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OPERATIONS GENERALIZED PRE-REGULAR CONTINUOUS MAPPINGS

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Abstract

In this paper, we defined (γ^*, β)- generalized-pre-regular-continuity and (γ^*, β^*)- generalized-pre-regular-continuity and obtained several characterizations and some properties of these mappings.

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1. Introduction

The concepts of pre-open sets and semi-pre-open sets were introduced, respectively, by Mashhour et al. [9] and Andrijevic [2]. Kasahara [6] defined the concept of operations α on topological spaces. Ogata [10] called the operations α (respectively, α-closed set) as γ-operations (respectively, γ-closed set) and introduced the notion of τ_γ which is the collection of γ-open sets in topological spaces. Ahmad et al. [1] introduced the concept of γ^*-regular spaces and explored their many interesting properties. Further, they initiated and discussed the concept of γ^*-semi-open sets which generalizes γ-open sets introduced by Ogata [10]. Sai Sundara Krishnan and Balachandran [13] introduced the concept of γ-pre-open sets and studied the separation axioms using γ-pre-open sets. Further, they generated a topology $\tau_{\gamma p}$ using γ-pre-open sets. Sai Sundara Krishnan et al. [14] introduced the concept of γ^*-pre-open sets and γ^*-semi-pre-open sets in topological spaces and investigated some basic properties. Further, they introduced γ^*-pre-$T_i(i = 0, \frac{1}{2}, 1, 2)$ spaces and studied the relationship between them. Saravanakumar et al. [15, 16] introduced the concepts of (γ^*, β)-pre-continuous and (γ^*, β^*)-pre-continuous mappings on topological spaces. Also we introduced the concept of γ^*-generalized-pre-open (closed) sets and defined the relationship between (γ^*, β)-generalized-pre-continuous and (γ^*, β^*)-generalized-pre-continuous mappings and investigated some of their basic properties. In this paper, we discussed the notions of (γ^*_g, β)-pr.continuous and (γ^*_g, β^*_g)-pr.continuous on topological spaces and investigated some of their basic properties.

2. Preliminaries

Throughout this paper, we represent the topological spaces (X, τ), (Y, σ) and (Z, η) as X, Y and Z, respectively, unless otherwise no separation axiom mentioned. An operation γ [6] on the topology τ is a mapping from τ into the power set $P(X)$ of X such that $V \subseteq V^\gamma$ for each $V \in \tau$, where V^γ denotes the value of γ at *V*. It is denoted by $\gamma : \tau \to P\ (X)$. A subset *A* of *X* is γ-open [10], if for each $x \in A$, there exists an open neighborhood *U* such that $x \in U$ and $U^{\gamma} \subseteq A$. Its complement is called γ-closed and τ_{γ} denotes set of all γ-open sets in *X*. For a subset *A* of *X*, γ-interior [10] of *A* is $\text{int}_{\gamma}(A) = \{x \in A : x \in N \in \tau$ and $N^{\gamma} \subseteq A$ for some $N\}$ and γ-closure [10] of *A* is $cl_{\gamma}(A) = \{x \in X;\ x \in U \in \tau$ and $U^{\gamma} \cap A \neq \emptyset$ for all $U\}$. An operation γ on τ is regular [10], if for any open neighborhoods U, V of each $x \in X$, there exists an open neighborhood *W* of *x* such that $U^{\gamma} \cap V^{\gamma} \supseteq W^{\gamma}$; open [10], if for every neighborhood *U* of each $x \in X$, there exists a γ-open set *B* such that $x \in B$ and $U^{\gamma} \supseteq B$. A space *X* is γ-regular [10], if for each $x \in X$ and for each open neighborhood *V* of *x*, there exists an open neighborhood *U* of *x* such that $U^{\gamma} \subseteq V$. A subset *A* of *X* is called γ^*-dense (resp., γ^*-nowhere dense, γ^*- regular-open, γ^*-pre-open, γ^*-semi-pre-open (briefly γ^*-sp.open)) [14], if $cl_{\gamma}(A) = X$ (resp., $\text{int}_{\gamma}(cl_{\gamma}(A)) = \emptyset$, $A = \text{int}_{\gamma}(cl_{\gamma}(A))$, $A \subseteq \text{int}_{\gamma}(cl_{\gamma}(A))$, $A \subseteq cl_{\gamma}(\text{int}_{\gamma}(A)))$. The set of all γ^*-pre-open (resp., γ^*-regular-open, γ^*-sp.open)) sets is denoted by $PO_{\gamma^*}(X)$ (resp., $RO_{\gamma^*}(X)$, $SPO_{\gamma^*}(X)$ *A* is γ^*-pre-closed (resp., γ^*-regular-closed, γ^*-semipre-closed (briefly γ^*-sp.closed)) [14] in *X* if and only if $X - A$ is γ^*-pre-open (resp., γ^*- regular-open, γ^*-sp.open)) in *X*. *A* is γ^*-pre-clopen [14], if *A* is both γ^*- pre-open and γ^*-pre-closed in *X*. For a subset *A* of *X*, γ^*-pre-interior [14] of *A* is $pint_{\gamma^*}(A) = \bigcup\{U : U \in PO_{\gamma^*}(X)$ and $U \subseteq A\}$ and γ^*-pre-closure [14] of *A* is $pcl_{\gamma^*}(A) = \bigcap\{F : X - F \in PO_{\gamma^*}(X)$ and $A \subseteq F\}$. For a subset *A* of *X*, γ^*-sp.interior [14] of *A* is $spint_{\gamma^*}(A) = \bigcup\{U : U \in SPO_{\gamma^*}(X)$ and $U \subseteq A\}$ and γ^*-sp.closure [14] of *A* is $spcl_{\gamma^*}(A) = \bigcap\{F : X - F \in SP\ O_{\gamma^*}(X)$ and $A \subseteq F\}$.

Throughout this paper, let X, Y and *Z* be three topological spaces and operations $\gamma : \tau \to P(X)$, $\beta : \sigma \to P(Y)$ and $\rho : \eta \to P(Z)$ on topologies τ, σ

and η, respectively. Here $PO_{\gamma^*}(X)$, $PO_{\beta^*}(Y)$ and $PO_{\rho^*}(Z)$ denote the family of γ^*- pre-open sets, β^*- pre-open sets and ρ^*- pre-open sets, respectively.

Definition 2.1. A subset *A* of a topological space *X* is said to be

(i) γ- generalized closed (briefly γ- g.closed) if $cl_\gamma(A) \subseteq U$ whenever $A \subseteq U$ and *U* is γ- open;

(ii) γ^*- regular-generalized closed (briefly γ^*- rg.closed) if $cl_\gamma(A) \subseteq U$ whenever $A \subseteq U$ and *U* is γ^*- regular-open;

(iii) γ^*- pre-generalized closed (briefly γ^*- pg.closed) if $pcl_{\gamma^*}(A) \subseteq U$ whenever $A \subseteq U$ and *U* is γ^*- pre-open;

(iv) γ^*- generalized-pre-closed (briefly γ^*- gp.closed) if $pcl_{\gamma^*}(A) \subseteq U$ whenever $A \subseteq U$ and *U* is γ- open;

(v) γ^*- semi-pre-generalized closed (briefly γ^*- spg.closed) if $spcl_{\gamma^*}(A) \subseteq U$ whenever $A \subseteq U$ and *U* is γ^*- sp.open;

(vi) γ^*- generalized-semi-pre-closed (briefly γ_g^*- sp.closed) if $spcl_{\gamma^*}(A) \subseteq U$ whenever $A \subseteq U$ and *U* is γ- open;

(vii) γ^*- generalized-pre-regular-closed (briefly γ_g^*- pr.closed) if $pcl_{\gamma^*}(A) \subseteq U$ whenever $A \subseteq U$ and *U* is γ^*- regular-open.

We denote the set of all γ- g.closed (resp., γ^*- regular-closed, γ^*- pre-closed, γ^*- sp.closed) γ^*- rg.closed, γ^*- pg.closed, γ^*- gp.closed, γ^*- spg.closed, γ_g^*- sp.closed and γ_g^*- pr.closed) sets by $GC_\gamma(X)$(resp., $RC_{\gamma^*}(X)$, $PC_{\gamma^*}(X)$, $SPC_{\gamma^*}(X)$, $RGC_{\gamma^*}(X)$, $PGC_{\gamma^*}(X)$, $GPC_{\gamma^*}(X)$, $SPGC_{\gamma^*}(X)$, $SPC_{\gamma_g^*}(X)$ and $PRC_{\gamma_g^*}(X)$).

Example 2.1. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{d\}, \{a, d\}, \{a, c, d\}\}$ and define operation $\gamma : \tau \to P(X)$ by

$$\gamma(A) = \begin{cases} A \cup \{b\} \text{ if } A = \{a\}, \\ A \cup \{c\} \text{ if } A = \{d\}, \{a, d\} & \text{for every } A \in \tau, \\ \quad \mathrm{A} \quad \text{if } A \neq \{a\}, \{d\}, \{a, d\}. \end{cases}$$

Then $RC_{\gamma^*}(X) = \{\emptyset, X, \{a, b, c\}, \{b, c, d\}\}$;

$GC_{\gamma}(X) = \{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$;

$RGC_{\gamma^*}(X) = \{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\},$

$\{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$;

$PC_{\gamma^*}(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}\}$;

$SPC_{\gamma^*}(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\},$

$\{a, b, c\}, \{b, c, d\}\}$;

$PGC_{\gamma^*}(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}\}$;

$GPC_{\gamma^*}(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\},$

$\{a, b, d\}, \{b, c, d\}\}$;

$SPGC_{\gamma^*}(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\},$

$\{a, b, c\}, \{b, c, d\}\}$;

$SPC_{\gamma_g^*}(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\},$

$\{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$;

$PRC_{\gamma_g^*}(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\},$

$\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$.

Theorem 2.1. *Let X be a topological space and* $\gamma : \tau \to P(X)$ *be an operation* *on* τ. *Then*

(i) *every* γ- *closed set is* γ- *g.closed*, γ^* - *rg.closed and* γ^* - *pre-closed*;

(ii) *every* γ^* - *regular-closed set is* γ^* - *pre-closed*;

(iii) *every* γ^* - *pre-closed set is* γ^* - *sp.closed and* γ^* - *pg.closed*;

(iv) *every* γ^* - *pg.closed set is* γ^* - *gp.closed*, γ^*_g - *sp.closed and* γ^*_g - *pr.closed*;

(v) *every* γ^* - *gp.closed set is* γ^*_g - *sp.closed*;

(vi) *every* γ^* - *sp.closed set is* γ^* - *spg.closed*;

(vii) *every* γ^* - *spg.closed set is* γ^*_g - *sp.closed*.

Proof. Proof follows from Definition 2.1, Theorem 2.2 [14] and Remark 2.1 [15].

The complement of γ- g.closed (resp., γ^* - rg.closed, γ^* - pg.closed, γ^* - gp.closed, γ^* - spg.closed, γ^*_g - sp.closed and γ^*_g - pr.closed) set is called γ- g.open (resp., γ^* - rg.open, γ^* - pg.open, γ^* - gp.open, γ^* - spg.open, γ^*_g - sp.open and γ^*_g - pr.open) set and defined in the following lemma.

Lemma 2.1. *A subset A of a topological space X is*

(i) γ- *g.open iff* $int_\gamma(A) \supseteq F$ *whenever* $(A) \supseteq F$ *and F is* γ- *closed*;

(ii) γ^* - *rg.open iff* $int_\gamma(A) \supseteq F$ *whenever* $(A) \supseteq F$ *and F is* γ^* - *regular-closed*;

(iii) γ^* - *pg.open iff* $pint_{\gamma^*}(A) \supseteq F$ *whenever* $(A) \supseteq F$ *and F is* γ^* - *pre-closed*;

(iv) γ^* - *gp.open iff* $pint_{\gamma^*}(A) \supseteq F$ *whenever* $(A) \supseteq F$ *and F is* γ- *closed*;

(v) γ^* - *spg.open iff* $spint_{\gamma^*}(A) \supseteq F$ *whenever* $(A) \supseteq F$ *and F is*

γ^*- *sp.closed*;

(vi) γ_g^*- *sp.open iff* $spint_{\gamma^*}(A) \supseteq F$ *whenever* $(A) \supseteq F$ *and F is* γ- *closed*;

(vii) γ_g^*- *pr.open iff* $pint_{\gamma^*}(A) \supseteq F$ *whenever* $(A) \supseteq F$ *and F is* γ^*- *regular-closed*.

We denote the set of all γ- g.open (resp., γ^*- rg.open, γ^*- pg.open, γ^*- gp.open, γ^*- spg.open, γ_g^*- sp.open and γ_g^*- pr.open) sets by $GO_\gamma(X)$ (resp., $RGO_{\gamma^*}(X)$, $PGO_{\gamma^*}(X)$, $GPO_{\gamma^*}(X)$, $SPGO_{\gamma^*}(X)$, $SPO_{\gamma_g^*}(X)$ and $PRO_{\gamma_g^*}(X)$).

Theorem 2.1. *For any topological space X and* $\gamma : \tau \to P(X)$ *is an operation on* τ, $A \subseteq X$, *the following hold*:

(i) *If* $A \in GC_\gamma(X) \cap \tau_\gamma$, *then* $A \in \tau_\gamma^c$;

(ii) *If* $A \in RGC_{\gamma^*}(X) \cap RO_{\gamma^*}(X)$, *then* $A \in \tau_\gamma^c$;

(iii) *If* $A \in PGC_{\gamma^*}(X) \cap PO_{\gamma^*}(X)$, *then* $A \in PC_{\gamma^*}(X)$;

(iv) *If* $A \in GPC_{\gamma^*}(X) \cap \tau_\gamma$, *then* $A \in PC_{\gamma^*}(X)$;

(v) *If* $A \in SPGC_{\gamma^*}(X) \cap SPO_{\gamma^*}(X)$, *then* $A \in SPC_{\gamma^*}(X)$;

(vi) *If* $A \in SPC_{\gamma_g^*}(X) \cap \tau_\gamma$, *then* $A \in SPC_{\gamma^*}(X)$;

(vii) *If* $A \in PRC_{\gamma_g^*}(X) \cap RO_{\gamma^*}(X)$, *then* $A \in PC_{\gamma^*}(X)$.

Proof. Proof is straightforward.

3. (γ_g^*, β_g^*)- pr.continuous Mappings

Definition 3.1 [15]**.** A mapping $f : X \to Y$ is called:

(i) (γ^*, β)- pre-continuous if $\forall B \in \sigma_\beta^c$, $f^{-1}(B) \in PC_{\gamma^*}(X)$;

(ii) (γ^*, β)- pg-continuous if $\forall B \in \sigma_\beta^c,\ f^{-1}(B) \in PGC_{\gamma^*}(X)$;

(iii) (γ^*, β)- gp-continuous if $\forall B \in \sigma_\beta^c,\ f^{-1}(B) \in GPC_{\gamma^*}(X)$;

(iv) (γ^*, β^*)- pre-continuous if $\forall B \in PC_{\beta^*}(Y),\ f^{-1}(B) \in PC_{\gamma^*}(X)$;

(v) (γ^*, β^*)- pg-continuous if $\forall B \in PGC_{\beta^*}(Y),\ f^{-1}(B) \in PGC_{\gamma^*}(X)$;

(vi) (γ^*, β^*)- gp-continuous if $\forall B \in GPC_{\beta^*}(Y),\ f^{-1}(B) \in GPC_{\gamma^*}(X)$.

Lemma 3.1 [15]. *Let* $f : X \to Y$ *be a mapping. Then*

(i) (γ^*, β)- *pre-continuous* $\forall B \in \sigma_\beta,\ f^{-1}(B) \in PO_{\gamma^*}(X)$;

(ii) (γ^*, β)- *pg.continuous* $\forall B \in \sigma_\beta,\ f^{-1}(B) \in PGO_{\gamma^*}(X)$;

(iii) (γ^*, β)- *gp.continuous* $\forall B \in \sigma_\beta,\ f^{-1}(B) \in GPO_{\gamma^*}(X)$;

(iv) (γ^*, β^*)- *pre-continuous* $\forall B \in PO_{\beta^*}(Y),\ f^{-1}(B) \in PO_{\gamma^*}(X)$;

(v) (γ^*, β^*)- *pg.continuous* $\forall B \in PGO_{\beta^*}(Y),\ f^{-1}(B) \in PGO_{\gamma^*}(X)$;

(vi) (γ^*, β^*)- *gp.continuous* $\forall B \in GPO_{\beta^*}(Y),\ f^{-1}(B) \in GPO_{\gamma^*}(X)$.

Proposition 3.1 [15]. *Let* $f : X \to Y$ *be a mapping. Then*

(i) *If f is* (γ^*, β)- *pre-continuous, then f is* (γ^*, β)- *pg.continuous*;

(ii) *If f is* (γ^*, β)- *pg.continuous, then f is* (γ^*, β)- *gp.continuous*;

(iii) *If f is* (γ^*, β^*)- *pre-continuous, then f is* (γ^*, β)- *pre-continuous*;

(iv) *If f is* (γ^*, β^*)- *pg.continuous, then f is* (γ^*, β)- *pg.continuous*;

(v) *If f is* (γ^*, β^*)- *gp.continuous, then f is* (γ^*, β)- *gp.continuous*.

Note that the converse of the above proposition need not be true.

Definition 3.2. A mapping $f : X \to Y$ is called:

(i) (γ^*, β)- rg.continuous if $\forall B \in \sigma_\beta^c$, $f^{-1}(B) \in RGC_{\gamma^*}(X)$;

(ii) (γ^*, β^*)- regular-continuous if $\forall B \in RC_{\beta^*}(Y)$, $f^{-1}(B) \in RC_{\gamma^*}(X)$;

(iii) (γ_g^*, β)- pr.continuous if $\forall B \in \sigma_\beta^c$, $f^{-1}(B) \in PRC_{\gamma_g^*}(X)$;

(iv) (γ_g^*, β_g^*)- pr.continuous if $\forall B \in PRC_{\beta_g^*}(Y)$, $f^{-1}(B) \in PRC_{\gamma_g^*}(X)$.

Example 3.1. Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{2\}, \{2, 3\}\}$ and define operations $\gamma : \tau \to P(X)$ and $\beta : \sigma \to P(Y)$ by $\gamma(A) = A$ if $a \in A$; $cl(A)$ if $a \notin A$ for every $A \in \tau$ and $\beta(A) = cl(A)$ if $A = \{2\}$; A if $A \neq \{2\}$ for every $A \in \sigma$, respectively. Define $f : X \to Y$ by $f(a) = 2$, $f(b) = 3$ and $f(c) = 1$. Then the inverse image of every β- closed set is γ^*- rg.closed under *f*. Hence *f* is (γ^*, β)- rg.continuous.

Example 3.2. Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}\}$ and define operations $\gamma : \tau \to P(X)$ and $\beta : \sigma \to P(Y)$ by $\gamma(A) = A$ if $a \in A$; $cl(A)$ if $a \notin A$ for every $A \in \tau$ and $\beta(A) = A \cup \{3\}$ if $A = \{1\}$; A if $A \neq \{1\}$ for every $A \in \sigma$, respectively. Define $f : X \to Y$ by $f(a) = 1$, $f(b) = 2$ and $f(c) = 3$. Then the inverse image of every β^*- regular-closed set is γ- regular-closed under *f*. Hence *f* is (γ^*, β^*)- regular-continuous.

Example 3.3. Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{\emptyset, Y, \{1\}, \{3\}, \{1, 3\}\}$ and define operations $\gamma : \tau \to P(X)$ and $\beta : \sigma \to P(Y)$ by $\gamma(A) = A$ if $A \neq \{b, c\}$; $cl(A)$ if $A = \{b, c\}$ for every $A \in \tau$ and $\beta(A) = A$ if $A = \{1, 3\}$; $\text{int}(cl(A))$ if $A \neq \{1, 3\}$ for every $A \in \sigma$, respectively. Define $f : X \to Y$ by $f(a) = 1$, $f(b) = 3$ and $f(c) = 2$. Then the inverse image of every β- closed set is γ_g^*- pr.closed under *f*. Hence *f* is (γ_g^*, β^*)- pr.continuous.

Example 3.4. Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$, $\tau = \{\emptyset, X, \{b\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}\}$ and define operations $\gamma : \tau \to P(X)$ and

$\beta : \sigma \to P(Y)$ by $\gamma(A) = A$ if $a \in A$; $cl(A)$ if $a \notin A$ for every $A \in \tau$ and $\beta(A) = A \cup \{1\}$ if $A = \{2\}$; $cl(A)$ if $A \neq \{2\}$ for every $A \in \sigma$, respectively. Define $f : X \to Y$ by $f(a) = 3$, $f(b) = 2$ and $f(c) = 1$. Then the inverse image of every β_g^*- pr.closed set is γ_g^*- pr.closed under *f*. Hence *f* is (γ_g^*, β_g^*)- pr.continuous.

Remark 3.1. (i) From Examples 3.1, 3.2 and Definition 3.2 the concepts of (γ^*, β)- rg.continuous and (γ^*, β^*)- regular-continuous are independent.

(ii) From Examples 3.2, 3.4 and Definition 3.2 the concepts of (γ^*, β^*)- regular-continuous and (γ_g^*, β_g^*)- pr.continuous are independent.

Remark 3.2. Every (γ^*, β)- pg.continuous mapping is (γ_g^*, β)- pr.continuous. But converse need not be true.

Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$, $\tau = \{\emptyset, X, \{b\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}\}$ and define operations $\gamma : \tau \to P(X)$ and $\beta : \sigma \to P(Y)$ by $\gamma(A) = A$ if $A = \{b\}$; $cl(A)$ if $A \neq \{b\}$ for every $A \in \tau$ and $\beta(A) = cl(A)$ if $A = \{1\}, \{2\}$; A if $A \neq \{1\}, \{2\}$ for every $A \in \sigma$, respectively. Define $f : X \to Y$ by $f(a) = 2$, $f(b) = 1$ and $f(c) = 3$. Then f is (γ_g^*, β^*)- pr.continuous. $f^{-1}(\{1, 3\}) = \{b, c\}$ is not γ^*- pg.closed in *X* for the β- closed set $\{1, 3\}$ of Y. So *f* is not (γ^*, β)- pg.continuous.

Remark 3.3. Every (γ_g^*, β_g^*)- pr.continuous mapping is (γ_g^*, β)- pr.continuous. But converse need not be true.

Let $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{1\}, \{1, 2\}\}$ and define operations $\gamma : \tau \to P(X)$ and $\beta : \sigma \to P(Y)$ by $\gamma(A) = A$ if $A = \{a\}, \{b\}$; $A \cup \{c\}$ if $A \neq \{a\}, \{b\}$ for every $A \in \tau$ and $\beta(A) = A$ if $A = \{1, 2\}$; $cl(A)$ if $A \neq \{1, 2\}$ for every $A \in \sigma$, respectively. Define $f : X \to Y$ by $f(a) = 2$, $f(b) = 1$ and $f(c) = 3$. Then *f* is (γ_g^*, β)- pr.continuous. $f^{-1}(\{1\}) = \{b\}$ is not γ_g^*- pr.closed in *X* for the β_g^*- pr.closed $\{1\}$ of *Y*. So *f* is not (γ_g^*, β_g^*)- pr.continuous.

Theorem 3.1. *Let X be a topological space and* $\gamma : \tau \to P(X)$ *be an open operation on* τ.

(i) *If A is a* γ^**-regular-closed set in X, then A is* γ*-closed*;

(ii) *If A is a* γ^**-rg.closed set in X, then A is* γ_g^**-pr.closed*;

(iii) *If A is a* γ^**-gp.closed set in X, then A is* γ_g^**-pr.closed.*

Proof. (i) Let A be γ^*-regular-closed in X. Then $A = cl_\gamma(\mathrm{int}_\gamma(A))$. By Theorem 3.6 (iii) [10], we have that $cl_\gamma(A) = cl_\gamma(cl_\gamma(\mathrm{int}_\gamma(A))) = cl_\gamma(\mathrm{int}_\gamma(A))$ $= A$. Therefore A is γ-closed.

(ii) Let A be γ^*-rg.closed in X and $A \subseteq U$ where U is γ^*-regular-open. Then $cl_\gamma(A) \subseteq U$. Then by Theorem 3.6 (iii) [10], we have that $cl_\gamma(A)$ is γ-closed. Since every γ-closed set is γ^*-pre-closed, $cl_\gamma(A)$ is γ^*-pre-closed. This implies that $pcl_{\gamma^*}(A) \subseteq cl_\gamma(A)$ and hence $pcl_{\gamma^*}(A) \subseteq U$. Hence A is γ_g^*-pr.closed.

(iii) Let A be γ^*-gp.closed in X and $A \subseteq U$, where U be γ^*-regular-open. Then by (i), U is γ-open. Since A is γ^*-gp.closed, $pcl_{\gamma^*}(A) \subseteq U$. Hence A is γ_g^*-pr.closed.

Theorem 3.2. *Let X be a topological space and* $\gamma : \tau \to P(X)$ *be an open operation on* τ. *Then*

(i) *If f is* (γ^*, β)*-rg.continuous, then f is* (γ_g^*, β)*-pr.continuous*;

(ii) *If f is* (γ^*, β)*-gp.continuous, then f is* (γ_g^*, β)*-pr.continuous.*

Proof. (i) Let B be β-closed in Y. Then by hypothesis, $f^{-1}(B)$ is γ^*-rg.closed in X. Since γ is open and by Theorem 3.1 (ii), $f^{-1}(B)$ is γ_g^*-pr.closed in X. Hence f is (γ_g^*, β)-pr.continuous.

(ii) Let B be β- closed in Y. Then by hypothesis, $f^{-1}(B)$ is γ^*- gp.closed in X. Since γ is open and by Theorem 3.1 (iii), $f^{-1}(B)$ is γ_g^*- pr.closed in X. Hence f is (γ_g^*, β)- pr.continuous.

Theorem 3.3. *Let* $f : X \to Y$ *be a mapping. Then*

(i) (γ_g^*, β)- *pr.continuous* $\forall B \in \sigma_\beta$, $f^{-1}(B) \in PRO_{\gamma_g^*}(X)$;

(ii) (γ_g^*, β_g^*)- *pr.continuous* $\forall B \in PRO_{\beta_g^*}(Y)$, $f^{-1}(B) \in PRO_{\gamma_g^*}(X)$.

Proof. (i) Let f be (γ_g^*, β)- pr.continuous and let $B = Y - V$ be β- open in Y. This implies that V is β- closed in Y. Since f is (γ_g^*, β)- pr.continuous, $f^{-1}(V)$ is γ_g^*- pr.closed in X. Hence $f^{-1}(B) = X - f^{-1}(Y - B) = X - f^{-1}(V)$ is γ_g^*- pr.open in X. Conversely, let F be β- closed in Y. Then $B = Y - F$ is β- open in Y. Then by hypothesis, $f^{-1}(B)$ is γ_g^*- pr.open in X. Hence $f^{-1}(F) = X - f^{-1}(Y - F) = X - f^{-1}(B)$ is γ_g^*- pr.closed in X. Therefore, we obtain that f is (γ_g^*, β_g^*)- pr.continuous.

(ii) Let f be (γ_g^*, β_g^*)- pr.continuous and let $B = Y - V$ be β_g^*- pr.open in Y. This implies that V is β_g^*- pr.closed in Y. Since f is (γ_g^*, β_g^*)- pr.continuous, $f^{-1}(V)$ is γ_g^*- pr.closed in X. Hence $f^{-1}(B) = X - f^{-1}(Y - B) = X - f^{-1}(V)$ is γ_g^*- pr.open in X. Conversely, let F be β_g^*- pr.closed in Y. Then $B = Y - F$ is β_g^*- pr.open in Y. Then by hypothesis, $f^{-1}(B)$ is γ_g^*- pr.open in X. Hence $f^{-1}(F) = X - f^{-1}(Y - F) = X - f^{-1}(B)$ is γ_g^*- pr.closed in X. Therefore, we obtain that f is (γ_g^*, β_g^*)- pr.continuous.

Definition 3.3 [15]**.** *A* mapping $f : X \to Y$ is called:

(i) (γ^*, β^*)- regular-closed if $\forall F \in RC_{\gamma^*}(X)$, $f(B) \in RC_{\beta^*}(Y)$;

(ii) (γ^*, β^*)- regular-open if $\forall F \in RO_{\gamma^*}(X)$, $f(B) \in RO_{\beta^*}(Y)$;

(iii) (γ^*, β^*)- pre-closed if $\forall F \in PC_{\gamma^*}(X)$, $f(B) \in PC_{\beta^*}(Y)$;

(iv) (γ^*, β^*)- pre-open if $\forall F \in PO_{\gamma^*}(X)$, $f(B) \in PO_{\beta^*}(Y)$;

(v) (γ^*, β^*)- gp.closed if $\forall F \in GPC_{\gamma^*}(X)$, $f(B) \in GPC_{\beta^*}(Y)$;

(vi) (γ^*, β^*)- gp.open if $\forall F \in GPO_{\gamma^*}(X)$, $f(B) \in GPO_{\beta^*}(Y)$.

Theorem 3.4. *Let* $f : X \to Y$ *be a* (γ^*, β^*)-*regular-continuous and* (γ^*, β^*)-*pre-closed mapping. Then for every* γ_g^*-*pr.closed set A of* X, $f(A)$ *is* β_g^*-*pr.closed in Y.*

Proof. Let A be γ_g^*- pr.closed in *X*. Let $f(A) \subseteq U$, where U be β^*- regular-open in *Y*. Then $A \subseteq f^{-1}(U)$. Since f is (γ^*, β^*)- regular-continuous and A is γ_g^*- pr.closed, implies that $f^{-1}(U)$ is γ^*- regular-open in X and $pcl_{\gamma^*}(A) \subseteq f^{-1}(U)$. That is, $f(pcl_{\gamma^*}(A)) \subseteq U$. Now $pcl_{\beta^*}(f(A)) \subseteq pcl_{\beta^*}(f(pcl_{\gamma^*}(A))) = f(pcl_{\gamma^*}(A)) \subseteq U$, since *f* is (γ^*, β^*)- pre-closed. Thus $f(A)$ is β_g^*- pr.closed in *Y*.

Theorem 3.5. *Let* $f : X \to Y$ *be a bijective*, (γ^*, β^*)-*pre-continuous and* (γ^*, β^*)-*regular-open mapping. Then for every* β_g^*-*pr.closed set B of Y*, $f^{-1}(B)$ *is* γ_g^*-*pr.closed in X.*

Proof. Let U be a γ^*- regular-open set such that $f^{-1}(B) \subseteq U$. Then $B \subseteq f(U)$. Since f is (γ^*, β^*)- regular-open, $f(U)$ is a β^*- regular-open set containing B. Since B is β_g^*- pr.closed, hence $pcl_{\beta^*}(B) \subseteq f(U)$ and so $f^{-1}(pcl_{\beta^*}(B)) \subseteq U$. Since $f^{-1}(pcl_{\beta^*}(B))$ is a γ^*- pre-closed set containing $f^{-1}(B)$, implies that $pcl_{\gamma^*}(f^{-1}(B)) \subseteq f^{-1}(pcl_{\beta^*}(B)) \subseteq U$. Hence $f^{-1}(B)$ is

γ_g^* - pr.closed in X.

Theorem 3.6. *For any surjection mapping* $f : X \to Y$, *the following statements are equivalent.*

(i) *For any* γ^* *- pre-open G of* X, $f(G)$ *is* β^* *- gp.open in Y*;

(ii) *For any* $B \subseteq Y, U \in PO_{\gamma^*}(X)$ *such that* $f^{-1}(B) \subseteq U$, *there exists* $V \in GPO_{\beta^*}(Y)$ *such that* $B \subseteq V$ *and* $f^{-1}(V) \subseteq U$.

Proof. Let $B \subseteq Y, U \in PO_{\gamma^*}(X)$ such that $f^{-1}(B) \subseteq U$. Then by hypothesis, we have that $f(U) \in GPO_{\beta^*}(Y)$. Put $V = f(U)$. Since $f^{-1}(B) \subseteq U$, $B = f(f^{-1}(B)) \subseteq f(U) = V$ and $f^{-1}(V) = f^{-1}(f(U)) \subseteq U$. Conversely, let $G \in PO_{\gamma^*}(X)$, $f(G) \supseteq F$ such that $F \in \sigma_\beta^c$, then $G \supseteq f^{-1}\ (F)$, $F \subseteq Y$. This implies that there exist $V \in GPO_{\beta^*}(Y)$ such that $F \subseteq V$ and $f^{-1}(V) \subseteq G$. Since $V \in GPO_{\beta^*}(Y)$, $F \in \sigma_\beta^c$ and $F \subseteq V$. Consequently, $pint_{\beta^*}(V) \supseteq F$. Since $V \subseteq f(G)$, $F \subseteq pint_{\beta^*}(V) \subseteq pint_{\beta^*}(f(G))$. This implies that $f(G) \in GPO_{\beta^*}(Y)$.

Theorem 3.7. *For any surjection mapping* $f : X \to Y$ *the following statements are equivalent.*

(*i*) *For any* γ^* *- pre-open G of* X, $f(G)$ *is* β_g^* *- pr.open in Y*;

(*ii*) *For any* $B \subseteq Y, U \in PO_{\gamma^*}(X)$ *such that* $f^{-1}(B) \subseteq U$, *there exists* $V \in PRO_{\beta_g^*}(Y)$ *such that* $B \subseteq V$ *and* $f^{-1}(V) \subseteq U$.

Proof. Proof is similar to Theorem 3.6.

Note that if $f : X \to Y$ is (γ_g^*, β) pr.continuous and $g : Y \to Z$ is (β_g^*, ρ)- pr.continuous, then the composition $gof : X \to Z$ is not (γ_g^*, ρ)-pr.continuous mapping.

Example 3.5. Let $X = Y = Z = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, d\}\}$ and $\eta = \{\emptyset, Z, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}\}$ and define operations $\gamma : \tau \to P(X)$, $\beta : \sigma \to P(Y)$ and $\rho : \eta \to P(Z)$ by $\gamma(A) = A$ if $A = \{a\}, \{a, b\}$; $cl(A)$ if $A \neq \{a\}, \{a, b\}$ for every $A \in \tau$ and $\beta(A) = A$ if $A = \{a\}, \{b\}$; $A \cup \{c\}$ if $A = \{a, b\}$; $cl(A)$ if $A \neq \{a\}, \{b\}, \{a, b\}$ for every $A \in \sigma$ and $\rho(A) = A \cup \{b\}$ if $A = \{a\}, \{a, c, d\}$; A if $A \neq \{a\}, \{a, c, d\}$ for every $A \in \eta$, respectively. Define $f : X \to Y$ by $f(a) = d$, $f(b) = c$, $f(c) = b$ and $f(d) = a$ and define $g : Y \to Z$ by $g(a) = b$, $g(b) = c$, $g(c) = a$ and $g(d) = d$. Then f and g are (γ_g^*, β)- pr.continuous and (β_g^*, ρ)- pr.continuous, respectively. $\{d\}$ is ρ- closed in Z. $(gof)^{-1}(\{d\}) = f^{-1}(g^{-1}(\{d\})) = f^{-1}(\{d\}) = \{a\}$ which is not γ_g^*- pr.closed in X. Hence gof is not (γ_g^*, ρ)- pr.continuous.

Theorem 3.8. *Let* $f : X \to Y$ *and* $g : Y \to Z$ *be two mappings*. Then

(i) *If* f *is* (γ_g^*, β)- *pr.continuous and* g *is* (β, ρ)- *continuous*, *then* gof *is* (γ_g^*, ρ)- *pr.continuous*;

(ii) *If* f *is* (γ_g^*, β_g^*)- *pr.continuous and* g *is* (β_g^*, ρ_g^*)- *pr.continuous*, *then* gof *is* (γ_g^*, ρ_g^*)- *pr.continuous*;

(iii) *If* f *is* (γ_g^*, β_g^*)- *pr.continuous and* g *is* (β_g^*, ρ)- *pr.continuous*, *then* gof *is* (γ_g^*, ρ)- *pr.continuous.*

Proof. (i) Let V be ρ- closed in Z. Then $g^{-1}(V)$ is β- closed in Y since g is (β, ρ)- continuous. (γ_g^*, β)- pr.continuity of f implies that $f^{-1}(g^{-1}(V))$ is γ_g^*- pr.closed in X. That is, $(gof)^{-1}(V)$ is γ_g^*- pr.closed in X. Hence gof is (γ_g^*, ρ)- pr.continuous.

(ii) Let V be ρ_g^*- pr.closed in Z. Since g is (β_g^*, ρ_g^*)- pr.continuous, $g^{-1}(V)$ is

β_g^* - pr.closed in *Y*. As *f* is (γ_g^*, β_g^*)- pr.continuous $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is γ_g^* - pr.closed in *X*. Therefore *gof* is (γ_g^*, ρ_g^*)- pr.continuous.

(iii) Let *V* be ρ- closed in *Z*. Since *g* is (β_g^*, ρ)- pr.continuous, $g^{-1}(V)$ is β_g^* - pr.closed in *Y*. As *f* is (γ_g^*, β_g^*)- pr.continuous $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is γ_g^* - pr.closed in *X*. Therefore *gof* is (γ_g^*, ρ)- pr.continuous.

Theorem 3.9. *Let X be a topological space and* $\gamma : \tau \to P(X)$ *be an operation on* τ. *Then the following properties are equivalent*:

(i) *For any pair of disjoint* γ^* *- regular-closed sets* A, B *of X, there exist disjoint* γ^* *- pre-open sets* U, V *such that* $A \subseteq U$ *and* $B \subseteq V$;

(ii) *For any pair of disjoint* γ^* *- regular-closed sets* A, B *of X, there exist disjoint* γ_g^* *- pr.open sets* U, V *such that* $A \subseteq U$ *and* $B \subseteq V$;

(iii) *For any* γ^* *- regular-closed set A and any* γ_g^* *- regular-open set V containing A, there exists a* γ_g^* *- pr.open set U such that* $A \subseteq U \subseteq pcl_{\gamma^*}(U) \subseteq V$.

Proof. (i) Let A and B be γ^* - regular-closed and $A \cap B = \emptyset$. Then by hypothesis, there exist disjoint γ^* - pre-open sets U, V such that $A \subseteq U$ and $B \subseteq V$, this follows that disjoint γ_g^* - pr.open sets U, V such that $A \subseteq U$ and $B \subseteq V$.

(ii) $\Rightarrow$ (iii). Let *A* be any γ^* - regular-closed set and *V* a γ^* - regular-open set containing A. Since A and $X - V$ are disjoint γ^* - regular-closed sets of *X*, there exist γ_g^* - pr.open sets U, W of *X* such that $A \subseteq U$, $X - V \subseteq W$ and $U \cap W = \emptyset$. Therefore by Definition 2.1, we have that $X - V \subseteq pint_{\gamma^*}(W)$. Since $U \cap pint_{\gamma^*}(W) = \emptyset$, we have that $pcl_{\gamma^*}(U) \cap pint_{\gamma^*}(W) = \emptyset$ and hence $pcl_{\gamma^*}(U) = X - pint_{\gamma^*}(W) \subseteq V$. Therefore $A \subseteq U \subseteq pcl_{\gamma^*}(U) \subseteq V$.

(iii) $\Rightarrow$ (i). Let A and B be any disjoint γ^*-regular-closed sets of X. Since $X - B$ is a γ^*-regular-open set containing A, there exists a γ_g^*-pr.open set G such that $A \subseteq G \subseteq pcl_{\gamma^*}(U) \subseteq X - B$. By Definition 2.1, we have that $A \subseteq pint_{\gamma^*}(G)$. Put $U = pint_{\gamma^*}(G)$ and $V = X - pcl_{\gamma^*}(G)$. This implies that U and V are disjoint γ^*-pre-open sets such that $A \subseteq U$ and $B \subseteq V$.

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