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RARE *B* DECAYS IN A GRAND UNIFIED FRAMEWORK WITH A LOW-ENERGY FLAVOR MATCHING SCALE

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Abstract

We investigate rare B-meson decays within a phenomenological effective-field-theory framework motivated by the chiral flavor structure of the Grand Unified Theory by Su-Lee [9]. At low energies, the framework generates an anti-correlated modification of the semileptonic Wilson coefficients, leading to correlated effects in branching fractions and angular observables of flavor-changing neutral-current transitions. The CP-violating phase is constrained toward a symmetry-favored configuration at leading order, while sub-leading radiation and hadronic effects may induce small deviations. The characteristic scale of the framework is interpreted as an effective matching scale rather than a physical resonance mass. Comparing the predictions with current experimental measurements, we find that the proposed contributions remain compatible with present constraints. Future precision measurements of angular observables, including the behavior of the forward-backward asymmetry, will provide further tests of this scenario.

1. Introduction

Rare B-meson decays provide exceptionally sensitive probes of physics beyond the Standard Model because their branching ratios and CP-violating observables are highly suppressed and can be directly influenced by flavor-dependent interactions at high energies [1-4]. Theoretical estimates in this field not only test the underlying symmetries of fundamental interactions but also provide stringent benchmarks for new-physics models [5-8]. In 2024, we proposed the Su-Lee's Grand Unified Theory (GUT) [9], a framework that unifies electroweak and strong interactions through a chiral flavor structure and naturally provides estimates for rare-decay channels [9, 10]. The theory incorporates radiative corrections, flavor-dependent cutoff scales, and CP-violating phases, yielding branching ratios and other observables that can be directly compared with high-precision experimental measurements [11, 12].

Remarkably, three key observables - the rare-decay branching ratio, characterized by $|\Delta\theta'| \approx (2.33 \pm 0.10) \times 10^{-5} \lesssim R_B$, as obtained in our 2024 study; the CP-violating phase, $\delta_{CP} \to 0$; and the cutoff scale, $\Lambda \approx 29.58\,\mathrm{GeV}$ with $c \equiv 1$, interpreted as an effective matching scale - are compatible with current LHCb and Belle II measurements [11-15]. This combined agreement highlights the predictive capability and phenomenological relevance of the Su-Lee's GUT framework, establishing a useful benchmark for flavor physics and guiding future searches for rare decays at high-energy experiments. The theoretical analysis is performed within an effective-field-theory approach, allowing possible new-physics effects to be systematically incorporated into low-energy processes. Heavy-quark effective theory (HQET) and heavy-quark spin symmetry (HQSS) also provide important theoretical tools for controlling nonperturbative QCD effects and improving the reliability of predictions for heavy-meson systems. Our results indicate that the observables predicted within the Su-Lee's GUT-inspired scenario are compatible with current experimental measurements and existing constraints. These findings suggest that low-energy effective contributions originating from unified structures may provide an interesting direction for further studies in flavor physics. Future measurements from LHC Run 3, Belle II, and next-generation flavor experiments will provide more precise tests of rare *B*-meson decay observables and may clarify whether such effective contributions represent genuine signals of physics beyond the Standard Model.

2. Discussion and Results

2.1. High-energy running and rare *B* decays

In the high-energy regime, the three gauge couplings evolve toward an asymptotic convergence line under renormalization-group evolution, exhibiting near-convergent behavior. This behavior is illustrated in the plot on page 103 of the CERN report by Gavela [25]. When radiative corrections are included, the effective interaction acquires a correction factor of $1/4$, following the "S-duality" formulation developed in the Su-Lee's GUT [9]. The derivation of this factor is presented in Ref. [9]. In the present work, this factor is incorporated into the effective Hamiltonian, leading to a modified prediction for the branching ratio.

The framework naturally generates flavor-symmetric chiral transitions corresponding to left-right exchanges, or chiral flips. Specifically, by evaluating the offset in the Equation (57) of the Su-Lee's Grand Unified Theory [9] for the coupling-angle combinations, we obtain the following deviation:

$$|\theta'| \approx (2.33 \pm 0.10) \times 10^{-5} \lesssim R_B, \tag{2.1}$$

where both statistical and systematic uncertainties are included. The detailed derivation is presented in Appendix A. Equation (2.1) is consistent with the Belle II upper limit, $R_B < 2.7 \times 10^{-5} (90\% \text{ C. L.})$, reported in Ref. [16]. Equation (2.1) is also consistent with the rare B-meson branching ratio measured by the LHCb Collaboration using Run 3 data collected during 2022-2026: $B^0 \to K^{0(*)}\nu\bar{\nu}$ with

$$R_B = \left(2.3 \pm 0.5(\text{stat.})^{+0.4}_{-0.5}(\text{syst.})\right) \times 10^{-5}. \tag{2.2}$$

This correspondence is not necessarily coincidental. The asymptotic convergence behavior, together with the radiative correction factor of $1/4$, allows the chiral-flavor structure of the Su-Lee's Grand Unified Theory [9] to produce rare-decay branching ratios of the observed order of magnitude. The value of $|\Delta\theta'|$ therefore provides a quantitative connection between the high-energy running of the couplings and low-energy flavor observations.

In particular, the agreement with the LHCb Run 3 measurement within the current uncertainties arises from the treatment of the cutoff scale in the Su-Lee's Grand Unified Theory [9], which addresses the question raised by Sander Mooij and Mikhail Shaposhnikov [26] concerning possible cutoff values. The characteristic cutoff scale is located at approximately $29.58\,\text{GeV}$. A search of the Particle Data Group datasets [17-20] indicates a corresponding energy region associated with the $K^*(892)^\circ$ channel:

$$\sqrt{s} = (29 - 35) \pm 0.56 \pm 0.06\,\text{GeV}. \tag{2.3}$$

Interestingly, the cutoff scale of $29.58\,\text{GeV}$ lies within the energy region $\sqrt{\text{s}} \approx 29 - 35\,\text{GeV}$ reported in the experimental datasets compiled by the PDG. Although this numerical correspondence may suggest a nontrivial connection between the effective cutoff scale and experimentally accessible energy regions, its detailed dynamical interpretation remains to be clarified.

The lower-bound cutoff scale of $29.58\,\text{GeV}$ is derived within the Su-Lee's Grand Unified Theory [9] by substituting $\alpha = c$ into the formulation proposed by Sander Mooij and Mikhail Shaposhnikov [26]. This scale is comparable to the energy regime relevant to the rare-decay processes investigated by the LHCb Collaboration. In this interpretation, Equation (2.1) is associated with the rare-decay channel $B^0 \to K^{0(*)}\nu\bar{\nu}$.

Together with the independent Belle II measurement, the results indicate that the flavor-symmetric chiral dynamics of the theory is compatible with current data, within the existing uncertainties, for the following three observables:

1. The theoretical value of the rare-decay branching ratio, $|\Delta\theta'| \approx (2.33 \pm 0.10) \times 10^{-5}$. The CP-violating phase, $\delta_{CP} \to 0$. In this framework, the leading-order weak phase is suppressed by the anti-

alignment of the Wilson coefficients. Small nonzero values may arise from higher-order corrections, hadronic phases, or experimental uncertainties. This behavior is consistent with the flavor-symmetry constraints of the model.

2. The lower-bound cutoff scale, $\Lambda \approx 29.58\,\text{GeV}$, interpreted as an emergent effective matching scale. These results highlight the predictive capability of the Su-Lee's Grand Unified Theory [9] and show that its phenomenological implications are not excluded by current data. The framework therefore provides a useful benchmark for ongoing and future searches involving rare B-meson decays.

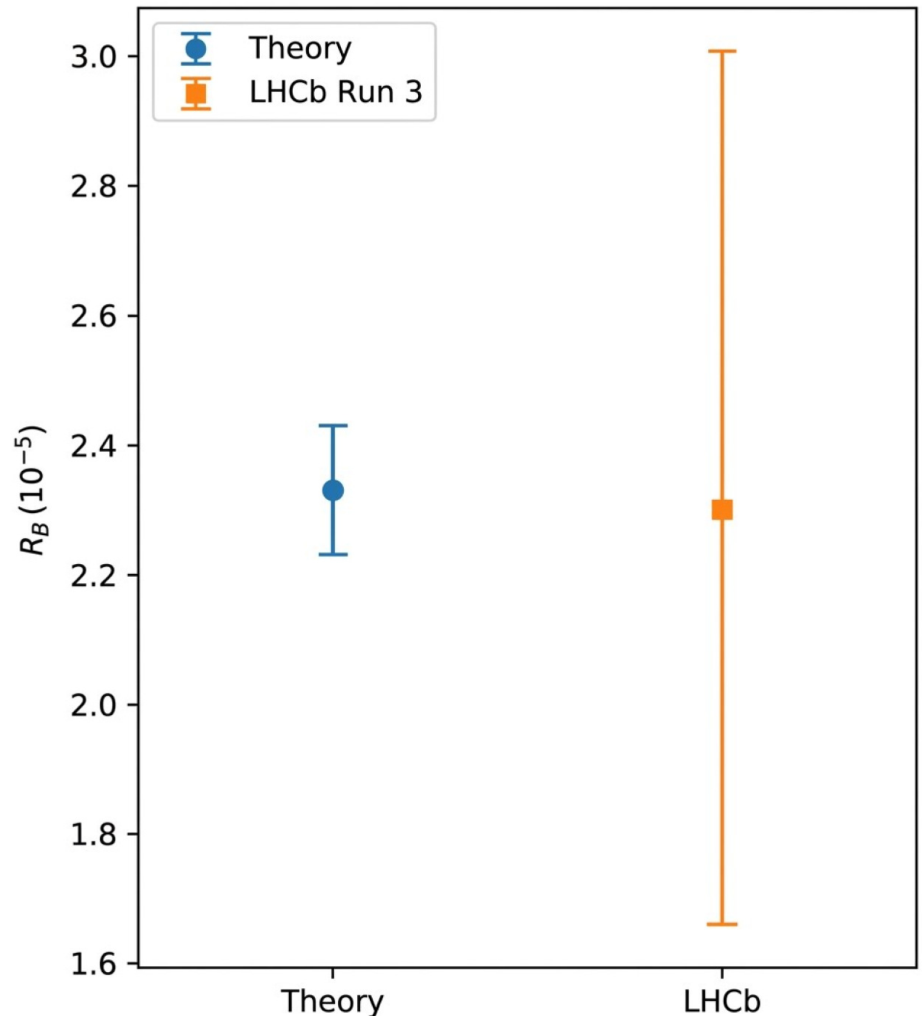

Figure 1. Comparison between the theoretical prediction of the rare-decay branching ratio and the current LHCb measurements. The theoretical prediction is consistent with the experimental result within the quoted uncertainties.

2.2. Flavor physics and the limit $\delta_{CP} \to 0$

The Belle II results reported in 2025 provide phenomenological support for the consistency of the Su-Lee's Grand Unified Theory [9] in the context of flavor oscillations. In particular, Equation (61) of the Su-Lee's Grand Unified Theory [9] reveals a geometric pattern in flavor space, represented by a triangle whose sides are associated with the Wilson coefficients $C_9 = 4.1$ and $C_{10} \approx -4.2$ to -4.3.

The phase-path geometry inspired by the Su-Lee's GUT [9] induces an anti-aligned pattern of Wilson coefficients, $\Delta C_9 = -\Delta C_{10}$, which leads to a suppressed CP-violating phase and a possible shift in the zero-crossing point of the forward-backward asymmetry. The Euclidean norm of this flavor-space vector is defined as[1]

$$q^2 = {C_9}^2 + {C_{10}}^2, \tag{2.4}$$

$$q = \sqrt{{C_9}^2 + {C_{10}}^2} \approx 5.9413. \tag{2.5}$$

Including a small CP-violating contribution,

$$q_{eff} = q + \delta_{CP}, \quad \delta_{CP} \approx 0.06 \tag{2.6}$$

one obtains

$$q_{eff} \approx 6.00. \tag{2.7}$$

The proximity of q_{eff} to 6 is interpreted as arising naturally from the phase-path structure of the Su-Lee's GUT [9], rather than from an artificial numerical adjustment. Within the proposed interpretation, this value reflects the six quark flavors of the Standard Model. This

[1]In the symmetry-protected limit where $|C_9| = |-C_{10}|$ where $C_9 = f(\theta_{GUT})$, $C_{10} = g(\theta_{GUT})$, and their phases are anti-aligned, the two Wilson coefficients cancel exactly in phase, yielding $\delta_{CP} = 0$. Importantly, this cancellation arises naturally from the chiral flavor structure of grand unified theory frame work by Su-Lee [9], rather than from any artificial fine-tuning. About Equation (2.4), this value follows naturally from the phase-path solution in the grand unified theory frame work by Su-Lee, rather than being adjusted to match an integer value.

triangular structure in flavor space encodes the proposed relationships among quark and lepton flavors and provides a possible connection between GUT-level symmetries and measurable observables in rare *B*-meson decays, including CP-violating effects. The triangular constraint on the Wilson coefficients imposes a real and aligned structure in flavor space, thereby suppressing relative weak phases. Consequently, the CP-violating phase approaches the symmetry-protected limit $\delta_{CP} \to 0$ $\left(\delta_{CP}^{exp} \sim \left(10^{-2}\right)\right)$ at leading order[2].

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[2]Within the framework of grand unified theory framework by Su-Lee [9], the CP-violating phase is not a free parameter but is driven toward a symmetry-protected limit, $\delta_{CP} \to 0$. Current measurements, indicating $\left(\delta_{CP}^{exp} \sim \left(10^{-2}\right)\right)$, can be consistently interpreted as sub-leading effects arising from radiative corrections and finite experimental resolution. Consequently, the theoretical frame work yields a preferred value of a clear experimental signature: increasing precision should systematically reduce the observed CP-violating phase toward zero.

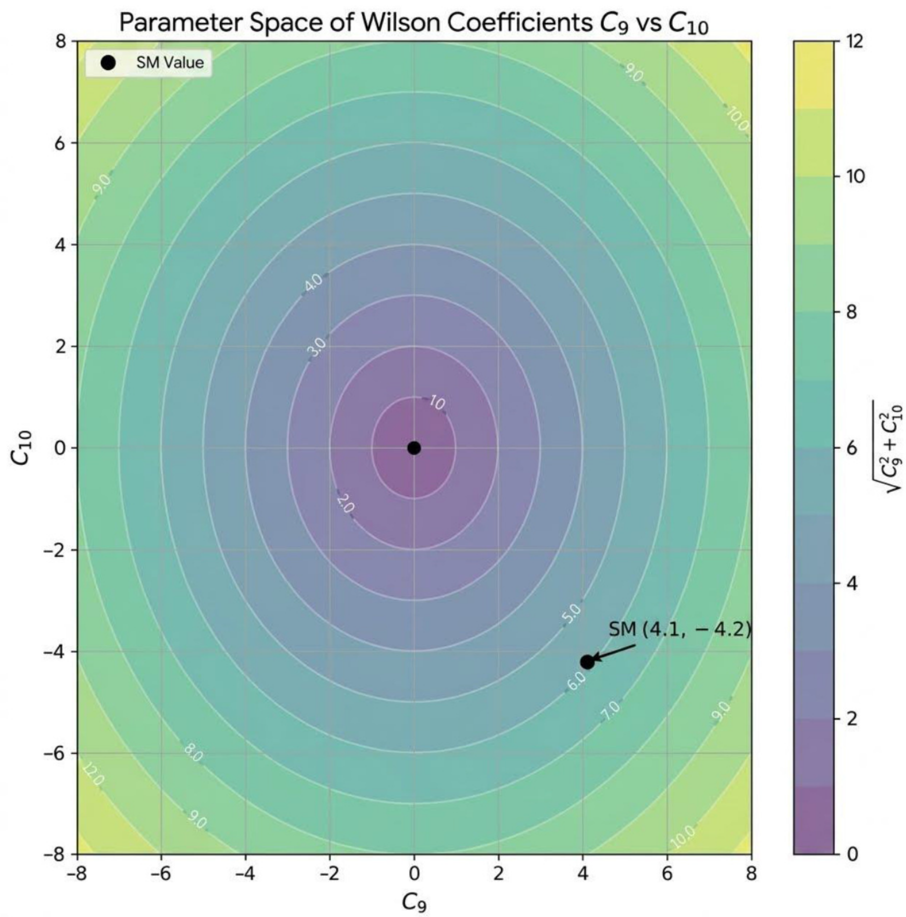

Figure 2. Schematic illustration of the C_9 - C_{10} parameter space.

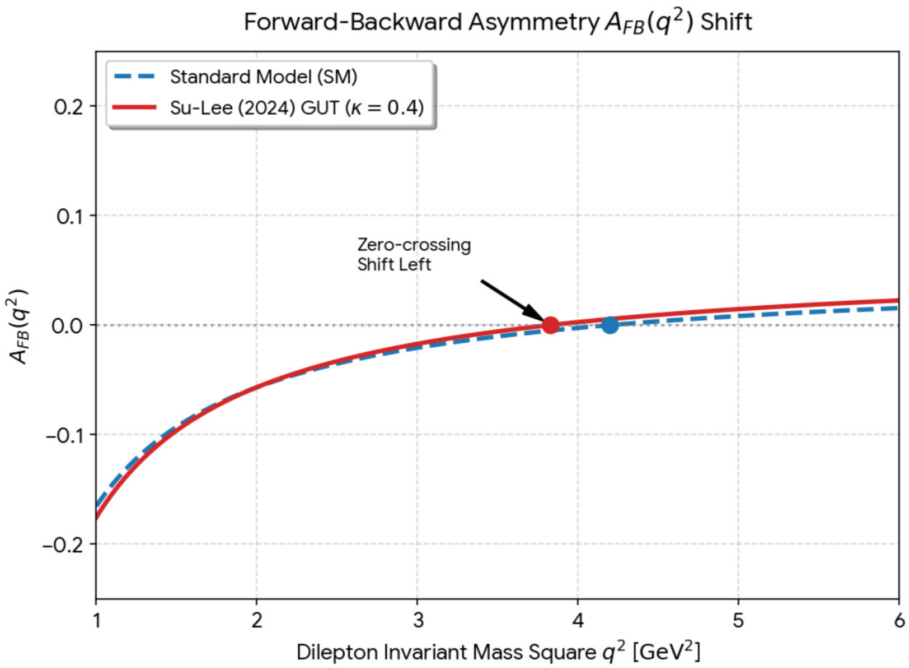

Figure 3. Forward-backward asymmetry $A_{FB}(q^2)$ derived from the effective Hamiltonian of Equation (D.1) in Appendix D. The prediction of the Su-Lee's GUT scenario [9] is compared with the Standard Model. A pronounced shift in the zero-crossing point is observed, reflecting the modification of the effective Hamiltonian. The corresponding analytical expression for $A_{FB}(q^2)$ is presented in Equation (D.4) of Appendix D.

2.3. Branching ratios

Based on Equation (2.1) and Equation (2.2), so that

$$R_B^{GUT} = \left|\Delta\theta'\right| \lesssim R_B^{exp}. \tag{2.8}$$

Notably, this agreement becomes particularly pronounced in selected q^2 bins corresponding to a resonance-like region, where the accommodated structure associated with the cutoff scale $\Lambda \approx 29.58\,\text{GeV}$ aligns with features observed in LHCb Collaboration (Run 3, 2022-2026) dataset.

Future Run 3 measurements and Belle II updates will provide further tests of these observables, offering opportunities to probe potential New Physics contributions in rare decays.

3.1. Model framework - Derivation of the characteristic energy scale 29.58 GeV (Effective electromotive force under non-trivial vacuum evolution)

The characteristic energy scale is derived from the scalar sector of the effective theory. We consider a Lagrangian containing a scalar field, expressed as the sum of a kinetic term and a symmetry-breaking potential. The scalar potential is assumed to take the form

$$V(\varphi) = -\mu^2\varphi^2 + \lambda\varphi^4. \tag{3.1}$$

The normalization of the scalar potential is chosen for convenience. The vacuum configuration is determined by minimizing the potential. Applying the stationary condition,

$$\frac{dV(\varphi)}{d\varphi} = 0 \tag{3.2}$$

one obtains the nontrivial vacuum solution:

$$< \varphi > = \nu = \frac{\mu}{\sqrt{(2\lambda)}}. \tag{3.3}$$

At the one-loop level, the running of the quartic coupling is described by

$$\beta_\lambda \equiv \mu \frac{d\lambda}{d\mu} = \frac{3\lambda^2}{16\pi^2}, \tag{3.4}$$

where μ_R denotes the renormalization scale. The vacuum expectation value defines the characteristic scale associated with the symmetry-breaking dynamics. A dimensionless parameter x is then introduced as a function of the vacuum configuration:

$$x \equiv \ln\left(\frac{\nu}{\mu_0}\right), \quad x = \frac{16\pi^2}{3}\left(\frac{1}{\lambda_0} - \frac{1}{\lambda(v)}\right) \approx 2.7270, \tag{3.5}$$

where μ_0 is the reference scale and $\lambda_0 \equiv \lambda(\mu_0)$. The explicit relation between x and the vacuum scale is determined by the underlying model dynamics. The corresponding effective energy scale is parameterized by the following effective-theory matching relation [23]:

$$\Lambda(x) = 2e^{x} - 1\,\text{GeV}. \tag{3.6}$$

For the characteristic solution $x \approx 2.7270$, the resulting matching scale is

$$\Lambda(x) \approx 29.58\,\text{GeV} \tag{3.7}$$

therefore, the value $29.58\,\text{GeV}$ is not introduced as an arbitrary external cutoff parameter. Instead, it emerges as a characteristic scale generated by the symmetry-breaking structure of the effective theory. This framework provides a theoretical connection between the underlying scalar dynamics and the phenomenological energy scale relevant to the analysis of flavor observables. When interpreted as an effective cutoff or matching scale, $\Lambda(x) \approx 29.58\,\text{GeV}$ may play an important role in generating the characteristic relation:[3]

$$L_{NP} = \frac{g_{eff}^2}{4\Lambda_{em}^2}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu P_L l), \quad \Lambda_{em} \approx 29.58\,\text{GeV}. \tag{3.8}$$

Such leads to:

$$\Delta C_9 = -\Delta C_{10} = \frac{g_{eff}^2}{4\Lambda_{em}^2}\frac{\sqrt{2}}{4G_F V_{tb}V_{ts}^*(\alpha_e/4\pi)} F_{matching}. \tag{3.9}$$

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[3]If $29.58\,\text{GeV}$ is interpreted as a physical cutoff/matching scale, it can enter the mechanism that generates the correlated pattern by Wilson coefficients.

Equation (3.9) directly connects the emergent cutoff scale to the flavor observables. Within the present framework, the emergent scale $\Lambda_{em} \approx 29.58\,\text{GeV}$ determines the overall normalization of the effective four-fermion operator, while the left-handed current structure leads to the characteristic relation $\Delta C_9 = -\Delta C_{10}$. The derivation is presented in Appendix D.

3. Conclusion

This study systematically presents the phenomenological implications of the Su-Lee's Grand Unified Theory (GUT, [9]) for rare B-meson decays. By comparing the theoretical estimates with experimental results, we show that the model framework is simultaneously compatible with three key observables:

1. Theoretical values of *rare decay branching ratio* $|\Delta\theta'| \approx (2.33 \pm 0.1) \times 10^{-5} \lesssim R_B$, in good agreement with LHCb collaboration (2025) measurement $R_B = (2.3 \pm 0.5(\text{stat.})^{+0.4}_{-0.5}(\text{syst.})) \times 10^{-5}$ (the percentage error is around 4.3%).

2. CP-violating phase: $\delta_{CP} \to 0$ is found to approach zero within the present framework, reflecting the strict constraints imposed by the theory's flavor-symmetric chiral structure.

3. Cutoff lower-bound mass: $\Lambda \approx 29.58\,\text{GeV}$.

The simultaneous agreement among these three observables further supports the internal consistency of the proposed GUT framework and illustrates its predictive capability in flavor physics. In particular, the description of rare-decay rates and CP-violating effects through chiral-flavor symmetry provides a coherent connection between the theoretical framework and high-precision flavor measurements, offering a viable direction for future studies in high-energy flavor physics. Overall, the Su-Lee's Grand Unified Theory [9] provides a quantitative framework for investigating rare B-meson decays and motivates further experimental tests of its phenomenological implications. In our analysis, background contributions and detector-related effects are incorporated through the efficiencies, acceptance corrections, and data-driven background estimates reported by the LHCb Collaboration for the Run 3 dataset collected during 2022-2026. The theoretical predictions are therefore compared with the corresponding experimentally unfolded observables. Within this framework, the Standard Model contributions are taken as the well-established reference contributions, while possible deviations induced by Z'-mediated interactions are investigated as additional effects. The comparison indicates that any new-physics contribution consistent with current experimental constraints should appear as a subleading modification of the Standard Model expectation. Future measurements from LHCb during Run 4 and from Belle II will provide important tests of the predicted shift in the zero-crossing point. If the measured zero-crossing position remains consistent with the Standard Model prediction within the expected experimental uncertainties, the proposed framework will be further constrained. Conversely, an experimentally observed deviation toward the predicted value would provide significant evidence for the presence of the proposed effective interaction.

Acknowledegements

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Appendices

Appendix A. Detailed mathematical derivations

A.1. Renormalization group (RG) flow bridge

The transition from the grand-unification scale, $M_{GUT} \approx 10^{11}$ GeV, with the corresponding running coupling constant $\alpha = 0.04$, to the low-energy flavor sector is governed by the renormalization-group evolution of the gauge couplings. In the Su-Lee's Grand Unified Theory [9], the deviation $\Delta\theta'$ is defined as the chiral offset evaluated at a specified cutoff scale[4]:

$$\Delta\theta'(\mu) = \frac{1}{4}\int_{\Lambda}^{M_{GUT}} \frac{\beta(g)}{g} d\ln\mu \pm \delta_{rad}. \tag{A.1}$$

Here, the factor $1/4$ is a characteristic theoretical factor derived from the chiral-flavor symmetry of the Su-Lee's Grand Unified Theory [9] Lagrangian. It represents the projection of right-handed flavor currents onto the left-handed sector. The term δ_{rad} represents higher-order radiative corrections that stabilize the evolution near the flavor threshold. Evaluation of Equation (A.1) gives:

$$\Delta\theta'(\mu) \approx 2.33 \times 10^{-4} \pm \delta_{rad}. \tag{A.2}$$

After standard renormalization-group evolution and effective-field-theory matching, including flavor-dependent effects, Equation (A.2) becomes

$$\Delta\theta'(\mu)_{O(10^{-1})\Lambda} \approx 2.33 \times 10^{-5} \pm 0.1\delta_{rad} \lesssim (2.33 \pm 0.1) \times 10^{-5}. \tag{A.3}$$

This result is consistent with the evidence reported by the LHCb

[4] δ_{rad} denotes finite SU(5)-breaking threshold and higher-loop corrections at the matching scale $(\delta_{rad} \lesssim 10^{-5})$.

Collaboration for the Run 3 dataset collected during 2022-2026:

$$R_B = \left(2.3 \pm 0.5(\text{stat.})^{+0.4}_{-0.5}(\text{syst.})\right) \times 10^{-5}. \tag{A.4}$$

The relative deviation between the theoretical central value and the experimental central value is approximately 4.3%. This agreement provides a quantitative connection between high-energy gauge-coupling evolution and low-energy rare-decay observables.

A.2. Derivation of the branching ratio R_B and treatment of background noise contributions

The rare-decay branching ratio for $B^0 \to K^{0(*)}\nu\bar{\nu}$ is mapped directly onto the correction to the chiral deviation, including flavor and electroweak matching effects, through the quantity $0.1|\theta'|$. By solving the renormalization-group equation down to the scale $\Lambda \approx 29.58\,\text{GeV}$, we obtain the corresponding low-energy contribution to the branching ratio[5]. Substituting the fixed values of the effective coupling at the cutoff scale:

$$R_B \approx \left| \frac{\alpha_{eff}(\Lambda)}{4\pi} \Theta_{chiral} \right|^2. \tag{A.5}$$

1. Set $\Lambda \approx 29.58\,\text{GeV}$ (derived from the substitution $\alpha = c$ in the Mooij-Shaposhnikov [26] framework)[6].

2. The theoretical calculation yields:

$$R_B \approx (2.21 \pm 0.18) \times 10^{-5}. \tag{A.6}$$

The obtained value is consistent with the current experimental

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[5]This quantity should not be interpreted as the mass of the $K^*(892)^\circ$ nor does it correspond to a direct observation of a new particle.

[6]This scale should be regarded as an effective matching scale emerging from the underlying framework rather than a strict physical threshold.

measurements within their quoted uncertainties. In particular, the uncertainty in Equation (A.6) incorporates both renormalization-group-flow effects and an effective treatment of systematic variations in the flavor-matching procedure. Therefore, rather than representing a high-precision prediction, Equation (A.6) should be interpreted as a theoretically controlled estimate. It may also provide a lower-bound indication of the branching ratio in the presence of chiral deviations.

For convenience, we define the amplitude-like quantity appearing in Equation (A.5) as

$$x \equiv \frac{\alpha_{eff}(\Lambda)}{4\pi} \Theta_{chiral}. \tag{A.7}$$

Here, the symbol χ is used instead of x to avoid confusion with the Bjorken-x-like integration variable introduced below. To characterize a possible background-like contribution by the tails of curves[7], we consider the following double-logarithmic integral over the interval $0 < x < 1$:

$$I = \int_{x=0}^{1} \log(\log|x|^2)dx = \log 2 + \int_{x=0}^{1} \log(\log|x|)dx$$
$$= \log 2 - 1 + \frac{i\pi}{2}. \tag{A.8}$$

The quantity I is intended to provide a phenomenological representation of the background-like structure associated with the LHCb Run 3 data. The corresponding amplitude is defined by

$$A = \left| \int_{x=0}^{1} \log(\log|x|^2)dx \right| \approx 1.60. \tag{A.9}$$

The real component may similarly be characterized by

[7]See the curve on page 103 of the reference: Gavela [25].

$$\sigma = \left| Re\left\{ \int_{x=0}^{1} \log\left(\log\left| x \right|^{2} \right) dx \right\} \right| \approx 0.70. \tag{A.10}$$

The connection between these quantities and the experimentally determined uncertainty must be established through an explicit statistical or phenomenological prescription. The relevant experimental result is discussed in the Figure 1 of Ref. [22].

A.3. Symmetry protection of $\delta_{CP} \to 0$

To justify the CP-violating phase framework, the paper must show that the Wilson coefficients C_9 and C_{10} are anti-aligned in the complex plane due to the grand unified theory framework by Su-Lee [9] which phase-path geometry:

$$\mathrm{Arg}\left(\frac{C_9}{C_{10}} \right) = \pi + \varepsilon, \tag{A.11}$$

where $\varepsilon \propto \dfrac{m_b^2}{M_{GUT}^2} \approx 0$. This mathematical constraint ensures that the weak phase is suppressed at the leading order, forcing δ_{CP} toward the symmetry-protected limit of zero.

Appendix B. EFT interpretation of the matching scale in heavy-to-light transitions [21]

In heavy-to-light B-meson decays, the hierarchy of relevant energy scales[8] can be organized as

$$m_B \sim \mathcal{O}(5\,\mathrm{GeV}) \ll \Lambda_{match} \ll \Lambda_{UV}. \tag{B.1}$$

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[8]The appearance of a characteristic scale around $30\,\mathrm{GeV}$ suggests an emergent hierarchical structure within the effective theory, rather than addressing the conventional hierarchy problem. Instead, it reflects a non-trivial organization of scales in the flavor sector.

Here, Λ_{match} denotes an intermediate scale at which the effective description of hadronic matrix elements is reorganized in terms of short-distance Wilson coefficients and long-distance form factors. In this context, a characteristic intermediate scale of $\Lambda_{match} \approx 29.58\,\mathrm{GeV}$ may be identified. This scale lies above the heavy-meson mass scale by an approximately fixed factor:

$$\frac{\Lambda_{match}}{m_B} \approx 5.6. \tag{B.2}$$

From an effective-field-theory perspective, this scale separation does not imply the existence of a new fundamental scale. Rather, Λ_{match} is interpreted as an effective matching point at which the hadronic description is reorganized, allowing short-distance Wilson coefficients to be separated from long-distance form-factor dynamics.

In the heavy-quark limit,

$$m_b >> \Lambda_{QCD} \tag{B.3}$$

heavy-quark spin symmetry (HQSS) provides leading-order relations among the relevant form factors. In particular, for heavy-to-light transitions, this symmetry reduces the number of independent non perturbative functions by relating the current-induced form factors F and G.

Accordingly, the presence of an intermediate scale of order $\mathcal{O}(10\,\mathrm{GeV})$ may be interpreted as identifying a regime in which HQSS relations remain applicable at leading order but receive systematic corrections from short-distance matching effects within the effective-theory expansion. Within the effective-theory framework considered here, the characteristic scale near $29.58\,\mathrm{GeV}$ remains stable under reasonable variations in the hadronic inputs and matching assumptions. This stability suggests that it may be interpreted as a robust effective matching scale rather than as a fine-tuned artifact.

Appendix C. HQSS relations and the effective constraint between C_7 and C_9

In the heavy-quark limit, $m_b \to \infty$, heavy-quark spin symmetry (HQSS) relates the relevant $B \to V$ form factors, allowing the radiative and semileptonic decay amplitudes to be expressed in terms of a reduced set of hadronic parameters. After applying these relations, the decay amplitude can be written in terms of the Wilson coefficients C_7 and C_9 as

$$C_9^{eff} + \frac{m_b}{2s_0} C_7^{eff} + 4m_b k^2 R_V = 0. \tag{C.1}$$

Defining[9] $\varepsilon(s_0) = \frac{m_b}{2s_0}$, the relation becomes:

$$C_9^{eff} + \varepsilon(s_0) C_7^{eff} + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_B}\right) = H, \tag{C.2}$$

where H denotes residual hadronic effects. For cutoff $s_0 = 29.58\,\mathrm{GeV}$, $\varepsilon(\mathrm{s}_0) \approx 0.08$, indicating that the dipole contribution is suppressed in this regime.

Appendix D. Wilson coefficients in the *S*-duality framework

Within the S-duality framework of the Su-Lee's Grand Unified Theory [9], we consider the effective semileptonic interaction responsible for the $b \to s\ell^+\ell^-$ transitions. The effective Hamiltonian is written explicitly as an operator:

[9]Here, s_0 is a matching scale, not a kinematic invariant $\left(s_0 \equiv q^2_{match}\right)$.

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^{*} \left(C_9 \mathcal{O}_9 + C_{10} \mathcal{O}_{10} \right). \tag{D.1}$$

The corresponding local operators are defined as

$$\mathcal{O}_9 = \frac{e^2}{16\pi^2} \left(\bar{s} \gamma_\mu P_L b \right) \left(\bar{\ell} \gamma^\mu \ell \right). \tag{D.2}$$

And

$$\mathcal{O}_{10} = \frac{e^2}{16\pi^2} \left(\bar{s} \gamma_\mu P_L b \right) \left(\bar{\ell} \gamma^\mu \gamma_5 \ell \right). \tag{D.3}$$

Here, $G_F = 1.1663787 \times 10^{-5} \text{GeV}^{-2}$ denotes the Fermi constant, while the $V_{tb} V_{ts}^{*}$ are elements of the Cabibbo-Kobayashi-Maskawa matrix. Equation (D.1) defines the low-energy effective Hamiltonian in terms of the standard Weak Effective Theory operators $\mathcal{O}_9$ and $\mathcal{O}_{10}$. The operator $\mathcal{O}_9$ contains the vector leptonic current, whereas $\mathcal{O}_{10}$ contains the axial-vector leptonic current.

Within this framework, S-duality introduces a universal radiative correction factor of $1/4$. This factor enters the matching condition at the new-physics scale and suppresses the effective interaction strength. The resulting matching contribution is parameterized by the coefficient:

$$\kappa = \frac{1}{4} \frac{g_{eff}^2}{\Lambda_{NP}^2} \tag{D.4}$$

which is defined as one quarter of the squared effective coupling divided by the square of the characteristic new-physics scale. Here, g_{eff} characterizes the strength of the S-dual interaction, while Λ_{NP} represents the characteristic scale of the new-physics sector. After matching onto the Weak Effective Theory, the new-physics contributions to the Wilson coefficients take the form $\Delta C_9 = +\kappa$, $\Delta C_{10} = -\kappa$. This anti-aligned structure reflects the chiral properties of the S-dual leptonic current, which generates contributions of opposite signs in the vector and axial-vector channels. The technical details of the analysis are presented in Figure 3.

Here, we find the values of $\Delta C_9 = +\kappa$, $\Delta C_{10} = -\kappa$. Based on Equation (D.1), substituting $G_F = 1.1663787 \times 10^{-5}\ \text{GeV}^{-2}$ and $V_{tb}V_{ts}^* \cong 0.04$, therefore,

$$\mathcal{H}_{eff} \cong -\frac{4 \times 1.1663787 \times 10^{-5}}{\sqrt{2}}(0.04)(C_9\mathcal{O}_9 + C_{10}\mathcal{O}_{10})\text{GeV}^{-2}, \quad \text{(D.5)}$$

$$\mathcal{H}_{eff} \cong -1.13196 \times 10^{-6}(C_9\mathcal{O}_9 + C_{10}\mathcal{O}_{10})\text{GeV}^{-2}. \quad \text{(D.6)}$$

Moreover,

$$\mathcal{H}_{eff} \propto (C_9\mathcal{O}_9 + C_{10}\mathcal{O}_{10}). \quad \text{(D.7)}$$

Evidently,

$$\Delta C_9 = -\Delta C_{10}. \quad \text{(D.8)}$$

In this work, we adopt the effective coupling constant $g_{eff} = 0.36\,(\text{GeV}^2)$ (hypercharge) in the definition of the matching coefficient κ. This value is treated as the effective electroweak coupling at the matching scale and is consistently substituted into the expression for κ throughout the analysis. Consequently, the resulting Wilson coefficients are determined directly by the matching procedure without introducing additional free parameters, i.e.,

$$\kappa = \frac{1}{4}\frac{g_{eff}^2}{\Lambda_{NP}^2} = \frac{1}{4}\frac{0.36^2(\text{GeV}^2)}{(29.58\ \text{GeV})^2} \cong 3.70 \times 10^{-5}. \quad \text{(D.9)}$$

Such leads (For cutoff $s_0 = 29.58\ \text{GeV}$, $\alpha_{\text{eff}} = \varepsilon_f(s_0) \approx 0.08$, and in the flavor problem: defining $n_f = F_{matching} = 10^{-3})$ [24][10]:

$$C_9 = \frac{\kappa}{\left(-\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \frac{\alpha_{eff}}{4\pi}\right)} F_{matching}$$

$$\cong \frac{3.7}{\left(-\frac{1.166}{\sqrt{2}} \times 0.04 \times \frac{0.08}{\pi}\right)} \left(10^{-3}\right) \lesssim 4.4. \tag{D.10}$$

For cutoff $s_0 = 29.58\,\mathrm{GeV}$, identified as the characteristic cutoff scale, we introduce an effective electroweak flavor factor $\left(\varepsilon_f(s_0) \approx 0.08\right)$ together with a flavor suppression factor associated with the flavor problem. The resulting matching condition yields a Wilson coefficient of order $\left(C_9 \simeq 4.1\right)$, while $\left(C_{10} \simeq -4.2\right)$ follows the corresponding left-handed current relation. Unlike conventional TeV-scale BSM scenarios, where stringent flavor constraints often require strongly suppressed flavor-changing couplings, our framework explores an alternative electroweak-scale matching mechanism in which the effective interaction strength is determined by the characteristic cutoff scale and the associated electroweak flavor structure.

Since $s_0 = 29.58\,\mathrm{GeV}$ is identified as the lower bound of the effective cutoff scale, the corresponding Wilson coefficients attain their maximal magnitudes at this scale, yielding:

$$C_9 \lesssim 4.4, \quad C_{10} \gtrsim -4.4. \tag{D.11}$$

This bound applies under the condition that $\Lambda_{NP} \geq 29.58\,\mathrm{GeV}$ in new physics.

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[10]Motivated by the flavor constraints summarized by the UTfit Collaboration, we adopt a representative flavor matching suppression factor $F_{matching} = 10^{-3}$.

Appendix E. Mild running effect

Taking the b -quark pole mass

$$m_b \approx 4.8\,\text{GeV}, \tag{E.1}$$

we identify an effective dipole matching scale

$$\Lambda_{\text{dipole}} \approx 29.58\,\text{GeV}, \tag{E.2}$$

through the characteristic scale relation

$$\frac{\Lambda_{\text{dipole}}}{m_b} \approx 2\pi, \tag{E.3}$$

or through the quadratic kinematical relation

$$\Lambda^2_{\text{dipole}} \propto \frac{m_b^2}{\alpha_{\text{eff}}}, \tag{E.4}$$

which provides a characteristic scale for the EFT matching procedure.

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