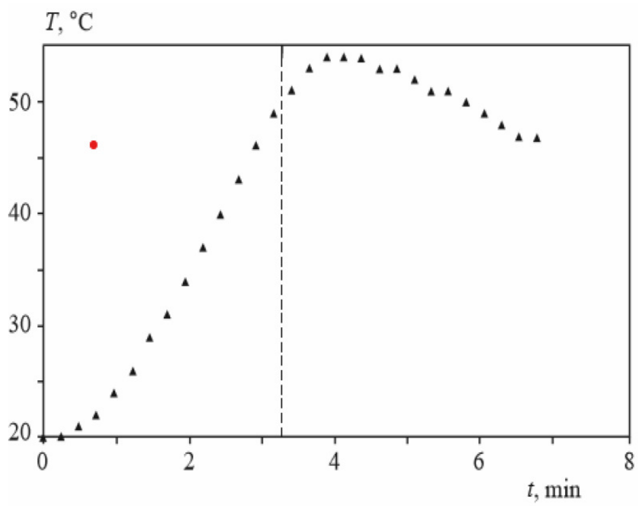

Figure 2. Time dependence of the temperature of a part of the surface of an ultrasonically heated brass rod (open holder). Ultrasound frequency 131.28 kHz. The dashed line indicates the moment when the ultrasound was switched off.

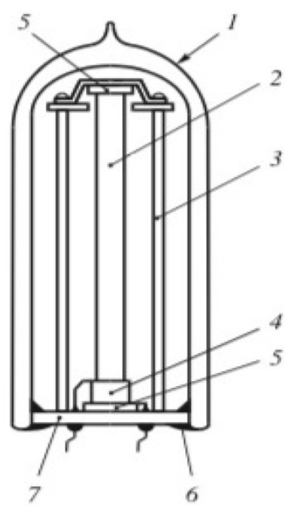

Figure 3. Arrangement of the air-tight container: (1) Dewar vessel; (2) Metal rod; (3) Holder pillar (textolite cloth-based laminate); (4) Piezoelectric transducer; (5) Foam plastic spacers; (6) Cold weld; (7) Holder base (ebonite).

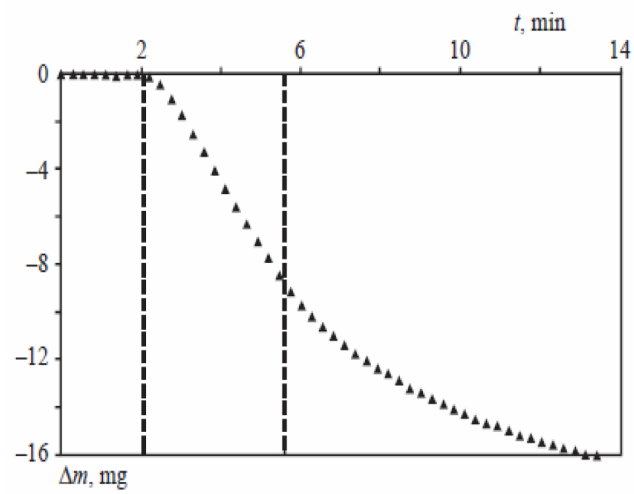

Figure 4. Change in mass of a brass rod mounted in a closed Dewar vessel. Ultrasound frequency 131.27 kHz. The dashed lines indicate the moments when the ultrasound was switched on and off.

Figure 1 shows the change of weight for the brass rod mounted in an open holder. Figure 2 shows the time dependence of the temperature of a part of the surface of an ultrasonically heated brass rod (open holder). Figure 3 shows the arrangement in an air-tight container. Figure 4 shows the change of weight for the brass rod in a closed Dewar vessel, which controls for the influence of outside heat. The brass rod weighed $58.5\,g$ before heating, with a length of $140.0\,mm$, and a diameter of $8.0\,mm$. These figures show that the Dewar vessel is not essential for the weight reduction experiment.

Dmitriev et al. [8] are confident that their observed results, the reduction of weight as temperature increases, is correct. They point out, "It is well known that the temperature regimes play an important role when weighing with high accuracy. The basic reasons for temperature influencing the results of such measurements are thermal expansion of the bodies, temperature changes in the magnetization of the weighed sample, adsorption of moisture by the surface of the sample (a change in the buoyancy), thermal convection of the air near the surface of the sample, the influence of the heated sample on the balance mechanism (through thermal radiation, heat conduction, or convection). These factors are quite well known in modern measurement technology and their contribution to the results of measuring the mass of samples can be estimated quantitatively."

It should also be noted that the temperature dependence of gravity also depends on the metal involved. Dmitriev et al. have measured such dependencies for lead, copper, brass, and duralumin, and found they are different. It would be interesting to find out the detailed rules for such dependencies. In 2010, Fan, Feng Jinsong and Liu [9] confirmed, using an electronic scale, that six kinds of metal have reduced weight after being heated.

Moreover, it has been verified by Lo [10], using a torsion balance scale, that lead balls have reduced gravitation after being heated. This confirms that it is, in fact, gravity that has been changed. In addition, a charged capacitor and a charged metal ball also have reduced weight [11]. Thus, an increase in electromagnetic energy need not mean an increase in mass, and thus also the weight.

3. The Inertial Mass and Einstein's Invalid Gravitational Mass

Although Dmitriev et al. [8] and Fan et al. [9] have shown that a piece of heated-up brass has reduced weight, they mistakenly believed that these experiments demonstrated the reduction of mass. It has been firmly established, however, from the atomic bomb, that mass is equivalent to energy [31]. Therefore, they needed to explain what became of the lost mass. As a result, their findings were incorrectly rejected by many physicists as errors.

Since physics is based on experiments, we must be able to explain the experiments consistently. According to experiments, although heat would increase energy, the increase of energy need not mean the increase of gravity [11, 12]. Apparently, some physicists do not understand that if repulsive gravitation is present, measuring mass through gravitation is no longer valid. Moreover, $E = mc^2$ is not generally valid since the electromagnetic energy is not equivalent to mass. Since David Gross won his Nobel Prize based on the general validity of $E = mc^2$ [32], proof for asymptotic freedom for the strong interaction is still incomplete.

As Einstein pointed out, the inertial mass is related to the resistance to acceleration, and gravitational mass is related to the attraction to a mass. Thus, acceleration mass and gravitational mass should be distinguishable. However, Einstein was able to identify them because the existence of repulsive gravity had not been recognized. Thus, Einstein's notion of gravitational mass is a misconception created by a failure to recognize repulsive gravity.

Unfortunately, the invalid notion of gravitational mass is currently very popular. Although, as Einstein pointed out, the notion of inertial mass is different from the notion of weight (gravitational mass), many theorists still cannot distinguish the difference between mass and weight. Nevertheless, the mass and gravity can be distinguished with the first approximation of a formula for the period T of a pendulum as follows [33]:

$$T = 2\pi\sqrt{\frac{l}{g}}, \tag{1}$$

where l is the length of the pendulum and g is the gravitational acceleration.(10) Thus, the change of mass of the pendulum would not change the pendulum period, but if the g changes, the period T of the pendulum will be changed.(11) Since a piece of metal is a solid, a reduction of its mass or gravity can be distinguished by using it as a pendulum. Apparently, Dmitriev et al. [8] and Fan et al. [9] did not measure the changes of the period T.

It has been verified by Liu [34] that the mass is essentially the same as that predicted by Einstein [7] and Lo [35], but the period is extended after being heated-up. Thus, from the above weight reduction experiments [8, 9], the repulsive gravitational force must exist. Moreover, it has been verified by Lo [10], using a torsion balance scale, that lead balls have reduced gravitation after being heated-up.

Thus, measuring mass through gravity is unreliable, and repulsive gravitation must exist.

4. Einstein's Incomplete Proof of $E = mc^2$

Einstein [36] has shown that the electromagnetic radiation energy L emitted from a body is equivalent to the mass L/c^2, where c is the light speed. In his approach, the energy L is due to two waves $W1$ and $W2$, in opposite directions, each with energy $(1/2)L$. His motivation is to assume the two waves as two groups of massless particles. Because these two waves are in opposite directions, their momentums cancel each other, and thus what remains is the sum of the energies of the massless particles, i.e., L. Then, he shows the energy L is equivalent to a mass L/c^2.

However, his proof is inconsistent with electromagnetism as follows: (1) the electromagnetic energy-stress tensor $T(E)_{\mu\nu}$ has a zero trace. (2) The sum of two electromagnetic energy-stress tensors is still an electromagnetic energy-stress tensor with zero traces. However, an energy-stress tensor of trace zero cannot be related to a mass whose energy-stress tensor has a non-zero trace. Thus, Einstein's proof is inconsistent with electromagnetism.(12)

Einstein [36] claimed, without proof, that the relation between the energy l for a wave measured in the co-ordinate (x, y, z) and the energy

l^* for the same wave measured in a new co-ordinate system (ξ, η, ζ) is

$$l^* = l\,\frac{1-(v/c)\cos\phi}{\sqrt{1-v^2/c^2}}\,, \tag{2}$$

where v is the relative velocity between these two coordinate systems, and ϕ is the angle between the propagating direction of the wave and the relative velocity v. He [36] also claimed that "The principle of the constancy of the velocity of light is of course contained in Maxwell's equations." However, we have just shown that the results being derived from eq. (2) are inconsistent with Maxwell's theory.

Note that the energy of massless particles is inconsistent with the electromagnetic energy-stress tensor related to an electromagnetic wave, although the energy-stress tensor of a massless particle is also traceless. On the other hand, Einstein's notion of photons that are quantized electromagnetic energy has successfully met all the experimental tests. In fact, Einstein obtained a Nobel Prize for his explanation of the photo-electric effects. Nevertheless, it is clear that Einstein did not understand Maxwell's theory well, and his intuition was not always logically valid.

Although $E = mc^2$ has been demonstrated with the conversion of mass to energy in nuclear physics, the atomic bomb for instance, [31], the reverse conversion of energy to mass has never been proved. In fact, Einstein failed to show the mass-energy equivalence in his efforts from 1905 to 1909 [1]. The radiation energy L [36], being the photons' energy, is Einstein's assumption whose validity must be proved. Thus, Einstein's proof is incomplete, and the claim of $E = mc^2$ as unconditional shows not an achievement as commonly believed, but certain of Einstein's shortcomings in physics.

Einstein [36] did not know that the electromagnetic energy-momentum tensor alone is incompatible with the energy of the massless particles [2]. In other words, Einstein used the energy-momentum tensor of massless particles for the photons without necessary proof, although such proof is needed as shown by Lo [4, 5] in 2006.

The omission of such proof shows that Einstein did not fully understand Maxwell's theory.

Nevertheless, Einstein's invalid derivation on the loss of mass L/c^2 was accepted without any question since it has been shown from special relativity that $E = m_0c^2$, where m_0 is the rest mass of a particle. Note that the energy-momentum tensor of the photons is compatible with the energy-momentum tensor of the mass [4, 5]. However, many just did not understand electromagnetism and gravity well enough to know the difference between the radiation L and electromagnetic energy. Thus, Einstein did not know that E/c^2 as mass may not always be valid.

Note that in 1912 Einstein invalidly changed the letter L to E in the formula to represent general energy [37]. As a result, Einstein's proof for $E = mc^2$ is not incomplete, as $L = mc^2$, but it is invalid.

5. The Conditional Validity of $E = mc^2$

The formula $E = mc^2$ appears in special relativity, but this only means that mass can be converted into energy. Einstein wants to have new content, $m = E/c^2$, i.e., any energy can be equivalent to mass. However, Einstein failed, although he made a great effort to prove this between 1905-1909 [1].

The truth is that, for the electromagnetic energy E, $E = mc^2$ is inconsistent with the Einstein equation [13, 14],

$$G_{\mu\nu} \equiv R_{\mu\nu} - (1/2)g_{\mu\nu}R = -KT_{\mu\nu}, \qquad (3)$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, $R = R_{\mu\nu}g^{\mu\nu}$ is the Ricci curvature, $T_{\mu\nu}$ is the sum of energy-stress tensor, and K is the coupling constant. Then, we have

$$R = KT_{\mu\nu}g^{\mu\nu}. \tag{4}$$

Note that eq. (4) is completely general.

For the case of electromagnetic energy E, the trace of the electromagnetic stress tensor $T(E)_{\mu\nu}$ is zero, i.e., $g^{\mu\nu}T(E)_{\mu\nu} = 0$. Thus, it cannot change the Ricci curvature. However, the mass m is able to do so since the trace for the massive energy-stress tensor is non-zero. Thus, electromagnetic energy and mass are not equivalent. Since eq. (4) was first derived by Einstein [14], the failure of seeing this inconsistency with $E = mc^2$ is Einstein's oversight.

Thus, Einstein's theory alone would show that $E = mc^2$ is not always valid, independent of the existence of repulsive gravitation. We shall show in the next section that this photonic energy is different from electromagnetic energy. It also includes the gravitational wave energy.

6. The Energy-Momentum Tensor of the Photons

Einstein proved that the energy of photons is equivalent to mass [36]. This does not mean, however, that electromagnetic energy is equivalent to mass, since it is actually based on Einstein's unproven assumption that photons are massless particles. Note that the energy-momentum tensor of the massless particles is incompatible with the electromagnetic energy-momentum tensor because a sum of electromagnetic energy-momentum tensors is always traceless, but a sum of the energy-momentum of massless particles can become massive. In fact, to derive the photonic energy-momentum tensor, general relativity must be used [2].

Consider a source of electromagnetic "plane wave." Einstein believed that the Einstein eq. (3) can be used for this case [38], and Penrose [39] obtained a solution as follows:

$$ds^2 = dudv + Hdu^2 - dx_i dx_i, \quad \text{where} \quad H = h_{ij}(u)x_i x_j, \tag{5}$$

where $u = ct - z$, $v = ct + z$. However, this metric is unbounded, and there are non-physical parameters (the choice of origin) that are unrelated to any physical causes. Thus Penrose [39], being primarily a mathematician, over-looked a violation of the principle of causality (Appendix B) in physics.

The verification of the bending of light rays made Einstein famous. Most of Einstein's followers, however, were not aware that the bending of light also exposed necessary modifications.

Einstein's calculation of the bending of light implicitly assumes that the gravity created by an electromagnetic wave is negligible. Einstein also claimed that any energy-momentum tensor could be the source of his equation; one should be able to obtain a gravitational solution for the electromagnetic wave. Since such gravity is physically very weak, many were in agreement with Einstein, and believed that such gravity could be calculated with the perturbation approach (although they did not do it).(13)

Mathematically, for a perturbation approach to be valid, a necessary condition is, however, that this problem has a bounded solution. This compatibility between mathematics and physics is crucial for the validity of a theory in physics.(14) Thus, it was natural for Einstein [14] to believe that his equation could be used for such a case. Although Einstein claimed that his equation was valid for any energy-momentum tensor, he solved only a few cases [40]. Nevertheless, Einstein insisted only on his Einstein tensor G_{ab} in eq. (3), but otherwise allowed modifications.

Moreover, explicit calculation shows that it is impossible to have bounded solutions for an electromagnetic wave's gravity. In order for Einstein's theory of general relativity to make sense, the related Einstein equation, with an electromagnetic wave as the source, must include a photonic energy-stress tensor with the anti-gravity coupling [4, 5]. For this case, the related modified Einstein equation is the following:

$$G_{ab} \equiv R_{ab} - (1/2)g_{ab}R = -K[T(w)_{ab} - T(p)_{ab}], \tag{6}$$

and

$$T_{ab} = -T(g)_{ab} = T(w)_{ab} - T(p)_{ab}, \tag{7}$$

where $T(w)_{ab}$ and $T(p)_{ab}$ are the energy-stress tensors for the electromagnetic wave and the related photons, which are massless particles. Thus, the photonic energy must also include the energy of its gravitational-wave component.

The energy, related to the photons, is clearly beyond special relativity. Further, the implicit assumption of a unique sign for all coupling constants in space-time singularity theorems is invalid. Thus, the claim of Hawking and Penrose that general relativity is not suitable for microscopic phenomena is simply incorrect.

Note that for a massive source to have a dynamic solution [15], the modified Einstein equation is as follows:

$$G_{\mu\nu} \equiv R_{\mu\nu} - (1/2)g_{\mu\nu}R = -K[T_{\mu\nu}(m) - t_{\mu\nu}(g)], \tag{8}$$

where $t_{\mu\nu}(g)$ is the energy-momentum tensor of the gravitational field. This equation was first obtained by Lorentz [27] and Levi [28], but Einstein objected to it on the mistaken grounds that his field equation implies $t(g)_{ab} = 0$. However, eq. (8) was recovered by Lo [15] with the support of Einstein's radiation formula. Thus, there are three important conclusions: (1) The antigravity coupling is necessary for a dynamic case,

(2) For the dynamic case, the Einstein equation has no bounded solution, and (3) The space-time singularity theorems, which require a unique sign for coupling constants, are invalid for physics.

Eq. (8) also explains that, for a dynamic case, the linearized equation does not have a compatible solution from the nonlinear Einstein equation. The linear equation is a valid linearization for eq. (8), but an invalid linearization of the Einstein equation. Thus, Einstein failed to see the need for an anti-gravity coupling for a dynamic solution.

Note that Einstein [36] uses massless particles to represent photons but from eq. (6) and eq. (7) it is clear that this cannot be done without the gravitational wave [4, 5]. Thus, Einstein failed to recognize that this energy problem is beyond special relativity. Between 1905-1909, Einstein also failed to show the general validity of $E = mc^2$ [1]. This failure to see the need for the anti-gravity coupling provides the basis for the space-time singularity theorems, which are based on the implicit assumption of a unique coupling sign.

If photons consist only of electromagnetic energy, then there is a conflict, since photonic energy can be equivalent to mass, but electromagnetic energy is not [2]. This conflict has now been resolved, since the photonic energy is the sum of electromagnetic energy and gravitational energy, and this confirms that $E = mc^2$ can be invalid.

The proof of photonic energy consisting of massless particles is a remarkable achievement of general relativity. This also shows an important example of the Einstein equation where a valid physical solution may not satisfy it. Thus, one cannot just conjecture a solution based only on “reasonable” physical considerations alone, without an explicit example. This is what is needed in the “Proof of the Positive Mass Theorem. II,” of Schoen and Yau.

7. The Repulsive Gravitation and the Necessary Extension of General Relativity

We should now address the nature of repulsive gravitation. In fact, a charge-mass repulsive force was derived from the Reissner-Nordstrom metric in 1916 for a particle with charge q and mass M [40] as follows:

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2, \qquad (9)$$

(with $c = 1$) where r is the radial distance (in terms of the Euclidean-like structure [41]) from the particle center.(15) In metric (9), the gravity components generated by electricity have not only a very different radial coordinate dependence but also a different sign that makes it a new repulsive gravity [12].(16) This repulsion implies that the basic assumption for black holes, gravity being always attractive, is invalid, and it will be shown that general relativity must be extended.

For an elementary charged particle, the repulsive force would be very small. However, a similar metric can be derived for a charged ball. The only changes are that r becomes R, the distance from the center of the ball, and q becomes Q, the total charge of the ball [42]. Thus, for a charged ball with a sufficiently large Q, the repulsive gravitational force can be macroscopically observed.(17) Nothing had been derived from this metric, however, until 1997 [43], because theorists did not acknowledge the repulsive gravitational force.

In 2005, Tsipenyuk and Andreev [44] discovered that a charged metal ball becomes lighter in weight, but they did not know why because repulsive gravitation was not included in Einstein's general relativity. Moreover, theorists such as Herrera, Santos and Skea [45] argued that M in metric (9) involves electric energy. Then they obtained a metric that would imply a charged ball would increase its weight as the charge

q increased [12], in disagreement with experiments [44]. Nevertheless, ‘t Hooft [46](18) and Wilczek [32](19) have also mistakenly assumed that $m = E/c^2$ is universally true. Since Wilczek used $E = mc^2$ for the asymptotic freedom without any justification [32], the proof is incomplete.

On the other hand, if the mass M is the inertial mass of the particle, the weight of a charged metal ball would be reduced [12]. Thus, experiments on two metal balls [44] support the conclusion that the mass M does not include electric energy since a charged ball has a reduced weight. It will be shown, based on the principle of causality (see Appendix A), that such a force leads to the necessity to extend the theoretical framework of general relativity.

To see the necessity of extending general relativity, we consider the force on a test particle with mass m, and

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0,$$

where

$$\Gamma^\mu{}_{\alpha\beta} = \left(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}\right)g^{\mu\nu} / 2$$

and $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, according to Einstein. Note, the gauge affects only the second-order approximation of g_{tt} [47].

Let us consider only the static case. For a test particle p with mass m at r, the force on p is

$$-m\frac{M}{r^2} + m\frac{q^2}{r^3} \tag{10a}$$

in the first-order approximation because of $g^{rr} \cong -1$. Thus, the second term is a repulsive force.

If the particles are at rest, then the force acting on the charged particle P has the same magnitude

$$\left(m\frac{M}{r^2} - m\frac{q^2}{r^3}\right)\hat{r}, \text{ where } \hat{r} \text{ is a unit vector} \tag{10b}$$

because the action and reaction forces are equal but in opposite directions. However, for the motion of the charged particle with mass M, if one calculates the metric according to the particle p of mass m, only the first term is obtained.

It is necessary then to have a repulsive force with the coupling q^2 to the charged particle P in a gravitational field generated by mass m. Thus, force (10b) to particle P is beyond the framework of gravitation + electromagnetism. As predicted by Lo, Goldstein and Napier [48], general relativity would lead to the necessity of its extension.

The repulsive force in metric (9) comes from electric energy [12]. An immediate question would be whether such a charge-mass repulsive force mq^2/r^3 is subjected to electromagnetic screening. This force, being independent of a charge sign, should not be subjected to such screening. Moreover, the existence of the repulsive force mq^2/r^3 means also that Maxwell's theory is actually inadequate.

Note that this force can be considered a result of q^2 interacting with a field created by the mass m. Thus, such a field is independent of electromagnetism and is beyond general relativity, and the need for unification is established. To test such a possibility, one can measure whether there is such a repulsive force outside a charged capacitor.(20) Thus, to include the repulsive gravitational force, general relativity must be extended to a five-dimensional space.

A necessary step to test is to measure the force (10a). However, for a charged particle, force (10a) is too small. Nevertheless, we could test a similar force for a ball with charge Q. For a sufficiently large charge Q, the force $-m\frac{M}{R^2}+m\frac{Q^2}{R^3}$, where R is the distance from the center of the ball should be verifiable experimentally. However, no experiment on this has been performed so far because physicists were not aware of the existence of repulsive gravitation. Thus, general relativity has not been properly tested.

8. Einstein's Conjecture of Unification and the Five-dimensional Relativity

The coupling with q^2 would lead to a five-dimensional space of Lo et al. [48], since such coupling does not exist in a four-dimensional theory, or the five-dimensional theories of Kaluza [49] or Einstein and Pauli [50]. It would be difficult to derive the five-dimensional field equation. To find out such a possibility, we shall consider the geodesic equation first.

Now let us give a brief introduction of the five-dimensional relativity. The five-dimensional geodesic of a particle is

$$\frac{d}{ds}\left(g_{ik}\frac{dx^k}{ds}\right)=\frac{1}{2}\frac{\partial g_{kl}}{\partial x^i}\frac{dx^k}{ds}\frac{dx^l}{ds}+\left(\frac{\partial g_{5k}}{\partial x^i}-\frac{\partial g_{5i}}{\partial x^k}\right)\frac{dx^5}{ds}\frac{dx^k}{ds}$$

$$-\Gamma_{i,55}\frac{dx^5}{ds}\frac{dx^5}{ds}-g_{i5}\frac{d^2x^5}{ds^2}, \tag{11a}$$

$$\frac{d}{ds}\left(g_{5k}\frac{dx^k}{ds}+\frac{1}{2}g_{55}\frac{dx^5}{ds}\right)=\Gamma_{k,55}\frac{dx^5}{ds}\frac{dx^k}{ds}$$

$$-\frac{1}{2}g_{55}\frac{d^2x^5}{ds^2}+\frac{1}{2}\frac{\partial g_{kl}}{\partial x^5}\frac{dx^l}{ds}\frac{dx^k}{ds}, \tag{11b}$$

where $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, $\mu, \nu = 0, 1, 2, 3, 5$ $(d\tau^2 = g_{kl}dx^k dx^l$;

$k, l = 0, 1, 2, 3$).

If instead of ds, $d\tau$ is used in (14), for a test particle with charge q and mass M, the Lorentz force suggests

$$\frac{q}{Mc^2}\left(\frac{\partial A_i}{\partial x^k} - \frac{\partial A_k}{\partial x^i}\right) = \left(\frac{\partial g_{i5}}{\partial x^k} - \frac{\partial g_{k5}}{\partial x^i}\right)\frac{dx^5}{d\tau}. \tag{12a}$$

Thus,

$$\frac{dx^5}{d\tau} = \frac{q}{Mc^2}\frac{1}{K}, \quad K\left(\frac{\partial A_i}{\partial x^k} - \frac{\partial A_k}{\partial x^i}\right) = \left(\frac{\partial g_{i5}}{\partial x^k} - \frac{\partial g_{k5}}{\partial x^i}\right) \text{ and } \frac{d^2x^5}{d\tau^2} = 0, \tag{12b}$$

where K is a constant. It thus follows that (11) is reduced to

$$\frac{d}{d\tau}\left(g_{ik}\frac{dx^k}{d\tau}\right) = \frac{1}{2}\frac{\partial g_{kl}}{\partial x^i}\frac{dx^k}{d\tau}\frac{dx^l}{d\tau} + \left(\frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k}\right)\frac{q}{Mc^2}\frac{dx^k}{d\tau} - \Gamma_{i,55}\left(\frac{q}{Mc^2}\right)\frac{1}{K^2}, \tag{13a}$$

$$\frac{d}{d\tau}\left(g_{ik}\frac{dx^k}{d\tau} + \frac{1}{2}g_{55}\frac{q}{KMc^2}\right) = \Gamma_{k,55}\frac{q}{KMc^2}\frac{dx^k}{d\tau} + \frac{1}{2}\frac{\partial g_{kl}}{\partial x^5}\frac{dx^l}{d\tau}\frac{dx^k}{d\tau}. \tag{13b}$$

One may ask what the physical meaning of the fifth dimension is. Our position is that the physical meaning of the fifth dimension is not yet very clear [48], except some physical meaning is given in the equation, $dx^5/d\tau = q/Mc^2K$ where M and q are, respectively, the mass and charge of a test particle, and K is a constant. We shall denote the fifth axis as the w-axis. Our approach is to find out the full physical meaning of the w-axis as our understanding gets deeper.

For a static case, we have the forces on the charged particle P in the

ρ - direction

$$-\frac{mM}{\rho^2} \approx \frac{Mc^2}{2}\frac{\partial g_{tt}}{\partial \rho}\frac{dct}{d\tau}\frac{dct}{d\tau}g^{\rho\rho}, \text{ and } \frac{mq^2}{\rho^3} \approx -\Gamma_{\rho,55}\frac{1}{K^2}\frac{q^2}{Mc^2}g^{\rho\rho} \quad (14a)$$

and

$$\Gamma_{k,55}\frac{q}{KMc^2}\frac{dx^k}{d\tau} = 0, \text{ where } \Gamma_{k,55} \equiv \frac{\partial g_{k5}}{\partial x^5} - \frac{1}{2}\frac{\partial g_{55}}{\partial x^k} = -\frac{1}{2}\frac{\partial g_{55}}{\partial x^k} \quad (14b)$$

in the $(-r)$- direction. The meaning of (14b) is the energy-momentum conservation. Thus

$$g_{tt} = 1 - \frac{2m}{\rho c^2}, \text{ and } g_{55} = \frac{mMc^2}{\rho^2}K^2 + \text{constant}$$

$$\left(\text{or } \frac{1}{MK^2c^2}g_{55} = \frac{m}{\rho^2} + \text{constant.}\right) \quad (15)$$

In other words, g_{55} is a repulsive potential, and g_{55} / M is also a function of a distance mass m. Because g_{55} is independent of q, this force would penetrate electromagnetic screening.

Thus, general relativity can be extended to accommodate the charge-mass interaction. For this, a five-dimensional relativity is a natural candidate. According to Lo et al. [48], the charge-mass interaction would penetrate a charged capacitor. To verify the five-dimensional theory, one can simply test the repulsive force on a charged capacitor. This has been experimentally confirmed [12]. However, since particle p is neutral, there is no charge-mass repulsion force on p.

9. The Weight Reduction of a Charged Capacitor

The repulsive gravitational force was first discovered from measuring a charged capacitor. Thomas T. Brown initiated the study of charging a capacitor and later was joined by Paul A. Biefeld [51, 52]. Since the B-B effects cannot be explained with current theories, many regarded such effects as experimental errors.

For instance, it is known that a charged capacitor has a reduced weight. Moreover, after being charged with a high voltage (about 40 kilovolts), without a continuous supply of electric energy, the lifter (a light capacitor) is able to lift its own weight plus a payload hovering over Earth. Also, a lifter could work by charging the wire to either a positive or a negative potential. It has been determined that the lift is not due to ion wind effects [51]. Thus, the lift is generated by changing something inside the lifter with a high voltage charge.

In a charged capacitor, the only change is the state of motion of some electrons that have become statically concentrated instead of moving in orbits. Then, a repulsive force appears. Since such a force did not appear before, it is clear that such a force was canceled out by the force created by the motion of the electrons. In other words, the repulsive force generated by the charges of protons and the electrons was canceled by the force generated by the motion of the initially moving charges of the electrons. Note that string theorists still do not acknowledge repulsive gravitation.

This repulsive force, however, cannot be proportional to the charge density. We have equal numbers of negatively charged electrons and positively charged protons with equal charge. This would lead to the cancellation of the forces generated by particle charges. However, if such a force is proportional to the charge density square, these two kinds of forces would be added together instead of canceled out. Moreover, since the lifter has a limited height, one should expect that this repulsive gravitational force would diminish faster than the attractive gravitational force. Thus, if we assume that the force is proportional to mass, as usual, the static charge-mass interaction would be a repulsive force between particles with charge density D_q and another particle of mass m would have the following form,

$$F_r \approx KmD_q^2 / r^n \text{ where } n > 2, \qquad (16)$$

r is the distance between the two particles, and K is the coupling constant. In formula (16), the coupling constant K and n the power of r can be determined by experiments. The simplest case would be $n = 3$.

Formula (16) is derived from the observations with common physical sense. The experimental results are that the charged capacitors have reduced weight. If the lift force is large enough, it will hover over the Earth [51, 52] since the repulsive gravitation force reduces faster than the gravitational force.

According to general relativity, if the electric energy leads to a repulsive force toward a mass, the magnetic energy would lead to an attractive force from a current toward a mass [22]. Due to a charged capacitor having reduced weight, it is necessary to have the current-mass interaction canceled out by the effect of the charge-mass interaction. Thus, the existence of the current-mass attractive force would solve a puzzle, i.e., why a charged capacitor exhibits the charge-mass repulsive force since a charged capacitor has no additional electric charges. In fact, the charge-mass repulsive force would be canceled by the current-mass force as Galileo, Newton, and Einstein implicitly assumed.

The existence of such a current-mass attractive force has been discovered by Martin Tajmar and Clovis de Matos [53] from the European Space Agency. Martin et al. found that a spinning ring of superconducting material increases its weight more than expected. Thus, they believed that general relativity was wrong. However, according to quantum theory, spinning superconductors should produce a weak magnetic field. Thus, they also measured the current-mass interaction to the Earth! The current-mass interaction would generate a force that is perpendicular to the current.

Since the additional weight from a current-mass interaction is directional, the weight of a magnet is directionally dependent, as our experiment verified [54]. *This directional dependence of weight is a completely new phenomenon that verifies the existence of the current-mass interaction.*

One may ask what the formula for the current-mass force is. Unlike the charge-mass repulsive force, which can be derived from general relativity, this general force would be beyond general relativity and since a current-mass interaction would involve the acceleration of a charge, this force would be time-dependent and generate electromagnetic radiation. Moreover, when radiation is involved, the radiation reaction force and the fifth-dimension variable must be considered [48]. Thus, we are not yet ready to derive current-mass interaction. Nevertheless, we may assume that for a charged capacitor, the resulting force is the interaction of net macroscopic charges with the mass.

Experimentally, the repulsive force would be proportional to the potential square, V^2 where V is the electric potential difference of the capacitor ($Q = CV$, C is the capacitance and Q is the charge). This has been verified by the experiments of Musha [55, 56]. Thus, the factor of charge density square in heuristic Eq. (9) is correct. Moreover, the lifter's hovering shows that the repulsive force would diminish faster than the gravitational force. However, even if the $1/r^3$ factor in the repulsive force is verified, the calculation would still depend on the detailed modeling [57]. Although the initial thrust due to the electric field is directional, the weight reduction effect for charged capacitors is not directional, and it stays if the potential does not change. This was verified by Liu [34] with the rolled-up capacitors. Thus, the repulsive force on the charged capacitor is the same force that derived from general relativity [40].

One may ask what the weight of the charged capacitor would be after it is discharged. It takes time for a capacitor to recover its weight after being discharged [58]. A discharged capacitor needs time to dissipate the heat generated by discharging, and the motions of its charges would accordingly recover to the previous state. This was observed because rolled-up capacitors keep heat better. Thus, this also explains the weight reduction of a piece of heated-up metal [8, 9].

It follows that there are three factors that determine the weight of matter. They are: (1) the mass of the matter, (2) the charge-mass repulsive force, and (3) the attractive current-mass force. For a piece of a heated-up metal, the current-mass attractive force due to orbital electrons is reduced, but the charge-mass repulsive force increases. Therefore, a net result is a reduction of weight [12], instead of increased weight as predicted by Einstein. Thus, to test the inverse square law accurately, one must know exactly how temperature affects weight.

10. Problems in Newtonian Gravity and the Repulsive Gravitation

Experimental tests of gravity's distance-dependence define a frontier between our understanding of gravity and many proposed forms of new physics. As gravity is $\sim 10^{40}$ times weaker than electromagnetism, gravity remains hidden by experimental backgrounds at distances smaller than the diameter of a fine human hair. The recent talk of Charles Hagedorn [59] surveys the past, present, and near future of the experimental field, with emphasis on precision sub-millimeter laboratory experiments. However, Hagedorn did not know that gravity also depends on temperature [60].

Faller [61] was aware that error budgets in the measurements of the Newtonian coupling constant are fundamentally flawed because they cannot make allowances for error sources that have not been thought of. However, he did not know that the measurements to obtain the Big G

coupling constant could not be accurate, as he was unaware of the influence of heat on weight [60]. Thus, the Newtonian coupling obtained by J. Luo is questionable [62].

However, for obvious reasons, it is also desirable to continue these experiments on gravitation.

Einstein did not see that for the dynamic case, the Einstein equation does not have any bounded solution [6]. He was confident in his work since he was accurate in the remaining perihelion of Mercury, but he could not justify his calculation with a perturbation approach. For the dynamic case, the "linearized" equation is an independent equation [63]. This explains Einstein's puzzlement [64] over why his equation did not produce the gravitational wave solution.

Since the measured Newtonian gravity is actually temperature dependent [60], the temperature dependence must be understood before an accurate test of Newton's inverse square law. The temperature dependence of gravity is expected since an increase in temperature means an increase in energy. The problem is, however, that an increase in temperature leads to a reduction of weight [60].

What we measured is actually a combination of Newtonian gravity and a much weaker repulsive force [12], i.e.,

$$F = G\frac{m_1 m_2}{r^2} - k_2\frac{m_1}{r^3} - k_1\frac{m_2}{r^3}, \tag{17}$$

where k_1 and k_2 are functions of temperature, depending on the matter used to construct m_1 and m_2. The increment of gravitational reduction is due to the increment of the number of random electrons as the temperature increases [60]. However, we have not been able to establish details of the temperature dependence of k_1 and k_2.

11. The Oversights of Maxwell, Einstein, and the Nobel Committee

In the experiment on the photoelectric effect, it was assumed that the photons consist entirely of electromagnetic energy. However, there is no evidence that photons consist of electromagnetic energy alone. In fact, it would be natural to conjecture that photons also consist of gravitational wave energy since all charged particles are massive. Historically, the formula $E = mc^2$ was proposed by Einstein in 1912 [37], well before the publication of general relativity in 1916. Understandably, Einstein did not include the gravitational energy in the photons.

Moreover, Maxwell claimed that light is also an electromagnetic wave because light and the electromagnetic wave have the same speed. Maxwell did not consider that light could contain anything else since there was nothing that could attain the speed of light. Thus, it was natural for Einstein to follow Maxwell.

Einstein did not modify his proposal since he was not sure of the existence of the gravitational wave, although it could have the speed of light. He [65, 66] was puzzled that his equation implied no gravitational wave while the linearized equation showed its existence. In fact, Einstein concluded his talk on gravitational waves at Princeton University by saying [67] "If you ask me whether there are gravitational waves or not, I must answer that I do not know. But it is a highly interesting problem."

Recently LIGO announced that the gravitational wave had been detected. However, the exact equation that produces the gravitational wave remains to be investigated. Although the Lorentz-Levi-Einstein equation $R_{\mu\nu} - (1/2)g_{\mu\nu}R = -K[T(m)_{\mu\nu} - t(g)_{\mu\nu}]$, can produce the gravitational wave approximately [15], the exact gravitational energy-stress tensor $t(g)_{\mu\nu}$ remains to be investigated. However, Einstein rejected such a modification because he believed that his equation was correct for the dynamic cases [13, 14], although mathematics shows otherwise [15].

Apparently, the claim of the Nobel committee that the Einstein equation produced the gravitational wave is due to a calculation error. It is well-known that a calculation with a computer can be highly unreliable because in such a calculation the result depends on how the calculating points are taken. Such a calculation can only be wrong if it disagrees with an analytic result.(21) This is not the first time that the Noble Committee made errors in gravitation [68].

12. On the Question of Newtonian Theory as an Approximation for General Relativity

The physics community incorrectly accepted Einstein's claim [13] that Newton's theory was a first-order approximation of general relativity. However, the word approximation means moving toward an exact solution. A problem is that there is no bounded dynamic solution for the Einstein equation. Thus, one cannot claim that Newtonian theory provides an approximation.

Einstein derived from his geodesic equation [13]

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0, \tag{18}$$

where $\Gamma^\mu{}_{\alpha\beta} = \left(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}\right)g^{\mu\nu} / 2$ and $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$.

For $\alpha = \beta = 4$, he obtained $\frac{d^2x^\mu}{dt^2} = -\Gamma^\mu{}_{44}$, which implies $\frac{d^2x^\mu}{dt^2} = \frac{1}{2}\frac{\partial g_{44}}{\partial x_\mu}$ in which $\frac{1}{2}g_{44}$ plays the gravitational potential.

Then, the Einstein equation yields $\nabla^2 g_{44} = \kappa\rho$, where ρ denotes the density of matter, and $\kappa = 1.87 \times 10^{-27}$. In this derivation, a crucial implicit assumption is that the solution of g_{44} is bounded. But this is not necessarily true.

However, for a dynamic case, the solution for g_{44} is not bounded [15, 16]. On the other hand, it is known that there are two-body solutions for the Newtonian theory. Thus, Einstein's theory has not yet superseded that of Newton. In fact, most of the Nobel Laureates in general relativity, including Einstein, have made some theoretical errors in physics [2, 23].

In 1995, based on Einstein's radiation formula, we found that the Einstein equation for the two-body problem must be modified [15]. It is well known that Einstein's radiation formula is supported by the Taylor-Hulse experiment, but its derivation is not self-consistent [15, 16]. As suggested by Einstein's own remark, modifications to the source tensor are necessary. Based on the Taylor-Hulse experiments, a theory was developed within the theoretical framework of general relativity. However, the radiation formula remains the same for the binary stars. Because of the radiation, the source tensor is not zero in a vacuum, and the antigravity coupling [15] is necessary, but this is not related to Newtonian theory.

13. The Question of Compatibility of the Einstein Equation and Its Linearization

Many theorists have failed to understand that the Einstein equation and its linearization are unrelated equations [26]. This can be illustrated with the metric of Bondi, Pirani and Robinson [69] as follows:

$$ds^2 = e^{2\varphi}(d\tau^2 - d\xi^2) - u^2\begin{bmatrix}\cosh 2\beta(d\eta^2 + d\zeta^2) \\ + \sinh 2\beta \cos 2\theta(d\eta^2 - d\zeta^2) \\ - 2\sinh 2\beta \sin 2\theta d\eta d\zeta\end{bmatrix}, \qquad (19a)$$

where φ, β and θ are functions of $u(= \tau - \xi)$. It satisfies the differential equation (i.e., their Eq. [2.8]),

$$2\phi' = u(\beta'^2 + \theta'^2 \sinh^2 2\beta) \tag{19b}$$

which is a special case of $G_{\mu\nu} = 0$. They claimed this is a wave from of a distant source but is not a weak gravity. The metric is irreducibly unbounded due to the factor u^2. Linearization does not make sense since u is not bounded.

Moreover, when gravity is absent, it is necessary to have $\phi = \sinh 2\beta = \sin 2\theta = 0$. These would reduce (19a) to

$$ds^2 = (d\tau^2 - d\xi^2) - u^2(d\eta^2 + d\zeta^2). \tag{19c}$$

However, this metric is not equivalent to the flat metric. Thus, metric (19c) violates the principle of causality.

Now, let us consider the linearized equation $G_{\mu\nu} = 0$. Since for a massive source, the Einstein equation is

$$G_{\mu\nu} \equiv R_{\mu\nu} - (1/2)g_{\mu\nu}R = -KT(m)_{\mu\nu}, \tag{20}$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci curvature tensor, $T(m)_{\mu\nu}$ is the energy-stress tensor for massive matter, and $K(= 8\pi Gc^{-2}$, and G is the Newtonian coupling constant) is the coupling constant. Thus, for the harmonic gauge, the linearized equation is

$$\frac{1}{2}\partial^c\partial_c\bar{\gamma}_{\mu\nu} = -KT(m)_{\mu\nu}, \text{ where } \bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}(\eta^{cd}\gamma_{cd}) \tag{21a}$$

and

$$\bar{\gamma}_{\mu\nu}(x^i, t) = -\frac{K}{2\pi}\int \frac{1}{R}T_{\mu\nu}[y^i, (t - R)]d^3y, \text{ where } R^2 = \sum_{i=1}^{3}(x^i - y^i)^2. \tag{21b}$$

For $T(m)_{\mu\nu} = 0$, we have $\bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} = 0$. Thus, the linearized equation (3) has a bounded solution, but the Einstein equation $G_{\mu\nu} = 0$ violates the principle of causality because the energy-momentum of a wave cannot be zero.

Note that the dynamic solution (19) is unbounded, and therefore the linearization is not a valid mathematical operation. Einstein wondered why his equation did not have a gravitational wave solution. The reason is simply that the linearization is not a valid mathematical operation since there is no bounded dynamic solution.

14. The Errors of Christodoulou, Klainnerman, Yau, Witten, Atiyah, and Faddeev

Theoretical physics requires a sophisticated knowledge of both physics and mathematics. Typically, however, theorists tend to be trained more fully in one or the other of these fields. This section examines the influence of mathematicians on the development of theoretical physics, and shows that while well-versed in pure mathematics, their sometimes insufficient understanding of physics can often impede breakthroughs in general relativity. Similarly, however, physicists often lack a sophisticated understanding of the pure mathematics that is required to discern fully the validity of newly claimed discoveries. It is essential that we bring these two fields closer together in the study of theoretical physics. Unfortunately, we have not been sufficiently successful.

In 1981, Schoen and Yau published their "Proof of the Positive Mass Theorem II" [17] in Commun. Math. Phys. In their paper, they claimed that Einstein's theory is consistent and stable, and in 1982 Yau was awarded a Fields Medal.(22) In spite of this, the physics community was skeptical of their conclusions because they never provided an explicit solution to support their claims, which is of key importance (see Appendix C).

Nevertheless, the attitude of the Nobel Committee in Physics changed in 1993 after Christodoulou and Klainerman [70] claimed that they had constructed dynamic solutions for general relativity. Their claims, however, were based on an insufficient understanding of the pure mathematics involved. In 2000, it was discovered that Christodoulou and Klainerman never actually completed their construction [71]. (In fact, in 1989 the claim of Schoen and Yau had already been found to be erroneous by Logunov and Mestvirishvilli [72].) Moreover, they did not relate their "dynamic" solutions to dynamic sources. Thus, their claim of the existence of a dynamic solution has never been proven. Nevertheless, physicists did not uncover this serious error, and continued to be enthusiastic about the new claims of Christodoulou and Klainerman.

Professor P. Morrison of MIT, however, was an exception, in part because he was aware of my 1995 paper, "Einstein's Radiation Formula and Modifications to the Einstein Equation," [15]. In 1996, after a discussion that took place over the course of a month, Prof. Morrison went to Princeton to ask Prof. Joseph Taylor Jr. about the justification of his calculations on gravitational waves.(23) Prof. Taylor, however, was unable to justify his calculations [68] since they were invalid [15, 16]. Thus, the Nobel Committee in Physics overlooked errors in gravitational calculations for the 1993 prize.

In the "Proof of the Positive Mass Theorem II", Schoen and Yau [17] made an error of incorrectly assuming that all the physical solutions satisfy their asymptotically flat condition, yet again failed to support their results with an example. Had they tried, they would have found their errors.

The theory of Christodoulou and Klainerman, on constructing dynamic solutions, is actually on an empty set. However, they were unaware of this, even in 2011 when Christodoulou received half of the 2011 Shaw Prize for mathematics. This was an oversight on the part of the Shaw Committee, and one which shows the difficulty in fully understanding general relativity. I wrote a letter of objection to the Selection Committee for the Shaw Prize in Mathematical Sciences, but I received no response. However, I found out later, from Prof. Peter C. Sarnak, the Chairman of that 2011 Shaw Committee, when I met him in Toronto University at a meeting in 2016, that this was because "We cannot find someone who understands general relativity."

From the proof of positive mass theorem [17], Schoen and Yau assumed that all the physically meaningful solutions of the Einstein equation could be subjected to the requirements of asymptotically flat without proof. This turns out to be incorrect. Thus, Yau failed to see such requirements eliminate a whole class of importance solutions because his requirements cannot be met for a two-body problem.

Their boundary condition to be imposed on the space-time is that it should be asymptotically [17] flat, i.e.,

$$g_{ij} = \delta_{ij} + O(r^{-1}), \tag{22}$$

However, Yau did not know that whether or not a physical requirement is valid also depends on the field equation.(24) For instance, in an explicit calculation of a two-body problem, due to the Einstein equation's deficiency, the requirement of asymptotically flat (22) just cannot be satisfied [15, 16]. The net result is that the solution to a two-body problem is excluded. Thus, what remain are the gravity of a single mass such as the Schwarzschild solution, the harmonic solution, and the Kerr solution, etc. Had they tried to obtain a solution for a two-body problem, they would have found that it is impossible to satisfy conditions (22) for the Einstein equation.

In effect, the boundary conditions actually excluded an important class of dynamic problems, and then they mistakenly claimed the result of remaining trivial problems as the general result of the theorem [19]. Thus, the positive mass theorem of Schoen and Yau incorrectly led to the notion that general relativity is flawless, thus preventing further progress in relativity [19] for about 40 years.

If you rely on others to obtain the physical condition, you can easily make the same mistake as Yau if you do not use explicit solutions to check your results. In fact, such errors had been made by top mathematicians such as M. Atiyah(25) and L. D. Faddeev.(26) In particular, Faddeev's "natural" definition of energy has no valid basis in physics since no bounded dynamic solution has ever produced. An obvious answer for this problem is to find a dynamic solution for the Einstein equation, but this is impossible.

Consequently, the erroneous theorem of Scheon and Yau was cited as the main reason to award the Fields Medal to Yau (1982) and Witten (1990) (Appendix C: The International Mathematical Union (IMU) executives 1979-1990.). Moreover, this also led to awarding the 2011 Shaw Prize to Christodoulou.(27)

Schoen and Yau appeared not to see that, for a dynamic case, the linearized equation and the non-linear Einstein equation are not compatible [26]. They failed to consider the fact that, in the literature, the Einstein equation has no dynamic solution, which is bounded. Thus, they failed to see that the asymptotically flat implies the exclusion of the dynamic solutions, instead of including all physical solutions. Note that the proof for the nonexistence of a bounded dynamic solution was published in 2000 [16], about 20 years after their theorem.

It should be noted that D. Hilbert also made a mistake in approving Einstein's calculation of the perihelion of Mercury because he was not aware that this calculation requires a bounded solution of the many-body problem [6].

In fact, theorists such as Yau [17], Christodoulou [70], Wald [21], Penrose [39], and Hawking [21] make essentially the same error of defining a set of solutions that include no dynamic solutions. Until 1997, based on special relativity, $E = mc^2$ was considered unquestionably valid [43]. Their errors result from neglecting to provide explicit examples to support their claims. If they had done so, their errors would have become apparent.

Moreover, the same erroneous work [19] was cited in awarding the 2011 Shaw Prize to Christodoulou. This resulted from an accumulation of long-standing errors, often resulting from an insufficient understanding of the principle of causality.

There have been doubts about general relativity, but we were unsure because of the lack of experimental confirmation. Now, we have clear and simple evidence that Einstein's predictions can be wrong [8, 9, 10].

15. The Equivalence Principle and Invalidity of Einstein's Covariance Principle

Einstein's general relativity is based on three pillars: the Einstein equation, the equivalence principle, and the covariance principle. The Einstein equation is not perfect since it cannot deal with the dynamic problems. In fact, the linearized equation has far more applications. The two principles also have problems.

First, as Einstein acknowledged, the equivalence principle is valid only when repulsive gravitation is not present [3]. Moreover, Misner, Thorne and Wheeler [40], in disagreement with Einstein, incorrectly referred to Pauli [74](28) and the 1911 invalid assumption of Einstein [13] for Einstein's equivalence principle, and Wald [21] even abandoned it but accepted the invalid covariance principle. [75](29)

Einstein's covariance principle assumed that the relations between physical quantities are independent of the chosen coordinates. He proclaimed "So, there is nothing for it but to regard all imaginable systems, on principle, as equally suitable for the description of nature." Thus, there is no clear logical reason for the covariance principle. This led Zhou Pei-Yuan [76] of Peking University to point out in 1983 that this principle is invalid. Zhou's claim was later supported by Lo [75] with examples showing that the relation between impact parameters b of a light ray to the sun and the sun's closest distance r_0 from the ray is gauge coordinate dependent.(30) For the harmonic and the Schwarzschild metrics, we have respectively,

$$b \approx 2m + r_0, \tag{23a}$$

or

$$b \approx m + r_0. \tag{23b}$$

Thus, this relation is gauge coordinate dependent, and the covariance principle is invalid.

Moreover, Einstein justified a curve space by showing the ratio $U / D > \pi$, (where U and D are, respectively, the circumference and the diameter of a circle in the rotating frame) with applications of special relativity [77]. It is well-known that special relativity has nothing to do with gravitation. Thus, there must be some mistakes in Einstein's application; he considered a small piece of circumstance dX in a local space L^*,

$$dX = [1 - (r\Omega / c^2)]^{-\frac{1}{2}}[rd\phi]. \tag{24}$$

From Eq. (24), he integrated these pieces from different local space L^* under different accelerations, and obtained

$$U = 1/2\,[1 - (D\Omega / 2c)^2]^{-1/2} \oint Dd\phi = \pi D[1 - (D\Omega / 2c)^2]^{-1/2}. \tag{25}$$

Einstein's error is a result of adding up measurements from different local space L^* [77].

Physicists accepted Einstein's covariance principle because it was felt that Einstein could not make an error in the application of special relativity.

On the other hand, in the calculation the bending of light, Einstein [13; p. 162] defines the light velocity as

$$\sqrt{\left(\frac{dx_1}{dx_4}\right)^2 + \left(\frac{dx_2}{dx_4}\right)^2 + \left(\frac{dx_4}{dx_4}\right)^2} = \gamma$$

in the sense of Euclidean geometry, and this light speed is supported by observation. So, the distance in the physical Riemannian space is determined by its Euclidean-like structure, but not as the distance in a mathematical Riemannian space, which is not supported by observation. Thus, a physical space is different from a mathematical Riemannian space.

Note that the incorrect interpretation of the Doppler effects [25] was accomplished by using a measurement of a mathematical Riemannian space that would lead to wrong light speeds.

16. Discussions and Conclusions

We have found crucial errors in general relativity, some of which, such as the covariance principle, were made by Einstein himself. The life of an error is often limited because experiments can expose them. However, some errors cannot be verified by experiments directly. For example, the remaining perihelion of Mercury has been obtained, but one cannot justify it with a necessary perturbation approach because there is no bounded dynamic solution.

There is no bounded dynamic solution for the Einstein equation because the principle of causality is violated. Some physicists did not recognize this, even though they could not provide a dynamic solution to show otherwise. In fact, this can be further confirmed by explicit examples such as the metric of Bondi, Pirani and Robinson [69] for $G_{\mu\nu} = 0$, which violates the principle of causality because the energy-momentum tensor of a wave cannot be zero even in vacuum.

Einstein did not understand why the linearized equation can generate the gravitational waves, but the non-linear Einstein equation cannot. This is because Einstein did not delve into the real issue of whether there is a bounded solution.(31) While the gravitational wave has been experimentally detected, there is still no equation to generate such waves.(21) The results from computers are often incorrect and incompatible with analytic calculations. Nevertheless, it is likely that we will develop such an equation since the gravitational wave is necessary for the photons [4, 5].

Einstein often regarded some partial successes of his theory as evidence that it was fully correct. For example, he did not know that the energy of photons must include the energy of the related gravitational wave [4, 5]. Thus, the existence of the gravitational wave is assured. As Feynman commented, Einstein was followed by those who believed that general relativity is applicable only to large-scale problems.(32) This is incorrect since general relativity assures the existence of the gravitational wave for photons. Thus, a thorough review of general relativity is necessary.(33)

The gauge invariance is an incorrect notion derived from an incorrect interpretation of the Yang-Mill-Shaw theory [78, 79]. In fact, Einstein justifies the covariance principle with invalid applications of special relativity [77]. Gauge theory did not produce anything meaningful until the notion of broken symmetry was discovered [80]. Also, the equivalence principle was misunderstood even by Wheeler et al. [40].

The current academic review system often overlooks challenges to entrenched views, such as for example, the claims of Hawking, which are based on an invalid implicit assumption, or the idea that there are no bounded dynamic solutions. Yet new experiments provoke new insights. The speculation of black holes, for example, ignored repulsive gravitation, even though it was well-tested [12].(34) Progress in the current theory of general relativity is often stalled because physicists are not always sufficiently trained in mathematics, and mathematicians are not always sufficiently trained in physics [19]. Thus, it is essential to review our progress to date and to identify errors.

Einstein's other major error is that he failed to recognize repulsive gravitation and thus created an invalid notion of gravitational mass. We have provided a simple experiment showing that Einstein's thought experiment, for increased weight as temperature increases, is wrong. A common mistake among theorists is to neglect the need for supporting examples. This is often why some mistakes are not discovered.

In 1905, Einstein represented the energy of photons with the energy of massless particles without necessary proof [36]. Apparently, he did not know that the energy-momentum tensor of massless particles alone is incompatible with the electromagnetic energy-stress tensor in Maxwell's theory [2]. Since Einstein proposed general relativity in 1915, he could not possibly know that the inconsistency between the electromagnetic energy-stress tensor and the energy-momentum tensor of massless particles can be removed with general relativity.

Einstein did, however, brilliantly consider photons as massless particles. Nevertheless, his shortcomings were exposed in 1912 [37] when he changed the radiation energy L [36] to a more general energy E, and thus made his proof [36] change from incomplete to invalid. Ohanian [81] credited von Laue for a complete proof of the equivalence of mass and photonic energy. However, the fact is that both von Laue and Einstein failed, and $E = mc^2$ is not generally valid. Thus, Einstein did not actually fully understand Maxwell's theory.

Since the notion of photons is due to gravitation, the assertion that general relativity is unsuitable for microscopic phenomena is simply incorrect. Note that $E = mc^2$ is the basis of the assumption in Hawking's space-time singularity theorems. Further, $E = mc^2$ is responsible for the rejection of repulsive gravitation, which is crucial for the unification of gravitation and electromagnetism [12]. Also, it is clear from Einstein's paper [3], that the energy $h\nu$ is for the whole photon.

It is gravity that makes the notion of photons compatible with the electromagnetic wave. Einstein failed to include gravitational wave energy in the photons since he proposed the photons in 1905, well before he conceived general relativity in 1916. Since a charged particle is always massive, it is natural to include the gravitational wave energy in the photons. Since the charge-mass repulsive interaction is absent from quantum theory, it is clearly not a final theory.

Currently, the Einstein equation has no bounded dynamic solutions [15, 16]. Hawking and Penrose, as an example, follow their mathematical results, but fail to consider the physics adequately. Their space-time singularity theorem is the basis of the big bang theory and the existence of black holes, which are based on the assumption that gravity is always attractive. Their incorrect theorems have also led to the claim that general relativity is invalid for microscopic subjects.

Penrose and Hawking(35) have ignored the simple experiments confirming the existence of repulsive gravitation. Thus, some crucial experiments that are related to repulsive gravity have not been addressed for a long time [12].

Since the repulsive gravitational force is distinct from the other four known forces, it is also called the fifth force [11].(36) Such a name emphasizes its character as a new force, but neglected its relations with other forces.

Errors in general relativity have evolved for over 100 years. *We should learn from the teachings of Galileo that experimental confirmation is essential*.

It is clear that general relativity must be extended to a five-dimensional theory, although the nature of the fifth dimension is still not very clear. Philosopher Hu Shih once remarked that in sciences, one can have daring assumptions, but one must also be careful in one's proof. A problem of many physicists is that they often adhere only to the first part. This is why so many crucial errors in general relativity were not discovered but instead were perpetuated.

Acknowledgments

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Appendix A: Riemannian spaces in Mathematics & Einstein Spaces in Physics

In mathematics, the Riemannian space is often embedded in a higher dimensional flat space. Then the coordinates dx^{μ} are determined by the metric through the metric,

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}, \text{ or } -g_{tt}dt^2 + g_{ij}dx^i dx^j \tag{A1}$$

such as the surface of a sphere in a three-dimensional Euclidean space. For a physical space, however, there are insufficient conditions to do so. Since the metric is a variable function, it is impossible to determine the coordinates with the metric. Thus, the coordinates must be physically independent of the metric. In Einstein's theory, this is accomplished with a Euclidean structure as a frame of reference.

In a physical space of Einstein, it has been proven from the theoretical framework of general relativity that a frame of reference with the Euclidean-like structure must exist for a physical space.

For example, the Schwarzschild solution in quasi-Minkowskian coordinates is,

$$ds^2 = -(1 - 2M\kappa/r)c^2dt^2 + (1 - 2M\kappa/r)^{-1}dr^2 + r^2(d\theta^2 + \sin\theta^2 d\varphi^2), \quad \text{(A2a)}$$

where $\kappa = G/c^2$ $(G = 6.67 \times 10^{-8}\ \text{erg cm/ gm}^2)$, M is the total mass, and $r = \sqrt{x^2 + y^2 + z^2}$. And the Euclidean-like structure

$$x = r\sin\theta\cos\varphi, \quad y = r\sin\theta\sin\varphi, \quad \text{and} \quad z = r\cos\theta. \qquad \text{(A2b)}$$

Coordinate transformation (A2b) shows that the space coordinates satisfy the Pythagorean theorem. The Euclidean-like structure represents this fact, but avoids confusion with the notion of a Euclidean subspace, determined by the metric. Metric (A2a) and the Euclidean-like structure (A2b) are complementary to each other in the Riemannian space of physics. These space-time coordinates forms not just a mathematical coordinate system since a light speed $(ds^2 = 0)$ is defined in terms of dx/dt, dy/dt, and dz/dt.

Another example is a spherical mass distribution with the center at the origin, the metric with the isotropic gauge is,

$$ds^2 = -[(1 - M\kappa/2r)^2 / (1 + M\kappa/2r)^2]c^2dt^2$$

$$+ (1 + M\kappa/2r)^4 (dx^2 + dy^2 + dz^2) \tag{A3}$$

while the Euclidean-like structure is the same as (A2b). Then, if the equivalence principle is satisfied, the light speeds are determined by $ds^2 = 0$ [13, 14], i.e.,

$$\frac{\sqrt{dx^2 + dy^2 + dz^2}}{dt} = c\,\frac{1 - M\kappa/2r}{(1 + M\kappa/2r)^3}. \tag{A4}$$

However, such a definition of light speeds is incompatible with the definition of velocity from a Riemannian space,

$$\frac{dl}{dt} = \frac{\sqrt{g_{ij}dx^i dx^j}}{dt} = g_{tt} = c\,\frac{1 - M\kappa/2r}{1 + M\kappa/2r}, \tag{A5}$$

where the distance dl is $\sqrt{g_{ij}dx^i dx^j}$ in a Riemannian space. Since this light speed (A4) is supported by observations, (A5) is invalid in physics.

The above analysis also explains why many theorists insist that the light speeds are not defined, even though Einstein defined them clearly in his 1916 paper as well as in his book, “The Meaning of Relativity”.

They might argue that the light speeds are not well defined since diffeomorphic metrics give different sets of light speeds for the same frame of reference. However, they should note that Einstein defines light speeds after the assumption that his equivalence principle is satisfied [13, 14]. A different metric for the same frame of reference means at most that only one of such metrics is physically valid, and therefore the definition of light speeds are, in principle, uniquely well-defined.

However, since the problem of a physical valid metric has not been solved, whether a light speed would be valid could remain a question. Nevertheless, it has been proven that the Maxwell-Newton Approximation gives the valid first order approximation of the physical metric [15], and the first order of the physically valid light speeds are solved. Since metric (A3) is compatible with the Maxwell-Newton approximation, the first order of light speed (A4) is valid in physics.

Thus, the groundless speculation that local light speeds are not well defined is proven incorrect. In essence, the velocity definition (A5), which leads to the notion of the Doppler redshifts, has been rejected by experiments. Nevertheless, some skeptics might still prefer to accept formula (A5) after light speed (A4) is confirmed by the experiment of local light speeds. This is called academic freedom.

Note also that the Euclidean-like structure implies that the ratio of the circumference and the diameter of a circle is always π. Einstein's error, based on an invalid application of special relativity, is due to adding up pieces measured from different coordinate systems [77]. Apparently, Einstein's misconception was derived from examples in mathematics and thus was mistaken that a physical Riemannian space must be curved. He even justifies it with wrong applications of special relativity!

A problem in Einstein's theory, as pointed out by Whitehead [82] and Fock [83] is that the physical meaning of coordinates is ambiguous and confusing. In view of this, it is understandable that the notion of an embedded Riemannian space was used when the physical nature of the problem is not yet clear.

Since Einstein is not a mathematician, his natural inclination would be to utilize the existing theory of Riemannian space. However, as Whitehead [82] saw, this created a seemingly irreconcilable problem between coordinates of a curved space-time and physics. To settle this, we shall define an Einstein space as a Riemannian space with the Euclidean-like structure [41]. If an Einstein space also satisfies the equivalence principle, then it is a physical space.

Analysis [84] demonstrates that Hubble's Law is not necessarily related to the Doppler redshifts. It also shows that the notion of an expanding universe is related to contradictory assumptions and thus is unlikely a physical possibility.

If the observed gravitational redshifts are not due to an expanding universe, then what causes such redshifts that are roughly proportional to the distances from the observer. One possibility is the scattering of a light ray along its path to the observer. In physics, it is known that different scatterings are common causes for losing energy of a particle, and for the case of photons it means redshifts. Since such an effect is so small, it must be the scattering of a weak field. In fact, the inelastic scattering of light by the gravitational field has been speculated. Unfortunately, testing such a conjecture is not possible because no current theory of gravity is capable of handling the inelastic scatterings of lights.

At present, Einstein's equation does not have any dynamic solution. Thus, to solve this puzzle rigorously is a problem for the remote future. Nevertheless, the assumption that observed redshifts could be due to inelastic scatterings may help to explain some puzzles of observed facts. For instance, it is known that younger objects such as star-forming galaxies have higher intrinsic redshifts, and objects with the same path length to the observer have much different redshifts while all parts of the object have about the same number of redshifts.

A noted advancement of the Euclidean-like structure is that notions used in a Euclidean space could be adapted much more easily in general relativity. Many things would be calculated as if in a Euclidean space. On the other hand, the speculations related to the notion of an expanding universe would cease to function, and physics should return to normal. Nevertheless, when a transformation between different frames of reference is considered, the physical space is clearly Riemannian, as Einstein discovered.

Appendix B: The Principle of Causality in Physics

Physics is essentially a science for causality. There are two aspects in causality: its relevance and its time ordering. In time ordering, a cause event must happen before its effects. This is further restricted by relativistic causality that no cause event can propagate faster than the light speed in the vacuum. The time-tested assumption that phenomena can be explained in terms of identifiable causes will be called the principle of causality. This is the basis of relevance for all scientific investigations.

Normally causality means causes will lead to consequences. It should be emphasized that the principle assumed:

(1) From the consequences that causes must exist even we do not know what they are.

(2) The partial consequences of the cause are identified even its full consequences remain to be known.

Then, we can use such partial consequences as requirements to decide whether a solution or even an equation is valid in physics. This might often provide crucial steps to solve a problem correctly. For example, this is how the equation (6) for the electromagnetic wave as a source was modified.

Thus, this principle implies that any parameter in a solution for physics must be related to some physical causes. Moreover, the principle of causality implies that a weak source would produce a weak gravity. Here this principle will be elucidated first in connection with symmetries of a field, the boundedness of a field solution. Although this principle alone cannot derive a field equation or its solution, it can help determine whether they are valid in physics. This has made a difference in the investigation of gravitation [4, 5, 11, 12, 15, 16, 39].

In practice, when the considered field is absent, physical properties are ascribed to the space-time as in a "normal" state. For example, the electromagnetic field is zero in a normal state. Then, any deviation from the normal state must have physically identifiable causes. Thus, the principle of causality implies that the symmetry must be preserved if no cause breaks it. The implication of causality to symmetry has been used in deriving the inverse square law from Gauss's law. The normal state of a space-time metric is the flat metric in special relativity. Thus, if a metric does not possess a symmetry, then there must be a physical cause(s) which has broken such a symmetry. For a spherically symmetric mass, causality requires that the metric is spherically symmetric and asymptotically flat. Also, a weak cause can lead to only weak gravity. Thus, Einstein's weak gravity is a consequence of causality.

However, the physical cause(s) should not be confused with the mathematical source term in the field equation.

In general relativity, the cause of gravity is the physical matter itself, but not its energy tensors in the source term of Einstein's field equation. The energy-stress tensors (for example, the perfect fluid model) may explicitly depend on the metric. Since nothing should be a cause of itself, such a source tensor does not represent the cause of a metric. For the accompanying gravitational wave of an electromagnetic wave, the physical cause is the electromagnetic wave. Thus, one should not infer the symmetries of the metric based on the source term instead of its causes.

Moreover, inferences based on the source term can be misleading because it may have higher symmetries than those of the cause and the metric. For instance, a transverse electromagnetic plane-wave is not rotationally invariant with respect to the z-direction of propagation. But the related electromagnetic energy-stress tensor component $T(E)_{tt}$ for a circularly polarized wave is. Such an assumption violates causality and results in theoretical difficulties.

A reason that the Einstein equation did not have a bounded dynamic solution is its violation of causality. In the Einstein equation the left side is the Einstein tensor $G_{\mu\nu}$ and the right side are the energy-momentum tensors. For the dynamic case, the energy-momentum tensor of the gravitational waves should have been included. Thus, for the dynamic case, the Einstein equation violates the principle of causality and thus has no bounded dynamic solution. The modified Einstein equation (8) can have a dynamic solution because the missing energy-momentum tensor has been added back. It is surprising that physicists did not find this principle of causality for general relativity earlier.

Classical electrodynamics implies that the flat metric is an accurate approximation, caused by the presence of weak electromagnetic waves. This physical requirement is supported by the principle of causality, which implies such a metric to be a bounded periodic function. However, this required boundedness is not satisfied with many solutions in the literature [85-94]. If these authors understood the principle of causality, they would not have produced them.

Many theorists and journals do not understand the principle of causality adequately. For instance, the Physical Review accepted an unbounded solution as valid in physics. As well, the Royal Society (London) accepted Hawking, even though the space-time singularity theorems violate the principle of causality. A major problem is that the teaching of Galileo on the importance on experimental verification is often forgotten.

Appendix C: My 1993 Meeting with Prof. S. T. Yau

In 1993, I went to Hong Kong to present my paper, "Einstein's Radiation Formula and Modifications in General Relativity," in *The Second William Fairbank Conference On Relativistic Gravitational Experiments In Space* & *Related Theoretical Topics*. Dr. Daniel Tse, the

President of Baptist University, suggested that I should meet Prof. S. T. Yau who was considered as an expert in this area. Prof. Yau and I met in his office at the Chinese University of Hong Kong. Prof. S. Y. Cheng was also present at the beginning. Prof. Yau told me that the Einstein equation had bounded dynamic solutions, whereas I presented the arguments of my paper. After long discussions, Prof. Yau finally agreed that my calculation was correct. Subsequently, Yau told Christodouluo and Klainerman that he had lost his earlier interest on their work [70].

Appendix D: The International Mathematical Union (IMU) executives

The International Mathematical Union (IMU) Executives (1979-1990)

Terms	Presidents	Vice-Presidents	Secretary	Members
1987-1990	L. D. Faddeev	W. Feit, L. Hörmander	O. Lehto	J. Coates, H. Komatsu, L. Lovász, J. Palis Jr., C. S. Seshadri
1983-1986	J. Moser	L. D. Faddeev, J. -P. Serre	O. Lehto	S. Mizohata, G. D. Mostow, M. S. Narasimhan, C. Olech, J. Palis Jr.
1979-1982	L. Carleson	M. Nagata, J. V. Prohorov	J.L. Lions	E. Bombieri, J. W. S. Cassels, M. Kneser, O. Lehto, C. Olech

The IMU executives should have provided an example of the dynamic solutions to defend Schoen and Yau [17].

Endnotes

(1)Einstein actually justified his covariance principle with an invalid application of special relativity, which is unrelated to gravity [77].

(2)Gullstrand [95, 96] pointed out that Einstein had not justified the remaining perihelion with a necessary perturbation.

(3)Einstein's famous thought experiment on weight increment as temperature increased failed because repulsive gravity does exist.

(4)Einstein failed to deal with the mathematical difficulty because of inadequate training in mathematics.

(5)To help physicists, mathematicians must understand physics; otherwise they may make errors as Hawking [21] or Yau [17] did.

(6)This is a problem that a mathematician turned physicist, such Hawking, Penrose, and Yau, can cause.

(7)Now, the damage to physics is deepen to wrong applications.

(8)Physicists do not understand that a mathematical procedure can be incompatible with an invalid equation.

(9)They believed that the truth is always within their circle of theorists. They forget the teaching of Galileo on experiments.

(10)Some claim alternatively $T \approx 2\pi\sqrt{lm_i/m_a g'}$, this only means $g = g'm_a/m_i$, where m_a is the gravitational mass and m_i is the inertial mass.

(11)A Napier and I have done the experiment to measure the frequency changes due to heating-up with a torsion balance scale. We did obtain an increment of the period from the small brass balls and a reduction of gravity for the large lead balls after heating up. However, we cannot get stable readings of the frequencies due to the interference of passing subway trains nearby. We shall publish our results after data are significantly improved.

(12)It is a surprise that Einstein actually did not understand Maxwell's theory well. It is difficult to reconcile this with the fact that he proposed special relativity.

(13)It is a common practice of relativists for an author to claim something without adequate supports. For instance, Wald [21] claimed that he has a second order approximation of the Einstein equation, but never provided one.

(14)Such compatibility is not a problem for a linear equation but could be a major problem for a non-linear equation.

(15)The existence of the Euclidean-like structure in a physical space clarifies the difference between physical Riemannian space and a mathematical Riemannian space embedded in a higher-dimensional Euclidean space. This was the crucial point needed to settle the difference between Einstein and Whitehead [96].

(16)The repulsive gravitational force was first obtained from the metric of a static Einstein equation for a charged particle. However, this new force was first recognized by Lo [97] with other then unexplainable experiments.

(17)It is expected that the repulsive force is very small from a single particle otherwise Maxwell could have found it.

(18)G. t'Hooft [46] incorrectly believed that the mass of an electron includes its electric energy. He did not understand Newtonian mechanics or special relativity adequately.

(19)Frank A. Wilzcek [32] incorrectly believed that $E = mc^2$ is unconditional. Thus, their proof (Frank. A. Wilczek, along with David Gross and H. David Politzer) for asymptotic freedom is actually incomplete.

(20)In current four-dimensional theory such a force does not exist.

(21)The MIT team claimed that they got a dynamic solution from the Einstein equation with a computer. However, it is known that a computer calculation is fast but can be unreliable. In my opinion, since it has been shown analytically that there is no bounded dynamic solution [15, 16], their calculation must be wrong. Apparently, unlike Prof. P. Morrison of MIT, they do not understand the principle of causality. Since the Nobel

Prize Committee had been wrong in the gravitational calculation in 1993 [68], it would appear that they made another error again.

(22)At the time of awarding the Fields Medal, Yau was affiliated with the Institute for Advanced Study at Princeton. Now he is at Harvard as a professor in mathematics. Note that in 1990, a Fields medal was awarded again to E. Witten of the Institute for Advanced Study.

(23)After returning to MIT, Prof. Morrison suggested that I should write a book on relativity. However, I believed that such a time had not come since related problems had not been solved.

(24)Yau [17] has not considered whether his theory can be applied to a two-body problem. For a dynamic case, he did not see that the Einstein equation and its linearization are not compatible.

(25)Michael Francis Atiyah has been a leader of the Royal Society (1990-1995), master of Trinity College, Cambridge (1990-1997), chancellor of the University of Leicester (1995-2005), and President of the Royal Society of Edinburgh (2005-2008). Since 1997, he has been an honorary professor at the University of Edinburgh. However, because of his crucial role for the 1982 award of a Fields Medal to Yau, Prof. Peter C. Sarnak, Chairman of the 2011 Shaw Prize Committee for Mathematics found out that Atiyah does not understand general relativity. In fact, he found none of the executives of IMU understands general relativity.

(26)Ludwig D. Faddeev, the Chairman of the Fields Medal Committee, wrote ("On the work of Edward Witten"): "Now I turn to another beautiful result of Witten - proof of positivity of energy in Einstein's theory of gravitation. Hamiltonian approach to this theory proposed by Dirac in the beginning of the fifties and developed further by many people has led to the natural definition of energy. In this approach a metric y and external curvature h on a space-like initial surface $S^{(3)}$ embedded in space-time $M^{(4)}$ are used as parameters in the corresponding phase space. These data are not independent. They satisfy Gauss-Codazzi constraints - highly non-linear PDE, The energy H in the asymptotically flat case is given as an integral of indefinite quadratic form of ∇y and h. Thus, it is not manifestly positive. The important statement that it is nevertheless positive may be proved only by taking into the account the constraints - a formidable problem solved by Yau and Schoen in the late seventy as Atiyah mentions, 'leading in part to Yau's Fields Medal at the Warsaw Congress'. Witten [18] proposed an alternative expression for energy in terms of solutions of a linear PDE with the coefficients expressed through y and h …." Faddeev failed to see that the so-called 'natural definition of energy' is invalid. Thus, he failed to see that Yau's theory excludes all the two-body problems. This is clear since there is no such explicit example to support the positive mass theorem.

(27)D. Christodoulou, Ph.D. (1971) in Physics, Princeton University, Advisor John A. Wheeler, who also missed crucial calculations [98, 99]. Christodoulou's claim [70] on the dynamic solution construction actually has never been completed and thus is false [71]. However, it was incorrectly regarded as valid by those who do not sufficiently understand mathematics. The 2011 Shaw Prize committee also made a mistake by awarding a half prize to Christodoulou for his error, which was disagreed the honorable Gullstrand [95, 96] of the Nobel Committee. Subsequently Christodoulou was elected to the U.S. National Academy of Sciences (2012). Although Christodoulou made errors, which were not seen by the 1993 Nobel Committee [100], his errors are now well-established [71]. Christodoulou claimed in his autobiography that his work is essentially based on two sources: (1) The claims of Christodoulou and Klainerman on general relativity as shown in their book The Global Nonlinear Stability of the Minkowski Space [70]; (2) Roger Penrose had introduced, in 1965, the concept of a trapped surface and had proved that a space-time containing such a surface cannot be complete. However, this work of Penrose, which uses an implicit assumption of unique sign for all coupling constants, actually depends on the errors of Christodoulou and Klainerman [70].

(28)Pauli's "infinitesimal" principle of equivalence, was objected to by Einstein as inadequate, but is commonly and mistakenly regarded as Einstein's version of the principle [101].

(29)Wald's erroneous view on the covariance principle is very popular in China, due to the influence of C. N. Yang.

(30)The original purpose of this example is to show that the bending of light is gauge invariant up to the second order. However, it turns out to be also a perfect example to show that gauge invariance is invalid.

(31)Compatibility between mathematics and physics is usually not a problem for a linear equation, but could be a major problem for a non-linear equation.

(32)Because this repulsive force is inconsistent with Maxwell's theory and Einstein's theory, many ignore this weak repulsive gravitational force as an experimental error or as if it did not exist. For instance, Michael Green and Edward Witten still do not know that experiments on the existence of repulsive gravitation have been confirmed. The real problem is that the existence of repulsive gravitation can prove Einstein wrong.

(33)Such a review is necessary for Dr. Kate Kirby, the CEO of APS, to renew physics because erroneous theories have been in the dominating positions. For instance, even the Fields Medal and the Nobel Prize Committee have been mistaken. Moreover, erroneous theorists such as Hawking, Penrose, Wald [21] and Wheeler et al. [40] were incorrectly regarded as experts in general relativity. In addition, Einstein himself has made serious errors.

(34)The reduction of gravity can be observed in a vacuum tower. This error of Galileo can be verified at NASA, which has a vacuum chamber.

(35)Penrose and Hawking do not understand the principle of causality. Their space-time singularity theorems are based on the invalid implicit assumption that all the couplings have the same sign [21]. Since we still do not have the Einstein equation that can have a dynamic solution, it is unlikely that Penrose can validly prove the existence of a black hole. Although we have observed very massive objects, there is no evidence to verify that they are black holes. Moreover, theoretically the existence of a black hole is clearly questionable because gravitation is no longer always attractive. Nevertheless, based on out-dated assumption that gravity were always attractive, Penrose wrote two papers in 1963 [102] and 1965 [103] that showed a black hole could exist. Since repulsive gravitation was discovered in 1997 [43] and the necessity to extend general relativity to a five-dimensional space was published in 2015 [23], Penrose did not see them in 1965. Thus, he did not re-justify the claim such that a black hole could exist in spite of the existence of repulsive gravitational force. Moreover a five-dimensional theory is obviously beyond Penrose and the Nobel Committee. Thus, Penrose actually did not show the existence of a black hole, and the Nobel Committee is not yet ready to give an award in this area since this is also clearly beyond their understand in physics. It is also interesting to note that most theorists, except S. Weinberg, do not understand that the picture commonly used to represent a gravitational field with a net is incorrect.

(36)Recently, unaware of that the fifth force has been discovered before 1997, some British scientists claimed they possibly discovered the fifth force. If they have discovered a new force, it should be called the sixth force.

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