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ON TWO REMARKABLE TRIPLETS

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Abstract

A search is made on finding three non zero distinct integer triplet a, b, c such that

$$a + 2b = \alpha^3, \quad a + 2c = \beta^3, \quad b + c = \gamma^3, \tag{1}$$

$$a + 2b = \alpha^3, \quad a + 2c = \beta^3, \quad b - c = \gamma^3. \tag{2}$$

It is seen that there exist infinitely many such triplets.

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1. Introduction

Number theory is that branch of Mathematics which deals with properties of the natural numbers $1, 2, 3, \ldots$ also called the positive numbers. These numbers together with the negative numbers and zero form the set of integers. Properties of these integers have been studied since antiquity. Number theory is an out enjoyable and pleasing to everybody. It has fascinated and inspired both armatures and mathematicians alike. Diophantine problems have fewer equations than unknown variables and involve finding integers that work correctly for all equations. Certain Diophantine problems come from physical problems or from immediate mathematical generalizations and others come from geometry in a variety of ways. Certain Diophantine problems are neither trivial nor difficult to analyze [1-9].

In this context, one may refer [10-13]. This paper consists of two sections, A and B. In section A, we search for a special non zero integer triplet (a, b, c) such that $a + 2b = \alpha^3$, $a + 2c = \beta^3$, $b + c = \gamma^3$. In section B, we search for a non zero integer triplet (a, b, c) such that $a + 2b = \alpha^3$, $a + 2c = \beta^3$, $b - c = \gamma^3$.

2. Method of Analysis

Section A

Let a, b, c be three non zero distinct integers such that

$$a + 2b = \alpha^3, \tag{1}$$

$$a + 2c = \beta^3, \tag{2}$$

$$b + c = \gamma. \tag{3}$$

Subtracting (1) from (2), we get

$$2(b - c) = \alpha^3 - \beta^2. \tag{4}$$

Solving for b and c from (3) and (4), we have

$$b = \frac{1}{4}(\alpha^3 - \beta^3 + 2\gamma^3), \tag{5}$$

$$c = \frac{1}{4}(2\gamma^3 - \alpha^3 + \beta^3). \tag{6}$$

From (1), we get

$$a = \frac{1}{2}(\alpha^3 + \beta^3 - 2\gamma^3). \tag{7}$$

It is noted that a, b, c are integers when

(i) $\alpha = 2p, \quad \beta = 2q, \quad \gamma = 2r,$

(ii) $\alpha = 5\beta, \quad \gamma = 2r,$

(iii) $\alpha = 2k + 3, \quad \beta = 2k - 1, \quad \gamma = 2r.$

Substituting the above values of α, β, γ intern in (5)-(7), we obtain three different triples satisfying (1) to (3) and they are exhibited below.

Case (i)

Taking $\alpha = 2p, \quad \beta = 2q, \quad \gamma = 2r.$

The corresponding non zero distinct integers a, b, c are given by

$$a = 4p^3 + 4q^3 - 8r^3,$$

$$b = 2p^3 - 2q^3 + 4r^3,$$

$$c = 4r^3 - 2p^3 + 2q^3.$$

Case (ii)

Let $\alpha = 5\beta, \quad \gamma = 2r.$

The corresponding non zero distinct integers a, b, c are represented by

$$a = 63\beta^3 - 8r^3,$$

$$b = 31\beta^3 + 4r^3,$$

$$c = 4r^3 - 31\beta^3.$$

Case (iii)

Put $\alpha = 2k + 3, \quad \beta = 2k - 1, \quad \gamma = 2r.$

The corresponding non zero distinct integers a, b, c are found to be

$$a = 8k^3 + 12k^2 + 30k + 13 - 8r^3,$$

$$b = 12k^2 + 12k + 7 + 4r^3,$$

$$c = 4r^3 - 12k^2 - 12k - 7.$$

Section B

Let a, b, c be three non zero distinct integers such that

$$a + 2b = \alpha^3, \tag{1}$$

$$a + 2c = \beta^3, \tag{2}$$

$$b - c = \gamma^3. \tag{3}$$

Subtracting (1) from (2), we get

$$2(b - c) = \alpha^3 - \beta^3. \tag{4}$$

Using (3), we have

$$\alpha^3 - \beta^3 = 2\gamma^3$$

which is equivalent to system of double equations in three different ways as follows:

	System 1	System 2	System 3
$\alpha + \beta$	$2\gamma^2$	γ^3	γ^2
$\alpha - \beta$	γ	2	2γ

We solve inturn the above three systems of double equations for α, β, γ.

The corresponding values of a, b, c satisfying (1)-(3) are exhibited in Table below:

	a	b	c
System 1	$64k^6 + 48k^5 + 12k^4 - 15k^3 - 2c$	$c + 8k^3$	$2k$
System 2	$64k^9 + 48k^6 - 4k^3 + 1 - 2c$	$c + 8k^3$	$2k$
System 3	$8k^6 + 24k^5 + 24k^4 - 8k^3 - 2c$	$c + 8k^3$	$2k$

3. Conclusion

As Diophantine problems are rich in variety, one may attempt to construct triples whose elements satisfy other characterization.

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