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ON THE SIMILARITIES BETWEEN CLASSICAL SELF-INTERACTION DYNAMICS AND SCHRODINGER'S EQUATION

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Abstract

This paper presents the dynamics of a classical self-interacting particle under Hamilton-Jacobi formalism. The resulting Hamilton-Jacobi equations are compared to Schrodinger's equation under Madelung's transform representation. The comparison shows that the classical self-interacting particle dynamics are analogous to Schrodinger's dynamics with the difference being that the interaction potential of the former is

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more general while the latter has a very specific interaction potential. This study gives voice to a simplistic idea of the possibility of fundamental particles being self-interacting in nature while also obeying both Newtonian and Schrodinger's wave mechanics.

1. Introduction

It is well-known that a dissipative system can be modeled as two identical systems interacting from different conceptual worlds, with one system existing in the dual space of the other [1, 2]. This scenario can also be represented in the form of a complex Lagrangian [3] hence resulting in a notion of a complex action upon which the principle of stationary action can be applied to arrive at the system's governing dynamics. In this way, one can think of a dissipative system as a system interacting with its identical copy from a dual space thus making this a two-world interaction problem or a self-interacting system. In this paper, we argue that there is a sense in which quantum theory can be seen as a theory of classical self-interacting particles. The word "self-interaction" is not entirely unheard of in theoretical Physics, with one example being the self-interacting dark matter [4]. In [5], the same word is associated with the possibility that Schrodinger's zitterbewegung is key to understanding Dirac's theory of electrons. To some extent, one can also think of the pilot-wave dynamics as the dynamics describing a particle that interacts with its dual or mirror image through a guiding field.

The rest of this paper is organized as follows. Section 2 presents the self-interacting particle dynamics in the Hamilton-Jacobi formalism. In Section 3, we use the Madelung transformation to show that Schrodinger's equation results in particle dynamics which are similar in form to the Hamilton-Jacobi dynamics for a self-interacting particle. Section 4 concludes this paper with some remarks on this work.

2. Classical Self-interaction Dynamics

In this model, we consider a self-interacting particle as the one which is coupled to its image (or copy) through the environment such that looking at the particle without considering its image, it would appear as if the particle is a dissipative system. There are several ways of thinking about the self-interacting particle, one being a particle-field interaction in which a particle causes a disturbance in the external field and this disturbance influences how the particle behaves. This bears some resemblance to the pilot-wave dynamics. Another way could be a particle-(anti) particle interaction in which a particle seems to be interacting with its (charge-conjugated) mirror image. Both scenarios can be generalized to just a particle-environment interaction which can be modeled as a particle interacting with its dual or mirror image.

A. Complex action

We adapt the formulation outlined in [6] for two interacting particles and set the two-particle masses to be identical to emphasize the identity of the particle with its image or dual partner. The Lagrangians associated with the particle and its mirror image will then be,

$$L_1 = \frac{\boldsymbol{p}_1^2}{2m} - V_{g1} - V_{c1}, \tag{1}$$

$$L_2 = \frac{\boldsymbol{p}_2^2}{2m} - V_{g2} - V_{c2}, \tag{2}$$

where the potentials, V_{gk} and V_{ck} are the external guiding potential and the particle-particle coupling potential, respectively. The parameters $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ are the momenta for the particle and its image, respectively. Lastly, the mass m is the same for the particle and its image. The coupling potentials V_{c1} and V_{c2} are necessarily related to one another as described in [6]. The guiding potentials V_{g1} and V_{g2} are not necessarily related to one another since the particle and its image exist in different spaces (one being the dual of the other). The resulting complex action,

$$S = \int_{t_1}^{t_2} L dt = \int_{t_1}^{t_2} (L_1 + iL_2) dt = S_1 + iS_2 \tag{3}$$

with ∇S as the complex momentum $\boldsymbol{p} = \boldsymbol{p}_1 + i\boldsymbol{p}_2 = \nabla S_1 + i\nabla S_2$ whereby the momentum $\boldsymbol{p}_n = md\boldsymbol{r}_n / dt$ is for particle n of mass m and displacement $\boldsymbol{r}_n$.

B. Coupled Hamilton-Jacobi dynamics

It follows from equation (3) that the Hamilton-Jacobi equation will be as follows,

$$\frac{\partial S_1}{\partial t} + i\frac{\partial S_2}{\partial t} + \frac{1}{2}\left(\frac{\nabla S_1}{m} + i\frac{\nabla S_2}{m}\right) \cdot (\nabla S_1 + i\nabla S_2)$$

$$+(V_{g1} + iV_{g2}) + (V_{c1} + iV_{c2}) = 0. \tag{4}$$

Equation (4) above can be decomposed into two coupled Hamilton-Jacobi equations as shown below,

$$\frac{\partial S_1}{\partial t} + \frac{\nabla S_1 \cdot \nabla S_1}{2m} - \frac{\nabla S_2 \cdot \nabla S_2}{2m} + V_{g1} + V_{c1} = 0, \tag{5}$$

$$\frac{\partial S_2}{\partial t} + \frac{\nabla S_1 \cdot \nabla S_2}{m} + V_{g2} + V_{c2} = 0. \tag{6}$$

The two equations (5) and (6) describe the dynamics of a classical self-interacting particle. One of the symmetries pointed out in [6] shows that swapping the phase components in the action amounts to swapping the particle with its dual partner. The coupled Hamilton-Jacobi dynamics above are invariant to this particle exchange transformation thus one cannot distinguish the particle from its dual partner based on these dynamics alone. In the next section, we attempt to show some similarities between the coupled Hamilton-Jacobi equations above and Schrodinger's equation.

3. Similarity to Schrodinger's Equation

A. The phase equation

The Schrodinger's time-dependent equation for a particle of mass m under the influence of some guiding potential V_g is presented as shown below [7, 8],

$$i\hbar \frac{\partial \psi(\boldsymbol{r},\, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(\boldsymbol{r},\, t) + V_g \psi(\boldsymbol{r},\, t). \tag{7}$$

To rewrite Schrodinger's time-dependent equation in terms of the phase of the wave, we adapt Madelung's transformation [9, 8, 7] as shown below,

$$\psi(\boldsymbol{r},\, t) = e^{iS(\boldsymbol{r},\, t)/\hbar} = e^{iS_1(\boldsymbol{r},\, t)/\hbar} e^{-S_2(\boldsymbol{r},\, t)/\hbar}. \tag{8}$$

Substituting equation (8) in Schrodinger's equation (7) and rearranging terms we obtain,

$$\frac{\partial S_1}{\partial t} + \frac{\nabla S_1 \cdot \nabla S_1}{2m} - \frac{\nabla S_2 \cdot \nabla S_2}{2m} + V_g + \frac{\hbar \nabla^2 S_2}{2m} = 0, \tag{9}$$

$$\frac{\partial S_2}{\partial t} + \frac{\nabla S_1 \cdot \nabla S_2}{m} - \frac{\hbar \nabla^2 S_1}{2m} = 0. \tag{10}$$

This set of equations (9) and (10) has the same general form as the Hamilton-Jacobi equations derived in Section 2 for two coupled particles. Next, we look at the similarities between the equation sets.

B. Similarities to coupled Hamilton-Jacobi equations

Looking at the two sets of equations above, it is clear that one can recover the Schrodinger equation set (9) and (10) by making the following settings on the coupled Hamilton-Jacobi equation set (5) and (6),

$$V_{g2} = 0, \tag{11}$$

$$V_{c1} = \frac{\hbar \nabla^2 S_2}{2m}, \tag{12}$$

$$V_{c2} = -\frac{\hbar \nabla^2 S_1}{2m}, \tag{13}$$

which simply translates to saying the dual partner of the particle is freely moving in the dual space since the external guiding potential is zero. This indicates that the classical self-interaction dynamics give rise to the more general dynamics of which Schrodinger's wave dynamics seem to be the special case. In [6], it is shown that, for some selections of coupling potentials (specifically those in equations (9) and (10)), there exists a conserved real quantity $P = e^{-2S_2/\hbar}$ associated with the dual partner. As mentioned in [6] that according to the principle of stationary action, the allowed trajectories are those which are, at some time t, all normal to the wavefront corresponding to a stationary or constant phase term [8, 10]. In the absence of strong self-interaction, a particle would follow one particular trajectory, however, if some self-interaction is brought into the picture the particle can deviate from one allowed trajectory to another. We believe that the conserved quantity P can be thought of as a function (from the dual space) assigning weights to each of the allowed trajectories of the particle and as such, it takes the form of the probability density function associated with the allowed trajectories.

C. Speculations on double-path experiments

Double-path experiments such as double-slit experiments have been discussed quite extensively concerning classical waves and quantum particles in the literature. The question of how a particle passes through the double-slit to form an interference pattern on the screen is a matter of great debate in the interpretations of quantum mechanics. The Everettian interpretation [11, 12] would insist that the particle went through both slits but in different parallel worlds. The de Broglie-Bohm theory [7, 13] would say the particle went through one slit while the guiding wave went through both slits to form an interference pattern on the other side. There are other interpretations by the collapse theories [14] and quantum Bayesianism [15, 16]. Although not yet fully developed, the classical self-interaction approach would probably say the particle went through slit one while its dual partner went through slit two in the dual space, thus the two would still interact after passing through the two slits, thus leading to a possibility of an interference pattern.

4. Conclusion

In this paper, we presented the notion of a classical self-interacting particle and its associated coupled Hamilton-Jacobi dynamics. This notion of classical self-interaction gave rise to the single-particle dynamics with some resemblance to Schrodinger's one-particle wave dynamics. It seems Schrodinger's dynamics form a special case of the classical self-interaction dynamics and one wonders if there is a more interesting Physics of classical self-interaction dynamics beyond the limiting case of Schrodinger's dynamics. There is a need to carry out the analysis of the classical self-interaction dynamics concerning the double-path experiments.

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