

# PERIOD DOUBLING PHENOMENON AS THE ORIGIN OF ELECTRON PROPERTIES

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## **Abstract**

In the Standard Model of elementary particles, the rest mass, magnetic moment and electric charge of the electron are considered to be natural constants. In this article, it is proposed that these properties can be calculated in a simple way from the corresponding Planck units using the well-known period doubling phenomenon in nonlinear dynamical systems. Periods corresponding to the values of the electron and positron properties belong to a subset of stable periods. The periodic structures of the rest energy and magnetic moment consist of three internal degrees of freedom, whereas the Coulomb energy of the electric charge consists of four. The number of period doublings for the elementary charge determines the value of the fine structure constant  $\alpha$ .

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## 1. Introduction

The purpose of this article is to give attention to the possibility that the per se well known physical phenomenon, namely the period doubling process occurring in various nonlinear dynamical systems [1-6], can be directly related to the physical properties of matter.

The possible existence at the Planck scale of the period doubling phenomenon was discovered by experimental data analysis showing regular discrete peaks [7] interpreted as degrees of freedom.

In this article the properties of the electron-positron pair and the fine structure constant are numerically analyzed in Section 2 by using the simple period doubling formulas presented in Section 1 for three and four degrees of freedom.

Louis de Broglie [8] in 1924 proposed that particles behave as a wave with an associated wavelength  $\lambda = h/p$ , where  $p$  is the momentum of the particle and  $h$  the Planck constant. The Compton wavelength  $\lambda = h/mc$ , where  $m$  is the rest mass of the particle, corresponds to the rest momentum, or rest energy, of the particle.

George Thomson [9], Clinton Davisson and Lester Germer [10] verified the wave behavior of electrons in diffraction experiments in the nineteen twenties. The matter wave concept led to the invention and development of quantum mechanics by E. Schrödinger, W. Heisenberg, W. Pauli, M. Born and others. The physical nature of the matter wave has remained obscure, however.

The wave nature of matter raises the question whether the electron could possess a periodic structure related to the Compton wavelength. This question is especially intriguing because the electron is traditionally considered as point-like and structure-less.

It is suggested that such a periodic structure exists and that the structure creating process is the well-known period doubling cascade

taking place in nonlinear dynamical systems, and that the Planck time can be taken as the fundamental period.

The Planck units, defined by natural constants, have been of interest to both cosmologists and the elementary particle physicists as a possible base for fundamental quantum physical phenomena.

However, the Planck units are far too large or too small to be meaningful, e.g., in connection with the properties of an electron. For instance, the Compton wavelength of the Planck mass is  $c \cdot 10^{-35}$  m, which can be compared with that of an electron, which is  $c \cdot 10^{-12}$  m. The value of the Planck charge is not too far from the elementary charge, however.

Period doubling is a universal property of nonlinear dynamical systems, as shown by M. Feigenbaum [10]. If the natural constants are the same everywhere, then the Planck units can be used as a universal basis. The Planck energy, magnetic moment, and the Planck charge, listed in Table 1, have no correspondence with the real world. The Planck units have been defined here by  $h$  (not  $\hbar$ ), because energy is  $E = hf = h/\text{period}$ . The definitions of the relevant Planck units are shown in equations (1) to (4), where  $h$  is the Planck constant,  $G$  the gravitational constant,  $c$  the speed of light,  $e$  the elementary electric charge and  $\mu_0$  the vacuum permeability.

It is convenient to define the Planck time  $t_p$  ( $\approx 10^{-43}$  s) as the fundamental period due to its universality, and because energy (2) and magnetic moment (3) can be derived from it.

$$t_p = \sqrt{\frac{hG}{c^5}}, \quad (1)$$

$$E_p = \frac{h}{t_p}, \quad (2)$$

$$\mu_p = \frac{ec^2 t_p}{4\pi}, \quad (3)$$

$$q_p = \sqrt{\frac{4\pi\hbar}{\mu_0 c}}, \quad (4)$$

where  $E_p$  is the Planck energy,  $\mu_p$  the Planck magnetic moment and  $q_p$  the Planck charge.

**Table 1.** Planck units and electron properties

Planck unit	Value
Energy	$3.0604 \times 10^{22}$ MeV
Magnetic moment	$1.5485 \times 10^{-46}$ Am <sup>2</sup>
Charge	$4.7013 \times 10^{-18}$ As
Electron property	Value
Rest energy	0.51099 MeV
Magnetic moment	$9.2848 \times 10^{-24}$ Am <sup>2</sup>
Charge	$1.6022 \times 10^{-19}$ As

Comparison of the values shows that the electron properties are quite far away from the corresponding Planck units except for the electric charge. It is quite evident that many period doublings, or energy halvings, are needed to obtain the values for the electron properties from the corresponding Planck units.

The Planck scale magnetic moment is defined as a current loop  $\mu_p = iA$ , where the current  $i$  is the elementary electric charge divided by the Planck time (= period). The loop area is  $A = \pi r^2$ , where  $r = ct_p / 2\pi$ , i.e., the circumference of the loop is the Planck length and radius the Planck length over  $2\pi$  resulting in equation (3).

If an electron related period  $t_n$  results from a one-degree-of-freedom period doubling process at the Planck scale, then

$$t_n = 2^n t_p, \quad (5)$$

where  $t_p$  is the Planck time (period) and  $n$  the number of period doublings.

Equation (6) displays the normalized  $t_n$ , i.e.,  $t_n$  is presented as ratio to the reference value.

$$\frac{t_n}{t_p} = 2^n. \quad (6)$$

Consider a system with several degrees of freedom, each having a different number of period doublings. For a system with three periods  $t_i$ ,  $t_j$  and  $t_k$  the corresponding geometric phase-space shape is a straight parallelepiped, whose volume is  $V_{ijk} = t_i t_j t_k$ . The volume doubles if any one period doubles. If this structure represents, e.g., an elementary particle, one should, in principle, be able to observe the individual periods. Unfortunately, the particle collision and decay experiments do not separate the degrees of freedom from one another but return the particle rest energy  $E = h/t_c$  corresponding to one characteristic period  $t_c$  according to (7).

Conservation of energy requires that

$$t_c^3 = t_i t_j t_k \quad (7)$$

which means that

$$t_c = (t_i t_j t_k)^{\frac{1}{3}} \quad (8)$$

and so

$$\frac{t_c}{t_p} = 2^n = 2^{\frac{i+j+k}{3}} = 2^{\frac{N}{3}}, \quad (9)$$

where  $N = i + j + k$  is the total number of period doublings. The exponent of 2 in (9) is an integer divided by the number of the degrees of freedom, i.e., cube root of  $2^N$ .

For an observer, energy is a scalar quantity, which manifests itself as a geometric mean of the periods of the internal degrees of freedom according to (8) and (9).

It is also known that periods of the form

$$t_n = 2^{2^n} t_0 \quad (10)$$

are especially stable [10]. In (10)  $t_0$  is the fundamental period of the system and  $n$  the number of period doublings.

Numerical calculations in the next paragraph show that (9) and (10) work well with the properties of the electron-positron pair.

## 2. The Electron-Positron Pair

### 2.1. Numerical calculations

The electron and positron are always born together in the pair production process assumed nonlinear. The pair is a 1.022 MeV entity, and both 0.511 MeV particles carry the elementary charge. Applying (9) for the rest energy  $E_{ep}$  of the electron-positron pair, one obtains

$$n = \frac{\log\left(\frac{E_{ep}}{E_p}\right)}{\log(2)} = -74.665 = -\frac{223.99}{3} \cong -\frac{224}{3}. \quad (11)$$

The ratio of the pair Compton wavelength to the Planck length naturally

yields the same  $n$ -value. The magnetic moment  $\mu_{ep}$  corresponding to the pair Compton wavelength is

$$n = \frac{\log\left(\frac{\mu_{ep}}{\mu_p}\right)}{\log(2)} = 74.665 = \frac{224}{3}, \quad (12)$$

where  $\mu_{ep}$  is half of the electron magnetic moment (Table 1), because the current loop for the pair structure is smaller. Equations (11) and (12) show that  $n$  is an integer divided by three as suggested by (9) for three degrees of freedom.

The remarkable observation is that the number of period doublings is the same for the electron-positron pair rest energy and magnetic moment albeit the fact that the Planck units have been calculated from a different set of natural constants ( $G$  in (1) and (2),  $\mu_0$  in (4)).

The electron rest energy is half of the original pair energy. Therefore, the corresponding period for the 0.511 MeV energy is longer by factor of 2, and so the number of period doublings is one more, or  $n = -75.665 \approx -227/3$ .

The magnetic moment corresponding to the larger loop of the electron structure is also obtained by one more period doubling, or  $n = 75.666 = 227/3$  period doublings.

The total number of period doublings in three degrees of freedom is  $N = 224$  for the pair. The number of period doublings is negative for the rest energy, because the doubling process divides the Planck energy ( $h/period$ ) into precise smaller parts, while the number of doublings is positive for the magnetic moment, because the magnetic moment gets larger with longer periods according to (3).

The Planck charge is quite close to the elementary electric charge  $e$ .

This means that the total number of period doublings for  $e$  is much less than 224. Comparison of the squares of the charges, i.e., their Coulomb energies, yields

$$n = \frac{\log\left(\frac{e}{q_p}\right)^2}{\log(2)} = -9.7499 \cong -\frac{39}{4}. \quad (13)$$

The total number of period doublings is  $N = 39$ . The fourth root indicates that the number of degrees of freedom is four and the phase space volume correspondingly  $V_{lmnp} = t_l t_m t_n t_p$ . The characteristic period is  $t_c = (t_l t_m t_n t_p)^{1/4}$  analogously to (8). At first sight the  $N$ -values do not seem to be related to anything familiar.

## 2.2. Interpretation of the $N$ -values

For one degree of freedom, the  $n$ -values should be integers. Equations (11), (12) and (13) show that  $n = \pm N/3 = 224/3$  for the magnetic moment and the rest energy of the pair, and  $n = -N/3 = -39/4$  for the Coulomb energy.

Mathematically speaking  $N = i + j + k$  for (11) and (12) and  $N = l + m + n + p$  for (13), where integers  $i$ ,  $j$ ,  $k$  and  $l$ ,  $m$ ,  $n$ ,  $p$  represent the individual period doublings for each degree of freedom.

The electron and positron are known to be stable, and one may expect  $i$ ,  $j$ ,  $k$  and  $l$ ,  $m$ ,  $n$ ,  $p$  to be of the form of (10). Indeed  $N = i + j + k = 224 = 32 + 64 + 128 = 2^5 + 2^6 + 2^7$  and  $N = l + m + n + p = 39 = 1 + 2 + 4 + 32 = 2^0 + 2^1 + 2^2 + 2^5$ , showing that the stability condition (10) is fulfilled for each degree of freedom. Equations (12) and (13) show that the periodic nature of matter also applies to the magnetic moment and the Coulomb energy of the electric charge.

Table 2 compares measured values to those given by  $n = 224/3$  and

$n = 39/4$  in (5) for the electron-positron pair rest energy  $E_{ep}$ , magnetic moment  $\mu_{ep}$  and elementary charge  $e^2$  correspondingly ( $t_p$  replaced by  $E_p$ ,  $q_p$  or  $\mu_p$ ). For convenience Table 2 shows the value of  $e$  instead of  $e^2$ .

**Table 2.** Comparison of the calculated and measured values

Electron	Measured	Calculated
$E_e$ (MeV)	0.511	0.511
$e$ (As)	$-1.6022 \times 10^{-19}$	$-1.6021 \times 10^{-19}$
$\mu_e$ (Am <sup>2</sup> )	$-9.285 \times 10^{-24}$	$-9.286 \times 10^{-24}$

The  $N$ -values for the electric charge squared differ from those of the rest energy and magnetic moment except for period  $i = p = 2^5$ , which is common to all. Three degrees of freedom ( $l = 1, m = 2, n = 4$ ) of the Coulomb energy are independent of the degrees of freedom of the rest energy and magnetic moment of the electron. This means that mass and electric charge related period doublings are independent phenomena ( $G$  vs.  $\mu_0$ ).

The Planck mass-energy (2) does not contain the Coulomb-energy, which can be calculated knowing the ratio of the elementary charge squared to the Planck charge squared. The Coulomb-energy for the pair structure becomes  $2^{-74.667} \cdot (2^{-975} \cdot E_p) = 0.00119$  MeV and 0.00059 MeV for the electron. The combined electron rest energy becomes 0.5109 MeV, which differs from the measured value by 0.02%.

The numeric value of the elementary charge can be solved for from (13) using  $N = -39/4$ . One obtains  $e = 1.6021$  As, which differs from the accepted value by 0.003%.

Changing any one number  $i, j, k$  or  $l, m, n, p$  by  $\pm 1$  changes the

calculated  $3D$  values by factor of 1.26 and  $4D$  (energy) values correspondingly by 1.19, and the calculated values in Table 2 would be largely incorrect.

The calculated value of the elementary charge differs the least from the measured value. The reason may be the missing of the gravitational constant from the definition of the Planck charge. The gravitational constant is by far the least accurate of the natural constants.

### 3. Fine Structure Constant

Alpha is defined  $\alpha = 1/2 \mu_0 e^2 c / h = 2\pi (e / q_p)^2$ , which can be written, according to (13),  $2\pi \cdot 2^{-39/4} = 0.00730$  and  $1/\alpha = 137.045$ . The difference is 0.007% with respect to the CODATA value of alpha. In this context, the interpretation of alpha is rather straightforward: The volumetric period doubling process dilutes the initial Planck scale Coulomb energy, and the elementary charge represents the resulting value of the Coulomb energy.

### 4. Summary and Conclusions

It is assumed that the electron-positron pair creation is a nonlinear process, and that the periodic structure is simply created by the period doubling phenomenon, which is a universal property of nonlinear dynamical systems. The Planck time, considered as a period, is used as the universal basis for the calculations.

The numerical calculations in Section 2 yield correct values with integer number of doublings without additional parameters or tuning.

The small difference between the calculated and measured values in Table 2 show that the period doubling process is exact after many doublings. This means that the combination of the natural constants used to define the Planck scale quantities has remained constant over time.

The cube root and fourth root differ in that the cube root is positive (or negative), while the fourth root is  $\pm$  giving the dipolar property for the elementary charge vs. monopolar mass.

The electron-positron pair is superstable as shown by the observed number of period doublings fulfilling the stability condition.

Considering the electron as a structureless point-like particle excludes the associated periodic structure revealed by this analysis, which shows that the periodic nature of matter also applies to the Coulomb energy of the electric charge and magnetic moment of the electron.

It is therefore concluded that the period doubling process at the Planck scale can be taken as a hypothesis for the origin of the present values of the electron properties and the fine structure constant.

This hypothesis is further corroborated by the good numerical fits in other natural phenomena (e.g., the Cosmic Microwave Background temperature, Hydrogen 21 cm line, quantized galaxy redshifts [12] and planetary orbits) [7]. It should be noted that the periodic structure, or intrinsic wave property created by the period doubling process is stationary contrary to the de Broglie wave, which is kinematic.

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