Fundamental Journal of Modern Physics
ISSN: 2249-9768
Vol. 11, Issue 2, 2018, Pages 63-91
This paper is available online at http://www.frdint.com/
Published online December 12, 2018

PLASMA WAVEGUIDES AND ELECTRIC GAS BREAKDOWN

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Abstract

A nonlinear plasma-waveguide model for the wave propagation of electric gas breakdown has been proposed on the basis of the theory of solitons and the theory of plasma waveguides. A model of the avalanche-streamer transition is considered as a transition from the sine-Gordon breather solution to the kink-antikink solution. The experimental data obtained on streamer gas breakdown provide description of a waveguide part of the model. A relationship is established between the radial profile of the waveguide electron density with the frequency and the wave number of the waveguide. The proposed model makes it possible to determine the scale of the streamer skin-layer thickness and the scale of its linear resistance.

Keywords and phrases: electric gas breakdown, nonlinear plasma-waveguide model, streamer chamber.

This work was supported by the Russian Foundation for Basic Research (Grant No17-02-00366).

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Received February 22, 2018; Accepted March 6, 2018

1. Introduction

The contradictions between experiments and the theory of electric gas breakdown (EGB) at a pressure of the order of atmospheric and the contradictions of the existing theory were considered earlier in detail [1, 2]. These contradictions are reduced to the fact that the theoretical range of the dependence of the speed of the EGB wave on the external field, described by the continuity equation and the four equations for the medium ionization in the vicinity of the wave breakdown head, does not coincide with the experimental region of the analogous dependence. Two of these dependences might be combined at one point only. That is why the traditional EGB model describes the experiment satisfactorily only at some local point, which corresponds to an external field strength of $\sim 10\,\text{KV/cm}$ and $\sim 10^8\,\text{cm/s}$ velocity of a streamer discharge. If the experimental parameters differ significantly from these values, then the agreement between theory and experiment cannot be reached, as is observed in experiments on the production of neutrons in a spark discharge [3]. However, the greatest amount of disagreements between the EGB theory and the experiment is observed not in the analysis of energy parameters, but in analyzing the space-time characteristics of this process. According to the traditional model, the avalanche-streamer transition is due to the appearance a new ionization source [4], but it cannot explain why the avalanche is split into two streamers, why these streamers move symmetrically to different electrodes in the avalanche reference system [5], why a similar transition in the streamer chamber is symmetric in the laboratory reference system [6], and why the EGB wave diameter decreases by three times in this case [7]. Finally, in frames of the traditional model, one cannot explain the presence of a minimum breakdown wave velocity different from zero, and the azimuthally equidistance of the gigantic jet channels in discharges between the thundercloud and the ionosphere [8, 9].

To overcome these contradictions, we have proposed a new model [1, 2, 10], to be called the nonlinear plasma-waveguide model (NPWM) of EGB. It is based on three statements following from the experimental results. In the NPWM it is assumed that:

(1) All the EGB waves are the solitons,

(2) They propagate along the surfaces of plasma waveguides formed during some preliminary processes or during the propagation of the EGB wave,

(3) The motion of such solitons is described by the sine-Gordon equation for the electric potential.

Although almost all the EGB waves can be considered as solitary waves with relaxing tails, it is necessary to make a brief review of the solitonics in order to justify the applicability of NPWM for solitons. A nonlinear solitary wave was observed for the first time on the surface of a shallow-water channel in 1844 by J. S. Russell [11]. This wave propagated over long distances of the order of several kilometers without noticeable spreading due to dispersion. And only in 1895, the mathematicians D. G. Korteweg and G. de Vries [12] were able to derive the necessary one-dimensional wave equation for a wave in a nonlinear medium with dispersion, but without attenuation:

$$u_t - 6uu_x + u_{xxx} - 0, \tag{1}$$

where u is the normalized wave potential, and the lower indices denote time t and coordinate x differentiation.

This equation, called the KdV equation, formed the basis for further research in this field. However, until the middle of the twentieth century, these studies were not numerous. Beginning from 1965, mathematicians took up that issue by combining nonlinear one-dimensional wave equations, developing methods for their general solution, and arranging the collected data. Since 1975, the theoretical physicists have joined that work by finding the appropriate objects for solitonic applications. Then, almost without interruption, a stage of demonstrative experiments followed, where theoretical views were experimentally confirmed, primarily in plasma physics. The current state of the problem is basically reduced to the diagnostics of observed solitary waves and their classification in respect to the developed nonlinear wave equations. Two of these basic equations, the KdV equation and the nonlinear Schrödinger equation (NSE), produce analytic one-dimensional soliton solutions basing on the nonlinear properties of the medium, where the corresponding waves propagate without regard to the attenuation. The simplest interpretation of this process is given in [13]. For surface waves in plasma, the same question is theoretically studied in [14, 15]. In this case, for nonlinear media with a dispersion acoustic characteristic, one obtains:

$$\omega = \gamma k - \beta k^3, \tag{2}$$

where ω is the wave frequency; k, the wave number; γ and β, certain constants; the wave equation for the wave potential u is reduced to Eq. (1), and its solution is:

$$u(x,\, t) = \frac{v}{2ch^2[\sqrt{v}(x - vt)]}. \tag{3}$$

If a dispersive characteristic has a spectrum of the optical type:

$$\omega = \omega_0 + \beta k^2, \tag{4}$$

the wave equation should be reduced to the NSE, i.e.:

$$iu_t - \omega_0 u + \beta u_{xx} = 0 \tag{5}$$

and its solution, in the simplest form, will take the form:

$$u(x,\, t) = u_0(x,\, t)\exp i\omega t, \tag{6}$$

where $u_0(x, t)$ plays the role of a slowly varying amplitude and is determined by Eq. (3).

By comparing the parameters of an observed wave with Eqs. (3) and (6), one can define the types of the medium nonlinearity and draw some other conclusions. However, one should not overestimate such an approach, since it does not take into account the process of wave propagation along the periodic potential considered in [16]. It was described later on through the sine-Gordon equation [17]:

$$u_{tt} - c^2 u_{xx} + \omega_0{}^2 . \sin u + \sigma u_t = \lambda E(t), \tag{7}$$

where c is the characteristic velocity of the process; ω_0, eigenfrequency, and the terms σu_t and $\lambda E(t)$ define attenuation and the source function. Equation (7) has two soliton solutions for breather and kink-antikink pair. The breather takes the form [18]:

$$u(x, t) = 4 \frac{arctg[tg\Theta \sin[(t + t_0) \cos \Theta]]}{ch(x \sin \Theta)}, \tag{8}$$

where Θ is the parameter of the problem $(0 < \Theta < 0.5\pi)$ and t_0 is the initial phase of the process.

The kink-antikink pair takes the form:

$$u(x, t) = 4arctg\left\{\exp\left[\pm \frac{\omega_0(x - vt)}{c\sqrt{1 - v^2 / c^2}}\right]\right\}, \tag{9}$$

where v is the wave velocity, and c, the scale of the process velocity.

For the first time, an attempt to describe the EGB wave on the basis of the KdV equation was undertaken in [19], but that work did not provide an experimental verification of the theory and, above all, did not explain the avalanche-streamer transition. As applied to EGB, Eqs. (8) and (9) should be most appropriate.

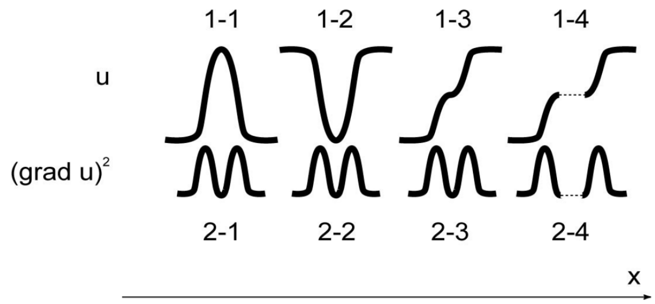

Figure 1. Evolution of the potential u of the EGB soliton, and its square gradient illustrating a longitudinal profile of the energy yield [20].

Figure 1 shows the scheme of this transition as verified experimentally in [20]. Here, the upper level illustrates the evolution of the breather potential (1-1 and 1-2) and its transition to a metastable state (1-3), which breaks up into a kink-antikink pair (1-4). The lower level illustrates the profile of the energy release of a traveling EGB wave without allowance for relaxation tails. At the stage of the breather evolution, the energy release profile shown in the lower level of Figure 1 should be double-humped, which can be used as the basis for the experimental diagnostic method. Such an experiment was necessary because a similar structure could be given by another nonlinear equation different from (7), which has not yet been investigated. The experimental verification was difficult for different reasons. The conventional Raether's avalanche [21] obtained for electropositive gas in the Wilson cloud chamber was a smooth cone with a rounded base, that is, it did not have a dip in the middle of its profile. And the avalanche obtained by Allen-Phillips [22] for electronegative gas had such a dip, but there had not been enough frames to diagnose that wave.

The difference in the types of the avalanches can be explained by analyzing the spatial resolution in the cloud chamber, where the avalanches had been recorded after cutoff of the applied voltage pulse. The full camera frame exposure of the wave under these conditions is determined by the lifetime of the oversaturated vapor, and the spatial resolution is determined by the characteristic diffusion time. For electrically positive gases, the diffusion is determined by fast electron-ion processes, and therefore the structure of the wave is smeared during the time of the frame exposure. In electronegative gases, diffusion is determined by a slow interaction of ions of different signs, and therefore the structure of the wave is resolved.

The diagnostics of the breather wave was complicated due to an interruption of the avalanche experiments after the appearance of work [22] due to their low applicability. The situation improved after a production of a similar wave during the EGB laser initiation [23]. Analyzing the dynamics of the distance between peaks of the wave during the transition from state (2-1) to state (2-2)) (see Figure 1), in [10] it was possible to establish the correspondence of the observed wave to the solution (8) for the breather wave. Since the kink mode of the sine-Gordon equation is treated in terms of the mass at rest, due to the theory [17], it becomes possible to determine its value experimentally, which was done in [24], basing on the fulfillment of the laws of energy conservation, charge, and momentum in the avalanche-streamer transition. It was obtained that $M_s \approx 3 \times 10^{-14}$g. This allowed one to solve the problems of the type [3].

As evidenced by the foregoing, the soliton positions of the NPWM prove to be fully justified [1, 2, 10, 24]. This does not mean that all issues here have been resolved. In particular, the parameters of the periodic potential in the EGB phenomena still require further investigation, which is beyond the scope of this paper devoted primarily to the NPWM waveguide postulate. The question of a three-fold changing of the EGB wave diameter for the avalanche-streamer transition will be considered below.

Let us analyze the presence of a waveguide in the process of EGB, which is observed experimentally in a streamer chamber [25]. We try to agree the existing theory of plasma waveguides (PW) developed for the plasma accelerators with the tasks of the EGB. It is assumed that the EGB wave propagation has a waveguide structure, which has found an experimental justification in [26]. It was observed that a high-density electron wave propagated along an earlier ionized channel due to the streamer beatings after the avalanche decay, at a sufficient time resolution.

We aim, in this paper, at finding new relationships between the PW theory and the EGB experiment to agree the ultimate parameters of a waveguide, that is, its electron temperature and radial electron density distribution, with its experimental characteristics, that is, the eigenfrequency and the wave number. The question of the formation of PW at the initial stage of the EGB will not be considered here. Obviously, this process proves to be connected with a superposition of electromagnetic waves in the plasma. However, the distribution of the field is considered to be predetermined, because solution of the problem is rather intricate. The collisional damping of the EGB wave is analyzed below only qualitatively due to an essentially new energy source of the propagation of the EGB wave in the framework of the NPWM.

2. Waveguide Experiment for the Electric Gas Breakdown

The distribution of streamers in a streamer chamber (SC) after the applied voltage cutoff [25] was not satisfactorily studied within the framework of the traditional approach. The process was studied experimentally when the equidistant streamer pairs had been initiated by a flow of outer shell electrons in the middle of an interelectrode gap. Not only a streamer running of the order of a several millimeter was recorded in the same directions, but also a radiation burst was observed of several tens nanosecond duration. The indicated time scale of the streamer run is much higher than that of the collisional relaxation of an electron pulse, which is less than 1 ps. Since the time scale of the streamer electron relaxation energy is three to four orders higher than that of the streamer pulse relaxation, the streamer should be stopped for 1 ns and irradiate the stored energy for 1 ns and more after the applied voltage cutoff, within the framework of the traditional approach, which contradicts [25]. In this case, the radiation intensity of the streamer itself should decrease monotonically.

The burst maximum coincided with the moment of complete stoppage of the streamer, which occurred velocity jump from $V \sim 10^7$ cm/s to zero. This leads to another problem connected with the need not to explain the streamer advancing in terms of neutral gas photoionization by the streamer self-radiation, finding the reasons for such synchronization and the streamer velocity jump in the end of propagation.

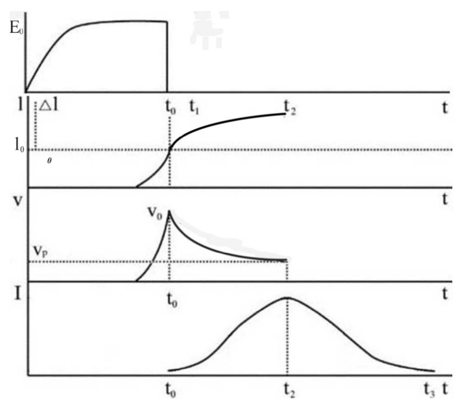

Figure 2. Experiment on the streamer propagation after the applied voltage cutoff [25].

Figure 2 illustrates the experimental conditions and results shown in the same time scale [25] with the data obtained by the same authors from works [6, 27]. The first two graphs demonstrate the dynamics of the field strength for the chamber $E(t)$ and the time dependence of average streamer length $l(t)$ due to elongation after shearing stress Δl. The two upper graphs illustrate the experimental conditions, and the two lower ones contain the basic information for analysis. They show the time variation of the streamer velocity $V(t)$ and the dynamics of the recorded radiation $I(t)$. Obviously, the streamer velocity after the applied voltage cutoff had to decrease. It did not reach zero, but achieved $V \approx 10^7$ cm/s, to which the authors of [25] did not give any explanation. The streamer propagation ceased immediately at the moment of the discharge gap emission maximum $I(t)$, which was recorded throughout the chamber. It continued after the streamer stoppage, and had a typical time shape corresponding to the power dynamics in the relaxing oscillatory circuit with a capacitive energy storage system.

To explain this phenomenon, the authors of [25] proposed a model for a streamer propagation and a model of longitudinal electromagnetic oscillations along a streamer pair channel [28]. For a streamer plasma, a collisionless approximation was assumed, and the frequencies of the corresponding electromagnetic field oscillations were estimated to be 10^{11} - $10^{12}\,\mathrm{s}^{-1}$ [28]. Some doubts are cast upon admissibility of this approximation in [29]. This objection seems to be justified, since the frequency of pulsations in the waist of the streamer pair in the SC is substantially lower than that indicated in the experiment [6], carried out by the same authors practically under the same conditions. However, the main objection to the model [28] is that it does not explain the radiation dynamics after cutting off the applied voltage, although it explains the anisotropy of the interelectrode space after the cutoff of the voltage. The authors of Ref. [25], to the model discussed in [28], they tried in [27] to explain the dependence of the streamer length increment Δl on the initial length l_0 and the initial field strength E_0 basing on the energy balance, that is, $\Delta l = f(E_0, l_0)$. Such an approach seems highly qualitative, because the terms of the energy balance in this problem cannot be determined with the necessary reliability and accuracy.

There are additional arguments in [30] in favor of the model [28], but the quantitative experimental data obtained in [25] are not used. The most perfect model of this phenomenon considered in [31] being based on a number of assumptions not confirmed by the experiment, establishes an approximate experimental dependence, $\Delta l \approx l_0 \cdot E_0^{1/3}$. The inaccuracy of this dependence is due to the fact that the experimentally established power of l_0 is less than unity. The unsolved problems of this phenomenon are listed most completely in [32]. Though the time dependence of the radiation power obtained in [25], as well as the reason for retaining the direction of the streamer motion after cutting the voltage, did not attract the attention of the authors.

It seems to be impossible to do this on the basis of the presently prevailing ideas about the mechanism of the EGB wave propagation, based on the ideas of neutral gas ionization in the vicinity of the head of the EGB wave. The NPWM assumes that at the time of voltage cutoff there was a plasma channel connecting the electrodes, and this explains the streamer continued distribution in the same direction after the voltage cutoff, and presents the first argument in favor of the NPWM. At the same time, not only the field longitudinal traveling wave (the wave responsible for the streamer propagation) was propagating along this channel, but the channel itself was emitting the light as an active element of the vibration contour of the SC.

In the experiment [25], such channels were of the order of 100, but the time scale of the oscillatory circuit was independent of their number and was equal to the time scale of one channel, which will be shown below.

The quantitative measurements of streamer length at different moments of time $\Delta l_1(t_1 - t_0) = 0.3\,\text{cm}$, $\Delta l_2(t_2 - t_0) = 0.5\,\text{cm}$ given in [25], as well as measurements of streamer average velocity in the process of deceleration during the first time period after the voltage cutoff

$V_m(t_1 - t_0) = 6\times10^7$ cm/s, allow one to make a system of three equations relative to the streamer initial velocity at the moment of voltage cutoff V_0, the velocity relaxation time τ, and streamer asymptotic velocity V_p.

Let us approximate the dependence between the streamer velocity and the time after the voltage cutoff in the form

$$V(t) = (V_0 - V_p)\cdot \exp\left(-\frac{t - t_0}{\tau}\right) + V_p. \tag{10}$$

This qualitatively agrees with the experiment, and, carrying out integration within the limits, which correspond to the experiment, we get a system of equations for V_0, V_p, and τ. Value V_0 for the case of waveguide propagation should be close to the electron thermal velocity (at normal temperature T)

$$V_T = \sqrt{\frac{2T}{m}} = 0.94\times10^7 \text{ cm/s}, \tag{11}$$

where m is the electron mass. The system takes the form:

$$\begin{cases} \Delta l_1 = (V_0 - V_p)\cdot\tau\cdot\left(1 - \exp\left(-\dfrac{t_1 - t_0}{\tau}\right)\right) + V_p\cdot(t_1 - t_0) = 0.3, \\ \Delta l_2 = (V_0 - V_p)\cdot\tau\cdot\left(1 - \exp\left(-\dfrac{t_2 - t_0}{\tau}\right)\right) + V_p\cdot(t_2 - t_0) = 0.5, \\ V_m = (V_0 - V_p)\cdot\left(1 - \exp\left(-\dfrac{t_1 - t_0}{\tau}\right)\right)\cdot\dfrac{\tau}{t_1 - t_0} + V_p = 6\times10^7. \end{cases} \tag{12}$$

Solving the system (12) at $t_1 - t_0 = 5$ ns, $t_2 - t_0 = 25$ ns, we have: $V_0 = 25.7\times10^7$ cm/s, $\tau = 1.034$ ns, and $V_p = 0.99\cdot10^7$ cm/s. The two first values are typical of the streamer process of EGB, and the obtained V_p value, which slightly exceeds the electron velocity in the maximum of Maxwell distribution at normal temperature, is considered to be the second argument to support the EGB waveguide theory. However, taking into account about 10% error in V_p, one can state only that $V_p \approx V_T$. In this case, the stop in the streamer motion can be due to the fact that the streamer, being a capacitive accumulator in the oscillatory circuit of the streamer chamber, changes the sign of the charge at the instant of the maximum of the current and the corresponding maximum of the emission of the channel. The double role of the streamer as an element of the oscillatory circuit and the channel of the plasma waveguide does not contain a contradiction. First, any electrophysical device is an oscillatory circuit, and, secondly, both the theory [33] and the experiment [26] allow one to consider a streamer as a charge density wave. This can be considered as the third argument in favor of the waveguide hypothesis of EGB. The fourth argument in favor of the waveguide hypothesis of EGB is the continued registration of the waveguide glow after the stop of the streamer. In this case, the streamer intrinsic emission, as indicated above, slightly distorts the $I(t)$ dependence, which is reflected as an error of less than 10% in the initial section of this dependence, like all other background sources. Consequently, the experiment [25] gives four arguments in favor of the waveguide model of EGB, which completely explain this effect.

However, an additional discussion is needed. After cutting off the external voltage, it is doubtful to expect any nonlinear, including soliton, effects in the plasma, if one makes use of the equations KdV (1) and NSE (5), since dissipation is not taken into account in these equations. At the same time, the sine-Gordon equation used in the NPWM remains applicable as long as the structure of the periodic potential of the medium is preserved. Since the experiment [25] showed such an applicability, i.e., the streamer did not change its spatial structure after the voltage cutoff, then the chosen approximation $V(t)$ can be used in the form (10), which does not contradict equation (7).

In addition, it is necessary to evaluate the errors in the results obtained. Since the right-hand side of (12) is determined from the histograms [25] constructed with $1\,\text{mm}$ step, the error can be estimated as $0.3 - 0.5\,\text{mm}$. And accordingly, the right-hand sides of (12) have the errors of the order of 10%. The same error is obtained for the values of V_0 and V_p, and the error τ is determined by the necessary accuracy of matching all the equations (12). However, for V_0 the error is less than the order of magnitude, and this is not critical, since the very value V_0 just determines the reliability of the process in the preliminary stage of the process before the voltage cutoff, and this remains true for the comparison of this value with the data of the same authors in [6]. The error of τ is also of secondary importance as long as this value remains substantially shorter than the streamer propagation process after the voltage cutoff, and that is $25\,\text{ns}$. The error in the V_p parameter is of greatest importance since later on it is compared with the value of electron thermal velocity at the maximum of Maxwell distribution function. With a V_p error of 10%, the proximity of V_p to the electron velocity at the maximum of the Maxwell function is of no doubt, and this confirms the applicability of the model of the interaction between the traveling longitudinal field wave and plasma electrons.

This model is usually considered in the analysis of the Landau collisionless absorption [34, 35]. In this case, a monochromatic wave with the potential amplitude U moving in the laboratory reference frame with the velocity V_p is approximated by a potential well, in the reference frame of which the electron moves. From the condition for the capture of an electron by a well of the depth eU, that is, $0.5\text{mv}^2 < eU$, the plasma electrons are divided into the catching wave electrons, for which $V_p + (2eU/m)^{0.5} > V_p$, and the lagging wave electrons, for which $V_p - (2eU/m)^{0.5} < V_p$. Consideration of the elastic interaction of the electron with the wall of the well shows that the electrons catching the wave give energy to the wave, while the lagging ones consume its energy. The resulting effect is determined by the quantitative relationship between the catching and lagging electrons. Note that the experiment [25] and all the EGB experiments consider the electron velocity (11) as fundamental. And this means that the Maxwell distribution function should be considered in the form

$$F = Av^2 \exp\left(-\frac{mv^2}{2T}\right). \tag{13}$$

This function has a maximum at v, determined by (11). Accordingly, in the vicinity of this maximum, one can find such a value of V_p, at which the number of electrons catching up the wave and lagging behind the wave is equal, that is, the collisionless absorption in the first approximation will go to zero. This indicates the first fundamental importance of the experiment [25]. Under certain experimental conditions, a spatially limited plasma medium contains a packet of incident waves, and among them an almost monochromatic wave having a phase velocity close to the velocity at the maximum of the electron velocity distribution function (13).

From the experiment [25] follows the necessity of the existence of runaway electrons in the EGB plasma. The EGB plasma is known to be collisional, and, therefore, any model of the streamer propagation under these conditions should provide for compensation of the corresponding energy losses. There can be only two such models: an electrostatic model that connects the energy source to the spatial distribution of charges at the preliminary stage, and an electrodynamic model that connects the energy source with a certain generator of a traveling wave. Any electrostatic model will lead to an asymptotic streamer velocity equal to zero, and this is in contradiction with [25]. That is why we must assume the presence of a source of the traveling waves of the field. Such a source under EGB conditions can be the runaway electrons. The runaway electron theory [36] does not impose strict restrictions on the threshold for their appearance at small degree of ionization, which is typical of the EGB. Moreover, the mechanism of such a process is described in Ref. [37], but to date the experimental threshold for the appearance of runaway electrons under conditions close to the EGB is not defined.

The experiment [25] poses not only the fundamental problems indicated above, but also allows one, basing on the analysis of $I(t)$, to determine the resistance of a single waveguide channel, the average concentration of plasma waveguide electrons in a streamer process, compare it with the data of other authors, and establish other experimental parameters.

The oscillatory contour of streamer chamber [25] is described by two parameters defined by the form $I(t): \alpha = \sec^2[\pi(t_2 - t_0)/(t_3 - t_0)]$ and $R_c C_c = \frac{t_3 - t_0}{\pi} \cdot \sin\left(2\pi \frac{t_2 - t_0}{t_3 - t_0}\right)$. If $\alpha > 1$, the processes in the contour have a form of a damping sinusoid. The product of the resistance of the contour R_c to its capacitance C_c does not depend on the number of parallel channels N_s, i.e., $R_c C_c = \frac{r_s}{N_s} \cdot (c_s \cdot N_s) = r_s \cdot c_s$, where r_s is the resistance of a single channel, and c_s, the channel capacity. Then at $t_3 - t_0 = 59\,\text{ns}$, we have $\alpha = 17.8$, $R_c C_c = 8.66\,\text{ns}$. Calculating the capacity of a single channel from $c_s = 0.28 \cdot \frac{l_0 + \Delta l}{\ln(2l/D)} = 0.04\,\text{pF}$ [38], where $D = 0.01\,\text{cm}$ is the streamer diameter, we get the channel resistance $r_s = R_c C_c / c_s = 2.1 \times 10^5\,\text{Ohm}$. The average density of electrons n in the channel may be calculated from $n = \frac{4d}{\pi e \mu D^2 r_s}$, where $d = 3.8\,\text{cm}$ is the interelectrode spacing [6]; $\mu = 2 \times 10^3\,\text{cm}^2/(\text{B.s})$, the electron mobility. For the waveguide electron density, we get

$n = 7\times10^{14}\,\mathrm{cm}^{-3}$. Value n is in a good agreement with the experimental results and estimates given in other papers [26], which again confirms the reliability of the chosen EGB model. In this case, the scale of the streamer linear resistance, according to [25], is $r_s / d = 5.5\times10^4$ $\mathrm{Ohm/cm}$. If we take into account that the ionization density of streamer channel is in the range $10^{13} < n < 10^{16}\,\mathrm{cm}^{-3}$ [26], then the streamer resistance should be in the range $10^3 < r_s / d < 3.8\times10^6\,\mathrm{Ohm/cm}$.

3. The theory of Plasma Waveguides and the Electric Gas Breakdown

However, this does not yet provide a complete agreement between the theory and experiment in the field of EGB, since the theory of plasma waveguides developed for plasma accelerators does not directly agree with the problems of EGB. Among the problems that need to be worked out, we should note the choice of the analytical form of the dispersion characteristic, the choice of a working point on the radial profile of the electron density, and the technology of choosing a working point on the dispersion characteristic.

The theory of plasma waveguides was developed by 1976. According to [33], a plasma waveguide is considered to be the plasma with a surface allowing the propagation of surface waves. The most demanded plasma waveguide is cylindrical. Its theory solves two main problems: the problem of distribution of the field in space and the problem of eigenfrequencies of the waveguide, or the obtaining of the dispersion characteristic [33]. Depending on the direction of the electrical and magnetic components of the wave, E-waves and H-waves are distinguished with respect to the axis of the cylindrical plasma waveguide. The longitudinal surface E-wave is of greatest interest for the EGB problem, since the phase velocity of the wave can be lower than the speed of light, in addition to the geometric longitudinal direction of the electric field strength E along the waveguide axis. This allows the phase interaction of the wave with the electron with no use of the elements slowing the wave velocity, and this reflects the charge transfer in the EGB phenomena.

A qualitative form of dispersion characteristic of the surface E-wave [33] $\omega(k)$, is shown in Figure 3. The upper curve illustrates the electric component, and the lower curve the ion one. For the upper curve two asymptotes are distinguished: a short-wave $\Omega_e / \sqrt{2}$, where $\Omega_e = \sqrt{4\pi ne^2 / m}$ is the plasma frequency, and a long-wave c, that is equal to the light velocity. And for the lower curve: a short-wave one equal to $\Omega_i / \sqrt{2}$, where Ω_i is the ion plasma frequency, and the long-wave one, which is equal to the ion sound velocity C.

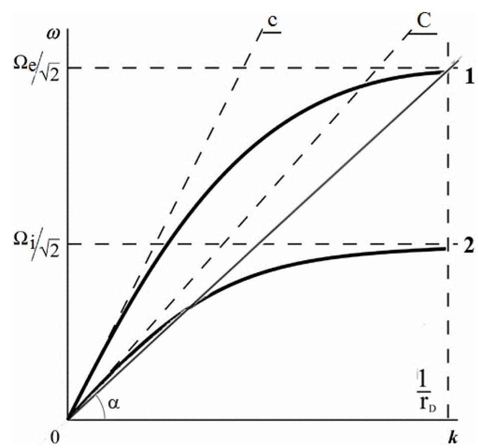

Figure 3. Dispersion characteristic of the E-wave in plasma waveguide [33]. Curve 1- the electron branch; curve 2- the ion branch.

Both characteristics are limited by the value of the Debye radius reciprocal r_D for the upper boundary of wave numbers. Thus, even from the asymptotic behavior of the dispersion characteristic follows the velocity boundaries of possible waves propagating along such a waveguide. The lower boundary of phase velocity of the electron wave is determined by $\mathrm{tg}\,\alpha$ (see Figure 3), and the magnitude of such a velocity will be of the order of the plasma electron thermal velocity, which at the temperature of $300\,\mathrm{K}$ gives $\sim 10^7\,\mathrm{cm/s}$, and this corresponds to the experiment.

In addition, it follows from Figure 3 that the group velocity of the *E*-wave is less than or equal to the phase velocity. The existence of an ion dispersion characteristic is extremely convenient, since it allows one to experimentally simulate most of the features of electronic characteristic with significantly lower requirements to the measuring equipment in terms of temporal and spatial resolution.

This was used in [10] for the wave motion equation diagnostics. Nevertheless, from Figure 3 follow the questions (mentioned at the beginning of this section), which made it difficult to apply this theory to the description of high-speed ionization waves [39], in spite of the fact that the theoretical design scheme [33] and the experimental scheme [39] practically coincided.

The application of telegraph equations, as was done in [39], is doubtful [40]. For the EGB problems, due to the absence of an external casing and magnetic field, the most suitable for the dispersion characteristic is the expression given in [41], in contrast to the problems of plasma accelerators. The expression has the form:

$$\varepsilon \cdot \frac{I_1(k_1 R)}{k_1 I_0(k_1 R)} + \frac{K_1(\chi R)}{\chi K_0(\chi R)} = 0, \tag{14}$$

where k_1 is the wave number along the waveguide radius in plasma, and $\chi = (k_3{}^2 - k^2)^{0.5}$, where k_3 is an axial wave number outside plasma; $k = \omega/c$, I_0, I_1, the modified Bessel functions; K_0 and K_1, the Macdonald functions.

The relation $\omega < \Omega_e / \sqrt{2}$ between the working frequency ω and the cutoff frequency of the surface wave $\Omega_e / \sqrt{2}$ (which follows from Figure 3), has an evident physical meaning. If an equality $1/\omega > \sqrt{2} / \Omega_e$ is multiplied by the wave phase velocity $V_p = \sqrt{2T/m}$, we have $\lambda_1 > 2$ $(2\pi r_D)$, i.e., the length of the transverse wave λ_1 must be greater than doubled circumference of the Debye radius.

In addition, it is necessary to determine the localization of the wave with respect to the radial profile of the electron density, and this will be done below. The concrete form of the electron radial density distribution with a "smooth boundary", not limited to a special shell, is determined by the geometry of the waveguide and by the technology of the corresponding distribution in it. In the case of EGB, the most probable distributions of electron density along the radius can be in the form of a sinusoid square, in the form of the Bessel function of zero and first order, and in the form of the Gaussian distribution. In Figure 4, this distribution (Figure 4b) is shown together with the ideal distribution used in [33] and the distribution formed after the relaxation of the axial part of the waveguide.

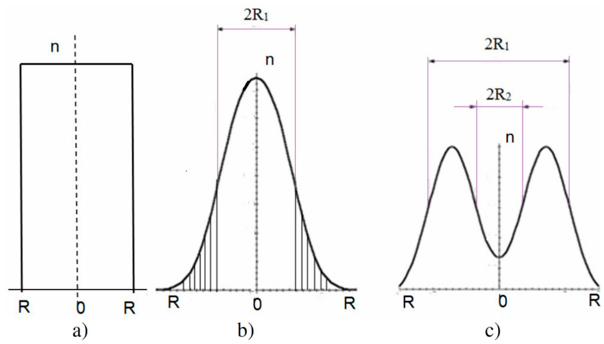

Figure 4. The transverse structure of the ionization density in the plasma waveguide (a- theoretical distribution of electron density [33], b- experimental distribution of electron density in the plasma waveguide; c- the electron density distribution in the plasma waveguide after its relaxation in the paraxial part). R_1 and R_2 are the waveguide radii corresponding to the points of inflection of the electron density profile.

For a plasma waveguide, the absorption has two components, the surface and the volume ones. In this case, the surface absorption is inversely proportional to the modulus of the radial electron density gradient [33]. This means that for a radial profile with a sharp boundary, the surface wave will have zero attenuation. And for a radial profile with a smooth boundary, the minimum attenuation has the surface wave propagating along the generatrix of the cylinder corresponding to the inflection point of the radial electron density profile, that is, at the point where $\partial^2 n / \partial r^2 = 0$, and where such a gradient is maximal. The electron density at this point will determine the limiting frequency $\Omega_e / \sqrt{2}$. In this case, the collision interaction of the wave and the medium is observed at the tail of such a distribution, and is shown by hatching in Figure 4b, where the wave field strength and the electron density decrease. And this may allow one to neglect this type of absorption in comparison with the surface one.

An integral experiment showing the possibility of such a neglect is the deviation of the wave asymptotic velocity from the electron thermal velocity. The insignificance of such a deviation, as observed in [25], indicates that collisionless absorption is much greater than the collision absorption, and this neglect is admissible.

The choice of the operating point on the dispersion characteristics, that is, the matching of the plasma elementary parameters of the plasma waveguide with the radial profile of electron distribution $n(r)$ and their temperature T with its measured parameters ω and χ, should begin with the choice of the limiting frequency in the above technique. Then the plot of dispersion characteristic turns to be single-valued, and its crossing with the wave phase velocity in Figure 3 will give the operating point.

In conclusion of this section one should make a few remarks. The choice of the expression (14) as the dispersion characteristic of the plasma waveguide is not devoid of shortcomings. This expression was obtained in the approximation of plasma hydrodynamic model, which does not always agree with the experiment [13]. For the EGB problems under consideration, these drawbacks, firstly, are due to the fact that this model satisfactorily describes the plasma waveguide in the collisionless regime, when the wave frequency substantially exceeds the frequency of electron-atom collisions. Secondly, this description requires that the phase velocity of the wave be substantially higher than the thermal velocity, that is, $V_p >> V_T$. Finally, thirdly, the structure (14) implies that the dielectric constant of the plasma is negative, since in the (14) all the remaining terms are positive, that is, $\varepsilon = 1 - \Omega^2 / \omega^2 < 0$. The latter means that there is no wave inside the waveguide. Nevertheless, at this stage of the study expression (14) does not contradict the NPVM. The reason for this lies in the fact that the collision absorption can be small as compared to the collisionless absorption for the reason indicated above.

The condition $V_p >> V_T$ is a mathematical condition, and, therefore, does not physically interfere with the consideration of the experimental situation $V_p \approx V_T$ on the basis of (14). In [33], the analysis of the dispersion characteristic is carried out up to $V_p \approx 0$, and this is incorrect. If we set the boundary for all the wave numbers $k < rD^{-1}$, then on the boundary of the dispersion characteristic for the EGB, an acceptable value of the wave phase velocity is obtained (see $\mathrm{tg}\,\alpha$ in Figure 3). The third circumstance, related to $\varepsilon < 0$, follows from the mathematical technology of obtaining the dispersion characteristic. This technology proceeds from the decoding of the boundary condition for the tangential field components on the cylindrical surface of the waveguide in the configuration of Figure 4a. This condition reduces to the equality of the tangent field components on both sides of the boundary. The presence of a field with the frequency below the plasma one inside the plasma is possible only in two cases [41]. Either it is due to the presence of a beam of external electrons in the plasma, or it is related to the skin effect. Each of these cases significantly complicates the consideration of the problem, but imposes only amplitude limits on its solutions, and, therefore, the technology of formulation of the problem chosen in [33] can be considered acceptable.

The resulting mathematical inconsistencies are eliminated by changing the structure of the plasma dielectric constant to the form $\varepsilon = 1 - \Omega^2 / \omega^2 - \varepsilon_0$, where ε_0 is the dielectric constant of the medium surrounding the waveguide [41-44]. The boundary frequency of the plasma waveguide is converted to the form $\Omega / (1 + \varepsilon_0)^{0.5}$. Physical contradictions are removed in this case due to the fact that the E-wave is converted into the plasma wave, the field does not penetrate into the waveguide, and the spectrum of the wave incident on the outer surface of the waveguide is contracted to the frequency of the plasma wave. One should also note that the structures of the transverse and longitudinal dispersion characteristics are similar, and in Figure 3 they have the same asymptotic behavior [33], and this means that the phase velocity of the E-wave is the same in both the longitudinal and transverse directions.

All this allows us to use (14) as the first mathematical approximation at this stage of the EGB study, but it requires additional experimental verification. Obviously, the final analysis of these contradictions must be carried out on the basis of a comparison with the experiment, but, unfortunately, it is not possible to use the experience gained in studying the beams in plasma [45], since in those works the plasma, as a rule, is in a strong magnetic field, the beams propagate in the volume, i.e., the skin-layer value is large.

In the EGB region, such an experiment is associated with a radial jump in the EGB wave accompanying the avalanche-streamer transition. To preserve the ionization profile of the waveguide $n(r)$ with one central maximum (see Figure 4b), it is necessary that the thickness of the skin layer be larger or of the order of the waveguide diameter. Otherwise, the wave can go to the inner cylindrical surface, as shown in Figure 4c, and observed in Figure 5 for the avalanche-streamer transition [7]. Since the next radial transition is not observed, it can be assumed that the diameter of the streamer is determined by the thickness of the skin layer and has the value of about $50\,\mu\text{m}$. As seen from Figure 4c, the diameter of the wave will change by 3 times in this transition. Accordingly, the frequency of the wave of the highest quality factor should change. Taking into account an increase in the energy density after such a transition, the temperature of the waveguide electrons and, consequently, the velocity of the EGB wave will increase.

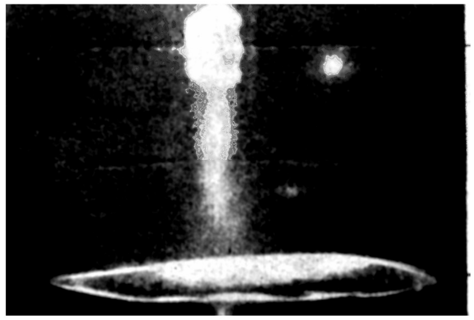

Figure 5. Change of the EGB wave diameter under avalanche-streamer transition [7].

To the top: the anode; to the bottom: the cathode. An avalanche to the top corresponds to R_1; the streamer to the bottom, R_2.

An additional verification was made in [46] in accordance with the experimental scheme given in Figure 6.

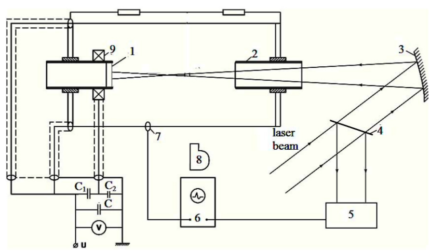

Figure 6. Schematic of the experimental setup for determining the plasma waveguide eigenfrequency [46]. 1- cathode; 2- anode; 3- focusing mirror; 4- splitter; 5- photo detector; 6- oscillograph; 7- Rogowski coil, 8- high-speed camera, 9- source of a longitudinal electric field.

When the electrode voltage is below the threshold of the electrical breakdown, a long laser spark generates a plasma waveguide between them. If in this channel, using the radiator of the longitudinal electric field 9 made in the form of a toroid, and changing the capacitance C_2, one tunes to the resonance frequency, then the oscilloscope will show the corresponding signal. In our experiment [46], two eigenfrequencies were generated, differing by a factor of three, i.e., $\nu_1 = 0.67\,\text{MHz}$ and $\nu_2 = 2\,\text{MHz}$. It turned out that the smaller frequency corresponded to the condition resulting from the NPWM, $\nu = V_p / 2\pi R_1$, which at $R_1 \approx 1\text{cm}$ see Figure 4), and corresponded to the range of the ionizing gas of ultraviolet radiation. This condition allowed one without frequency scanning the PW almost immediately obtain a resonant reaction of high-Q channel with the help of simple methods, changing only the capacitance C_2.

5. Conclusion

The soliton hypothesis of the NPWM, first tested in [19], is sufficiently substantiated when a mathematical description of the EGB wave motion goes from the KdV equations to the sine-Gordon equation. The reliability of the sine-Gordon equation is determined by the experiment [10], in which the nature of the avalanche-streamer transition was uniquely related to the transition from the breather solution of the sine-Gordon equation to the kink-antikink solution.

The waveguide part of the NPWM is sufficiently substantiated by experiment [25]. However, [25] establishes two fundamental requirements. A spatially bounded plasma medium generates on its surface a near-monochromatic longitudinal E-wave that is transformed into a plasma one and has the phase velocity close to the velocity of a thermal electron at the maximum Maxwell distribution function. A necessary condition for the propagation of the EGB wave is the presence of runaway electrons as a source of a longitudinal E-wave in the medium of a pre-prepared plasma waveguide.

The possibility to describe the EGB waves as the surface waves of the plasma waveguide was predicted in [14, 15]. The analysis of NPWM in the first approximation can be carried out on the basis of the existing theory of plasma waveguides, since it reflects the values of the EGB wave velocities at the boundaries of the ranges, and reflects, also, the condition that the phase velocity of the EGB wave exceeds the group velocity. The longitudinal E-wave in the plasma waveguide admits a transverse radial transition, which is associated with the formation of a dip on the axis of the radial electron distribution $n(r)$, and this is proved by the experiments [7, 46]. In this case, the NPWM sets the thickness of the skin layer in the avalanche and streamer mechanisms of the EGB as $50\mu\text{m}$. In addition, the experiment [25] determines the scale of the streamer resistance in the range from $3.8 \times 10^3 \Omega/\text{cm}$ to $3.8 \times 10^6 \Omega/\text{cm}$.

As seen from the analysis of collisionless surface absorption in the plasma waveguide with a smooth boundary, the diameter of the EGB wave coincides with the minimum of such absorption, where the condition $\partial^2 n / \partial r^2 \approx 0$ is satisfied for the radial electron density $n(r)$. Taking into account the fact that the phase velocity of the EGB wave is equal to the electron thermal velocity, the choice of the operating point on the dispersion characteristic of the waveguide $\omega(k)$ becomes unambiguous.

The behavior of the EGB wave makes it possible to model the waves in other plasma waveguides and resonators of a similar type, for example, in Ref. [39].

Acknowledgement

The authors are grateful to A. A. Rukhadze, Yu. A. Mikhailov and A. A. Rupasov for useful discussions.

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