Fundamental J. Thermal Science and Engineering, Vol. 1, Issue 2, 2011, Pages 83-99
Published online at http://www.frdint.com/

OSCILLATORY FLOW OF VISCOUS FLUID IN A HORIZONTAL CHANNEL FILLED WITH POROUS MATERIAL

ABIODUN O. AJIBADE* **and STEPHEN ISHAYA**

Department of Mathematics
Ahmadu Bello University
Zaria, Nigeria
e-mail: olubadey2k@yahoo.com

Abstract

The flow of viscous fluid in a horizontal channel filled with porous material is considered. The motion of the fluid is induced by one of the channel boundaries moving in its own plane with a sinusoidal variation of velocity. An analytic solution describing the flow is obtained by solving the partial differential equation in dimensionless form. The expressions for the velocity and skin-friction are derived using the method of substitution in the momentum balance equation that is used to describe the flow. The response of velocity and skin-friction to the different governing parameters is fully discussed with the aid of graphs. It is interesting to remark that fluid velocity increases as the Darcy number increases. Also, as the ratio of effective viscosity to that of the fluid viscosity increases, fluid motion responds with an increase in velocity within the channel.

1. Introduction

The motion of viscous fluid caused by the sinusoidal oscillation of flat plate is termed as Stokes' second problem (Schlichting [1]). The study of this type of flow is not only of fundamental theoretical interest but it also occurs in many applied

Keywords and phrases: porous medium, oscillating plate, periodic solution, Darcy number.

*Corresponding author

Received November 10, 2011

problems such as acoustic streaming around an oscillating body, an unsteady boundary layer with fluctuation (Tokuda [2]). Pop and Soundalgekar [3] studied the Stokes problem for a uniformly accelerated vertical plate. Erdogan [4] studied the flow of a viscous fluid induced by a plane boundary moving in its own plane with a sinusoidal variation of velocity. He concluded that the time required to reach steady-state for the sine oscillation of the wall is longer than that for the cosine oscillation of the wall. In a related work, Fetecau et al. [5] agrees with the conclusions of Erdogan [4] on the time required to reach a steady-state for sine and cosine oscillations of the wall. They, however, added that the time decreases as the oscillation frequency of the velocity of the wall increases. In a recent article, Ajibade and Bello [6] investigated the influence of transpiration and variable channel width on oscillatory Couette flow in a horizontal channel. The work reported that velocity of fluid increases as the channel is widened when flow is induced by cosine oscillation of the channel plate while the trend is reversed in case of flow induced by sine oscillation. Jha and Ajibade [7] studied the Couette flow of heat generating fluid between two vertical parallel plates when one plate is subjected to an impulsive motion. In another work, Jha and Ajibade [8] studied the hydrodynamics of viscous incompressible fluid in a vertical channel induced by an accelerated motion of a boundary plate. Muhuri [9] studied the Couette flow between two porous walls when one of the walls moves with a uniform acceleration with uniform suction and injection.

The flow of an incompressible viscous fluid caused by the oscillation of the plane wall is considered when the fluid motion is set up from rest. The velocity field contains transients determined by the initial conditions and these transients gradually disappear with time. The transient solution for the flow due to the oscillating plate has been given by Penton [10]. He has assumed that for large time, steady-state flow is set up with the same frequency as the velocity of the plane boundary. In order to obtain the periodic solution, since the problem is linear, a transient solution must be added to the steady-state solution. Penton [10] has presented a closed-form solution to the transient component of Stokes' problem using the steady state component as the initial profile; however, he has not given expression for the starting velocity field and the expression of the transient solution for the cosine oscillation of the plane boundary.

Flow through porous medium has been extensively studied because of its importance and applications in several industrial and engineering processes. Some of the works that studied flow through annular porous media include: Jha [11], Hossain et al. [10], Kou and Huang [13], Muralidhar and Kulacki [14]. Cheng [15] has found that the effect of modified Darcy number, the buoyancy ratio and annular gap on fully developed natural convection heat and mass transfer is to increase the flow velocity within the annulus. Tiwari et al. [16] investigated an oscillatory natural convection flow in a vertical annulus when the boundaries are subjected to periodic and isothermal heating. They used perturbation technique to obtain analytical expressions for velocity and temperature of the fluid. Other works found in literature include Geindreau and Auriault [17], Hayat et al. [18], Hill and Straughan [19]. Chaughan and Rastogi [20] investigated heat transfer and entropy generation in MHD flow through a porous medium past a stretching sheet while Chaughan and Kumar [21] investigated heat transfer and entropy generation in a channel partially filled with porous medium. In all the works mentioned above, none was found to investigate oscillatory flow in a horizontal channel filled with porous material, hence the motivation.

The purpose of this study is to present the unsteady flow of a viscous fluid in a horizontal channel filled with porous material. The fluid flow is caused by the oscillation of one of the boundary plates. In order to obtain the expression for the velocity of the fluid within the flow domain, the method of substitution is used.

2. Mathematical Analysis

Consider a viscous fluid in a horizontal channel filled with porous material. The fluid motion is set up due to the oscillatory motion of one of the channel boundary plates $(y' = 0)$. Two cases were considered in the present problem; (i) when the plate $y' = 0$ moves with cosine oscillation and (ii) when the plate $y' = 0$ moves with sine oscillation. In addition, the fluid is assumed to be hydrodynamically fully developed in the x'- direction and y'- direction is taken normal to the boundary plates. The schematic diagram of the present problem is presented in Figure 1. The governing equation for the present problem is presented in dimensional form as

$$\frac{\partial u'}{\partial t'} = \mu_{eff} \frac{\partial^2 u'}{\partial y'^2} - \mu \frac{u'}{K}. \tag{1}$$

The boundary conditions for the two cases considered are

$$\text{(i)}\ u' = u_0 \cos \omega' t', \quad y' = 0; \quad u' = 0, \quad y' = h,$$

$$\text{(ii)}\ u' = u_0 \sin \omega' t' \quad y' = 0; \quad u' = 0, \quad y' = h. \tag{2}$$

Using the non-dimensional quantities,

$$u = \frac{u'}{u_0}, \quad \gamma = \frac{\mu_{eff}}{\mu}, \quad \frac{t'\mu}{\rho h^2}, \quad \omega = \frac{\rho\omega' h^2}{\mu}, \quad y = \frac{y'}{h}, \quad \mathrm{Da} = \frac{K}{h^2}. \tag{3}$$

Equations (1) and (2) are presented in dimensionless form as

$$\frac{\partial u}{\partial t} = \gamma \frac{\partial^2 u}{\partial y^2} - \frac{u}{\mathrm{Da}} \tag{4}$$

with the corresponding boundary conditions:

$$\text{(i)} \ u = \cos \omega t, \ y = 0; \quad u = 0, \ y = 1,$$

$$\text{(ii)} \ u = \sin \omega t, \ y = 0; \quad u = 0, \ y = 1, \tag{5}$$

where γ is the ratio of the effective viscosity of the porous media to the viscosity of the working fluid and Da is the Darcy number that measures the degree of porosity of the porous medium, ω is the frequency of periodic oscillations of the plate $y = 0$ and t is dimensionless time. The physical quantities used in equation (3) are defined in the nomenclature.

3. Method of Solution

To solve equation (4) subject to the boundary conditions (5), we assume a periodic solution of the form

$$u(y, t) = F(y)\exp(i\omega t) \tag{6}$$

and use $u(y, t) = \mathrm{Re}(F(y)\exp(i\omega t))$ for case (i) while $u(y, t) = \mathrm{Im}(F(y)\exp(i\omega t))$ is used for case (ii).

Substituting equation (6) into (4) and (5), we obtain the following second order ordinary differential equation

$$\frac{d^2 F}{dy^2} - pF = 0, \tag{7}$$

where $p = (1 + i\omega Da)/(\gamma Da)$. The boundary conditions also reduces to

$$F = (0) = 1, \ F(1) = 0. \tag{8}$$

Solving equation (7) with the boundary conditions (8), we obtain an expression for F as follows

$$F(y) = \frac{\exp(-y\sqrt{p})}{1-\exp(-2\sqrt{p})} - \frac{\exp((y-2)\sqrt{p})}{1-\exp(-2\sqrt{p})}. \tag{9}$$

Using equation (9) in equation (6), the solutions of equation (4) subject to boundary conditions (5) are given as

$$\text{(i)}\ u_1(y) = \mathrm{Re}\left[\left(\frac{\exp(-y\sqrt{p})}{1-\exp(-2\sqrt{p})} - \frac{\exp((y-2)\sqrt{p})}{1-\exp(-2\sqrt{p})}\right)\exp(i\omega t)\right], \tag{10}$$

$$\text{(ii)}\ u_2(y) = \mathrm{Im}\left[\left(\frac{\exp(-y\sqrt{p})}{1-\exp(-2\sqrt{p})} - \frac{\exp((y-2)\sqrt{p})}{1-\exp(-2\sqrt{p})}\right)\exp(i\omega t)\right]. \tag{11}$$

Using expressions (10) and (11), we obtain the skin-friction (τ) on the channel plates as follows;

On the plate $y = 0$

$$\tau_0 = \left.\frac{\partial u}{\partial y}\right|_{y=0} = \left[\left(\frac{-\sqrt{p}}{1-\exp(-2\sqrt{p})} - \frac{\sqrt{p}\exp(-2\sqrt{p})}{1-\exp(-2\sqrt{p})}\right)\exp(i\omega t)\right] \tag{12}$$

while on plate $y = 1$

$$\tau_1 = \left.\frac{\partial u}{\partial y}\right|_{y=1} = \frac{-2\sqrt{p}\exp(i\omega t - \sqrt{p})}{1-\exp(-2\sqrt{p})}. \tag{13}$$

The real part of equations (12) and (13) gives the skin-friction for case (i) while the imaginary part gives the skin-friction for case (ii). To be able to clearly discuss flow formations and give a clear distinction between cases (i) and (ii), equations (10-13) are separated into real and imaginary parts by the use of scientific computing software: MATLAB.

4. Result and Discussion

An unsteady flow of a viscous incompressible fluid is considered in a horizontal channel filled with porous materials. One of the boundary plates is subjected to oscillatory motion and the other plate is stationary. The influence of the governing parameters on the velocity and the skin-friction is discussed under the two cases:

Case (i), when the plate $y = 0$ moves with cosine oscillation and Case (ii), when the plate $y = 0$ moves with sine oscillation.

The expressions for velocity and skin-friction (10-13) are presented graphically for selected values of the governing parameters in Figures 2-8 so that the influence of each governing parameter can easily be seen at a glance. Each of the figures has two parts: (a) is for case (i), while (b) is for case (ii). The dimensionless parameters governing the flow in the present problem are: omega (ω) representing the frequency of the oscillatory motion of the channel plate $y = 0,$ dimensionless time $(t),$ Darcy number (Da) and ratio of the effective viscosity of the porous media to the viscosity of the fluid (γ).

Figure 2 shows the spatial distribution of velocity at different level of porosity. It is revealed from the figure that fluid velocity increases as Da increases in both cases of cosine oscillation as well as sine oscillation of the plate $y = 0.$ This is the physical evidence of the fact that as Da increases, the degree of porosity of the porous media increases as well allowing free passage of the fluid within the channel, hence an increase in the fluid velocity. In addition, when fluid motion is induced by cosine oscillation of the plate $y = 0,$ the fluid velocity is maximum near the oscillating plate while point of maximum flow shifts towards the center of the channel as Da increases. The influence of the ratio of the effective viscosity of the porous medium to that of the fluid on the spatial distribution of velocity of fluid is presented in Figure 3. From this figure, it is observed that when fluid motion is induced by cosine oscillation of the boundary plate, fluid velocity decreases near the oscillating plate as γ increases. However, towards the center of the channel, the trend is reversed as velocity is observed to increase as γ increases. On the other hand, when the plate $y = 0$ moves with sine oscillation, fluid velocity increases with increasing. This is physically true since an increase in γ is caused by either a decrease in the viscosity of the working fluid or an increase in the effective viscosity of the porous media. In either case, free flow of fluid is allowed leading to an increase in the velocity of the fluid.

The effect of time on fluid velocity is shown in Figure 4 for different values of oscillation frequency. From this figure, it is observed for both cases of cosine as well as sine oscillation of the plate $y = 0$ that fluid velocity is oscillatory in nature. It is also observed that the period of oscillation of fluid velocity within the channel is dependent on the frequency of oscillation (ω). That is, the period of oscillation of velocity decreases as ω increases. In addition, velocity is observed to decreases as ω increases. This is the physical evidence of the fact that, as ω increases, the frequency of the sinusoidal motion of the plate increase which decreases the impact of the channel boundary motion on fluid within the channel leading to a decrease in fluid velocity.

Figures 5 to 8 show skin-friction on the channel walls for different values of the governing parameters. Figures 5 and 6 present the skin-friction on the oscillating plate while the skin-friction on the stationary plate is shown in Figures 7 and 8. It is observed in Figure 5 that the skin-friction on the oscillating plate increases as Da increases when the plate moves with cosine oscillation while it decreases as Da increases in the case of sine oscillation of the plate $y = 0$. This is the result of increase in velocity with an increase in Da that was shown in Figure 2 which results to an increase in velocity gradient (Figure 2a) and a decrease in velocity gradient (Figure 2b) on the oscillating channel plate. From this figure also, it is observed that skin-friction decreases as γ increases. This is physically true since velocity gradient on the oscillating plate decreases with increase in γ as shown in Figure 3. Figure 6 shows the influence of ω on skin-friction on the oscillating plate. From this figure, the skin-friction is observed to oscillate as time increases with the period of oscillation decreasing as ω increases.

On the stationary plate $(y = 0)$, the skin-friction is observed in Figure 7 to increase as Da increases for both cosine as well as sine oscillation of the plate $y = 0$. This is attributed to velocity increase with an increase in Da as shown in Figure 2. Consequently, velocity gradient and hence the skin-friction increases as Da increases. From this figure, it is also observed in both cases that skin-friction increases as γ increases. This is physically true since velocity increases with an increase in γ as shown in Figure 3. As the velocity increases with γ, velocity gradient on the stationary plate increases as well so that the skin-friction increases as γ increases. Figure 8 shows the influence of ω on skin-friction on the stationary plate. From this figure, the skin-friction is observed to oscillate as time increases with the period of oscillation decreasing as ω increases.

A comparison between the skin-friction on the channel plates revealed that the skin-friction is higher on the oscillating plate in comparison with the stationary plate. In addition, skin-friction on both channel plates is higher when fluid motion is induced by sine oscillation of the channel plate in comparison to when fluid motion is generated by the cosine oscillation (see Figures 5 and 7).

5. Conclusion

The present article has considered an oscillatory flow of a viscous incompressible fluid in a horizontal channel filled with porous material. During the course of investigation, the following conclusions are drawn from the present problem. Fluid velocity increases with an increase in Darcy number in both cases of cosine oscillation as well as sine oscillation of the plate $y = 0.$ Also, fluid velocity increases within the flow domain when the effective viscosity of the porous medium increases. In addition, the period of oscillation of velocity decreases as ω increases. Finally, skin-friction on both channel plates is higher when fluid motion is induced by sine oscillation of the channel plate in comparison to when fluid motion is generated by the cosine oscillation

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Nomenclature

Da	Darcy number
h	width of the channel
K	permeability of the porous medium
t'	dimensional time
t	dimensionless time
u_0	dimensional amplitude of the periodic oscillation of the moving plate
u'	dimensional velocity of fluid
u	dimensionless velocity of fluid
x'	horizontal axis (direction of flow)
y'	dimensional co-ordinate perpendicular to the plate
y	dimensionless co-ordinate perpendicular to the plate

Greek symbols

μ	coefficient of fluid viscosity
μ_{eff}	meffective viscosity of the porous media
γ	ratio of fluid viscosity to that of the porous media
τ_0, τ_1	skin-friction on the plate $y = 0$ and $y = 1$ respectively
ω'	dimensional frequency of periodic oscillation
ω	dimensionless frequency of periodic oscillation

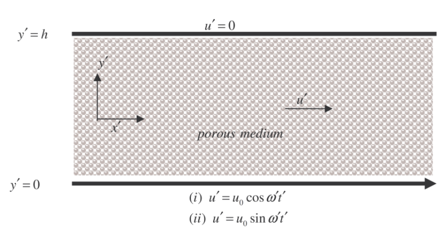
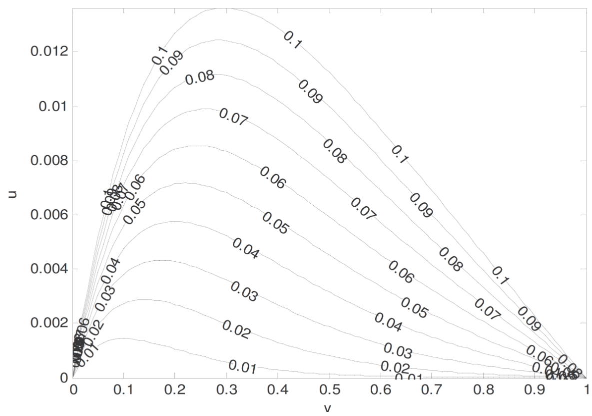


Figure 1. Flow configuration and coordinates system.



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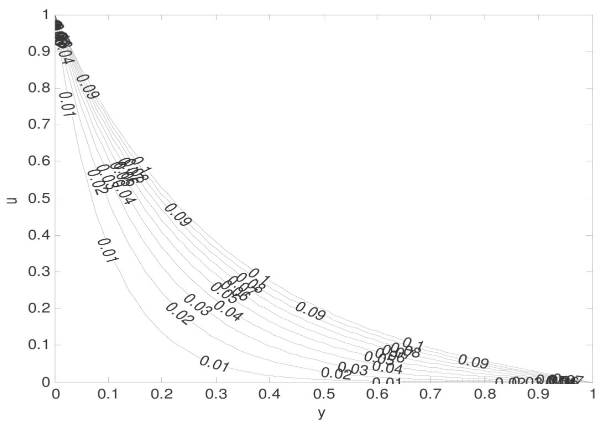
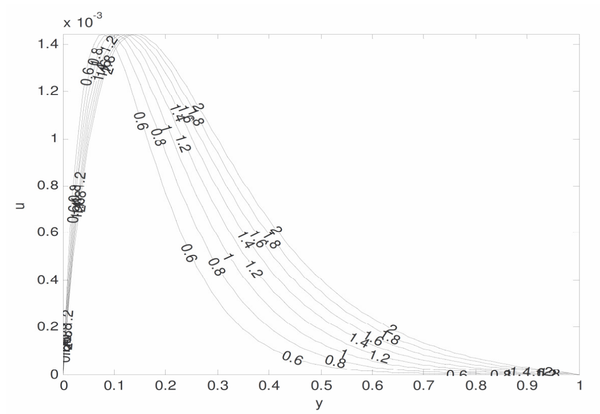


(b)

Figure 2. Velocity profile for different Da $(t = 2,\ \omega = \pi/4,\ \gamma = 1)$.



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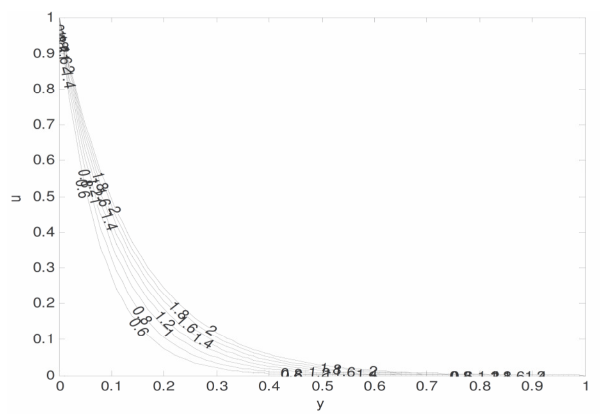
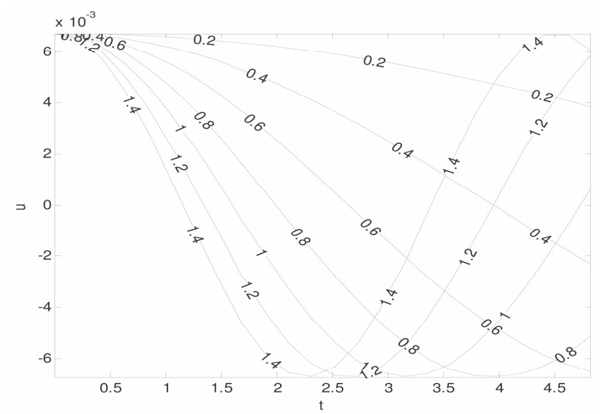


(b)

Figure 3. Velocity profile for different γ $(t = 2,\ \omega = \pi/4,\ \text{Da} = 0.01)$.



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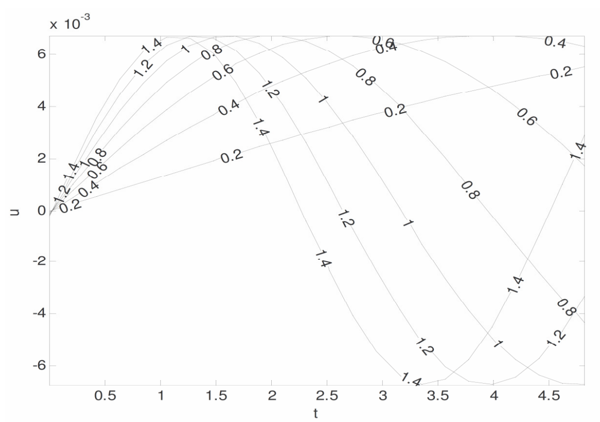
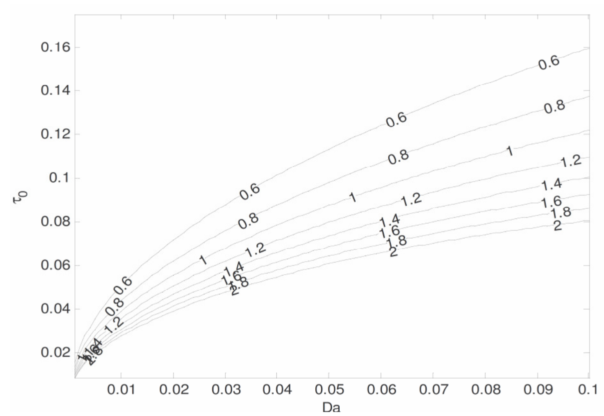


(b)

Figure 4. Velocity profile for different ω $(y = 0.5,\ \gamma = 1,\ \text{Da} = 0.01)$.



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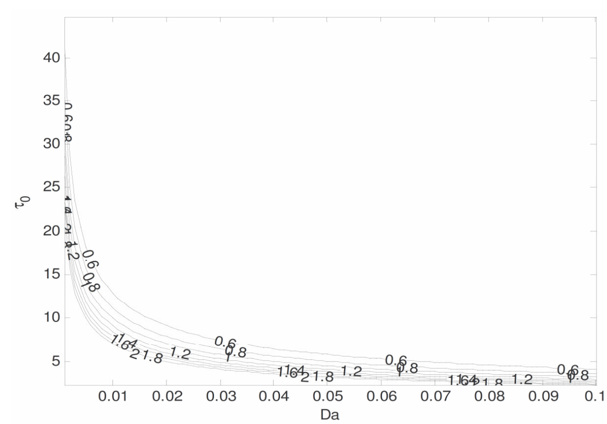
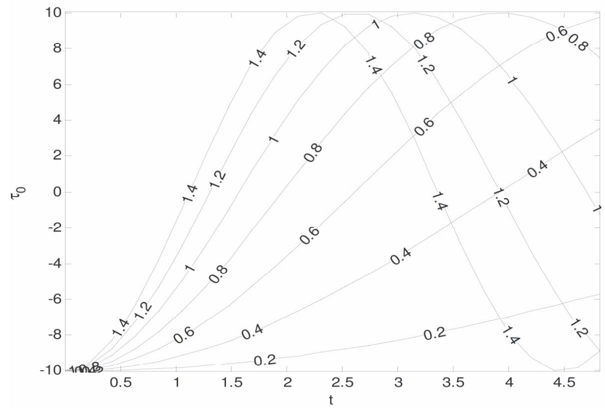


(b)

Figure 5. Skin-friction on the oscillating plate for different $\gamma \left(t = 2, \ \omega = \frac{\pi}{4} \right)$.



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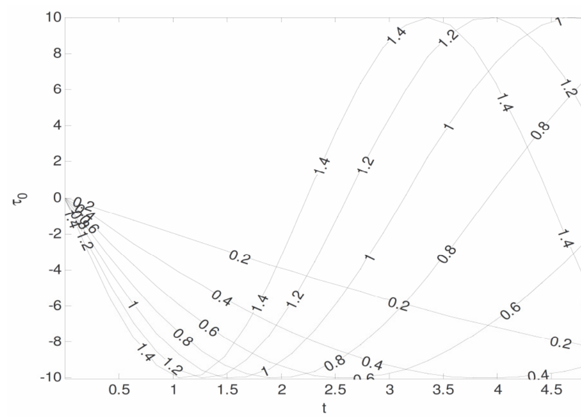

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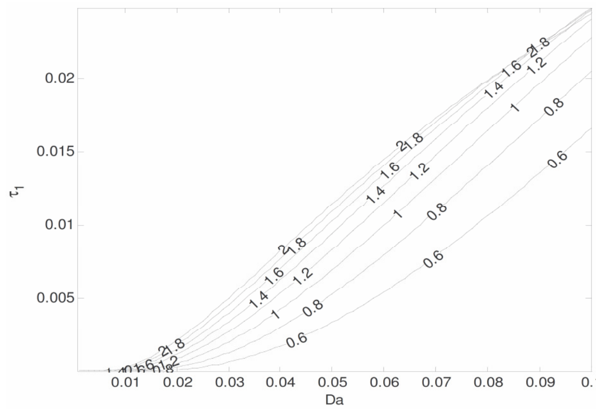
Figure 6. Skin-friction on the oscillating plate for different ω ($Da = 0.01$, $\gamma = 1$).



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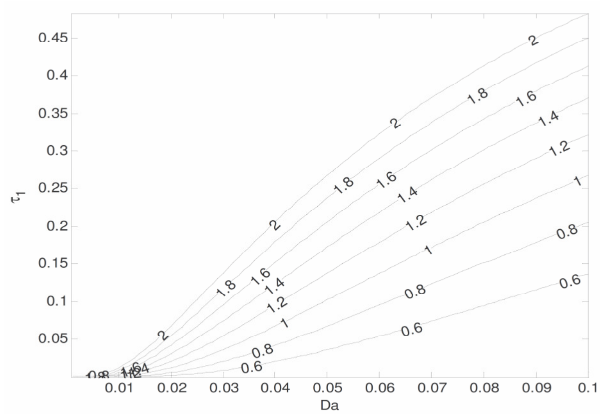
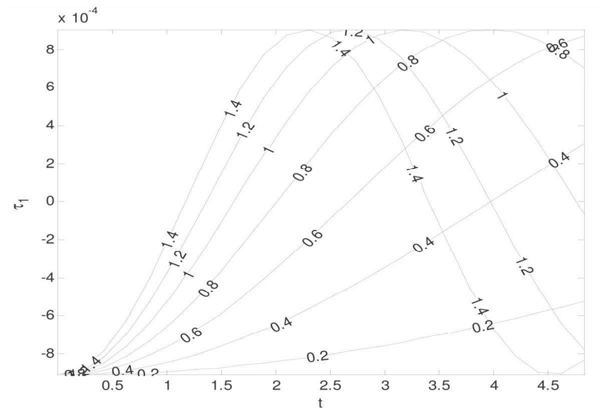


(b)

Figure 7. Skin-friction on the stationary plate for different $\gamma\left(t = 2,\ \omega = \frac{\pi}{4}\right)$.



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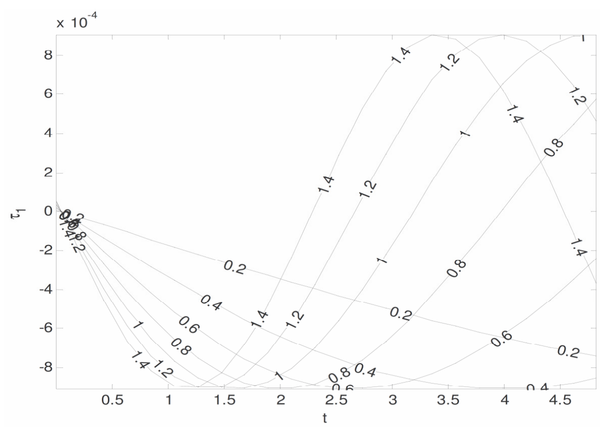

(b)

Figure 8. Skin-friction on the stationary plate for different ω ($\mathrm{Da} = 0.01$, $\gamma = 1$).