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ON A UNIFIED FIELD EQUATION IN GENERAL RELATIVITY

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Abstract

An equation linking the geometry of spacetime to the kinematics of the motion of matter opens a new path towards the unification of General Relativity and Quantum Mechanics.

1. Introduction

Einstein's field equation appeared in its complete form in a paper published on November 25, 1915. Since then, numerous experimental verifications have demonstrated its accuracy in the study of gravitation. Its major strength lies in the conservation of energy of matter, of which it is a mathematical consequence (the conservation of energy is no longer a principle). However, the inadequate form of the energy-momentum tensor, commonly established from a classical description of matter in terms of mass and pressure, is the reason why electrodynamics and quantum mechanics are inaccessible within the framework of the theory

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of general relativity. This is what we attempt to demonstrate in this article.

2. Total Field Equation

In the following study, we refer to a local frame of reference in Minkowski space, with coordinates denoted from 0 to 3 and signature $(1, -1, -1, -1)$.

We come from the belief that Einstein's equation must account for the quantum aspects of matter and all the forces acting on it.

Thus, we have 10 equations with 6 unknowns across 10 components of the metric g_{ij}, due to the free choice of the spatiotemporal frame of reference. To ensure the compatibility of the equations, it is necessary to associate with the metric a four-component entity, namely a four-vector, which we denote μ_i.

Let us a priori assume the simplest field equation: $R_{ij} = \mu_i\mu_j$ where R_{ij} denotes the Ricci tensor. Forming the Einstein tensor, we obtain:

$$R_{ij} - \frac{1}{2} R\delta_{ij} = \chi T_{ij} = \mu_i\mu_j - \frac{1}{2}\mu^2\delta_{ij}, \tag{1}$$

where χ denotes Einstein's gravitational constant and $\mu^2 = \mu_k\mu^k$.

The tensor T_{ij} represents the energy-momentum tensor in the form of a density. It is conservative in reason of no-divergence of the Einstein tensor.

By integrating T_{00} over a spatial volume at the boundaries of which all components of the tensor vanish, we obtain:

$$\int T_{00} dV = \frac{1}{2\chi}\int \left(\mu_0^2 + \mu_1^2 + \mu_2^2 + \mu_3^2\right) dV = \text{constant}. \tag{2}$$

This relationship expresses the conservation of energy of matter within a closed spatial volume. Its expression is strictly positive. Furthermore, if $\mu^2 \neq 0,\ T_{i,j}^{\ j} = 0$ implies, after all calculations:

$$\begin{cases} \mu^j(\mu_{i,j} - \mu_{j,i}) = 0, \\ \mu^i_{\ ,i} = 0. \end{cases} \qquad (3a, 3b)$$

The second equation is a conservation equation. The system completely defines the four-vector μ_i if $\int \mu_0 dV$ is known.

3. Wave Associated with the Movement of a Particle

Using the Levi-Civita symbol, knowing that $\varepsilon_{ijkl} = -\varepsilon^{ijkl}$ (4) in Minkowski space, we seek an approximate solution ψ_i to the system of equations (3a, 3b) by setting:

$$\psi_{i,j} - \psi_{j,i} = \frac{2\pi\nu}{c}\varepsilon_{ijkl}\psi^k u^l \Leftrightarrow \psi^{i,j} - \psi^{j,i} = -\frac{2\pi\nu}{c}\varepsilon^{ijkl}\psi_k u_l, \qquad (5)$$

where ν has the dimensions of a time frequency and u_i is a dimensionless four-vector, indeterminate at this stage of the calculations. Equation (3a) is thus satisfied. Taking into account equation (3b), and differentiating both sides of the above equality with respect to j:

$$\square\psi_i = \frac{2\pi\nu}{c}\varepsilon_{ijkl}(\psi^{k,j}u^l + \psi^k u^{l,j}),$$

$$\square\psi_i = \frac{\pi\nu}{c}\varepsilon_{ijkl}(\psi^{k,j} - \psi^{j,k})u^l + \frac{2\pi\nu}{c}\varepsilon_{ijkl}\psi^k u^{l,j}.$$

By hypothesis, let us identify u_i with the unit 4-velocity local and set ψ_i collinear with the 4-acceleration $\gamma^i = \dfrac{du^i}{ds}$. Then, since $u^{l,j} = \gamma^l s^{,j}$, it follows: $\varepsilon_{ijkl}\psi^{k}u^{l,j} = 0$ and:

$$\square\,\psi_i = \frac{\pi\nu}{c}\,\varepsilon_{ilkj}\left(\psi^{k,l} - \psi^{l,k}\right)u^{j},$$

$$\square\,\psi_i = -\frac{\pi\nu}{c}\,\varepsilon_{ijkl}\left(\psi^{k,l} - \psi^{l,k}\right)u^{j}.$$

Considering relation (5):

$$\square\,\psi_i = 2\left(\frac{\pi\nu}{c}\right)^2 \varepsilon_{ijkl}\varepsilon^{klmn}\psi_m u_n u^{j}.$$

Considering relation (4):

$$\varepsilon_{ijkl}\varepsilon^{klmn} = -2\left(\delta_i^m\delta_j^n - \delta_i^n\delta_j^m\right).$$

Therefore:

$$\square\,\psi_i = -\left(\frac{2\pi\nu}{c}\right)^2\left(\delta_i^m\delta_j^n - \delta_i^n\delta_j^m\right)\psi_m u_n u^{j},$$

$$\square\,\psi_i = -\left(\frac{2\pi\nu}{c}\right)^2\left(\psi_i u_j u^{j} - u_i\psi_j u^{j}\right),$$

$$\square\,\psi_i = -\left(\frac{2\pi\nu}{c}\right)^2\psi_i.$$

If, in addition: $\nu = \frac{mc^2}{h}$, where m, c and h denote, respectively, the particle's mass, the speed of light, and Planck's constant, the above equation becomes:

$$\square\,\psi_i + \frac{m^2c^2}{\hbar^2}\psi_i = 0, \tag{6}$$

where $\hbar = \frac{h}{2\pi}$.

This is a wave equation analogous to the Klein-Gordon equation governing a free particle in Quantum Mechanics, but with the difference that the wave function is real here. As an approximate solution to the system of equations (3a, 3b), the quadratic expression under the integral (2), which is strictly positive, does not represent an energy density, but a density that we can normalize to unity by integration over all physical space. This is analogous to the probability density of the particle's presence in Quantum Mechanics.

However, considering the particle as a whole by positing relation (5) excludes any possibility of describing its internal dynamics. This approach only allows us to show that there is a direct relationship between the densities of energy and of probability of the particle's presence.

Returning to the previous hypotheses, we must state:

$$\frac{R_{ij}}{R} = \frac{\mu_i \mu_j}{\mu^2} = \frac{\gamma_i \gamma_j}{\gamma^2}, \tag{7a}$$

where $R = \mu^2$ et $\gamma^2 = \gamma_k \gamma^k$.

These are 9 differential equations with 9 unknowns: 6 components of the metric and 3 components of the 4-speed, taking into account the free choice of the spatiotemporal frame of reference and the unitary nature of the 4-speed.

Note that in any given coordinate system, we have:

$$\gamma_i = u^j \nabla_j u_i = u^j \left(u_{i,j} - u_{j,i} \right) = u^j u_{i,j} + \frac{1}{2} u^j u^k g_{jk,i}. \tag{7b}$$

4. Relationship between Energy and Momentum

The following calculation shows that $\sigma = \frac{1}{c^2} T^{ij} u_i u_j = -\frac{\mu^2}{2\chi c^2}$

represents an eigenmass density.

Indeed, we verify a relationship, analogous to Einstein's equation linking the energy and momentum of a particle, in the form of a density:

$$T^i_k T^k_j = \frac{1}{\chi^2}\left(\mu^i \mu_j \mu^2 - \mu^i \mu_j \mu^2 + \frac{\mu^4}{4}\delta^i_j\right),$$

$$T^i_k T^k_j = \left(-\frac{\mu^2}{2\chi}\right)^2 \delta^i_j = \sigma^2 c^4 \delta^i_j,$$

$$T^0_k T^k_0 = \sigma^2 c^4 \delta^0_0. \tag{8}$$

In comparison, concerning a free particle considered as a whole and assimilated to a point of space:

$$\int T^0_i dV = \int \sigma c^2 u_i u^0 dV \simeq mc^2 u_i \Rightarrow \int T^0_i dV \int T^i_0 dV = m^2 c^4. \tag{9}$$

5. Law of Microscopic Movement of Matter

First, let us show that any antisymmetric tensor of order two, differentiable at least once, satisfies the tensor identity:

$$\left(\frac{\varphi^2}{4}\delta^j_i - \varphi_{ik}\varphi^{jk}\right)_{,j} = \varphi^{jk}_{,k}\varphi_{ij} + \frac{1}{2}\left(\varphi_{ij,k} + \varphi_{jk,i} + \varphi_{ki,j}\right)\varphi^{jk}, \tag{10}$$

where $\varphi^2 = \varphi_{jk}\varphi^{jk}$.

Indeed:

$$\left(\frac{1}{4}\varphi^2 \delta^j_i\right)_{,j} = \frac{1}{2}\varphi_{jk,i}\varphi^{jk},$$

$$\left(-\varphi_{ik}\varphi^{jk}\right)_{,j} = -\varphi_{ik,j}\varphi^{jk} - \varphi_{ik}\varphi^{jk}_{,j}.$$

But:

$$-\varphi_{ik,\,j}\varphi^{jk} = \varphi_{ki,\,j}\varphi^{jk} = \varphi_{ij,\,k}\varphi^{jk},$$

$$-\varphi_{ik}\varphi^{jk}_{,\,j} = \varphi^{jk}_{,\,k}\varphi_{ij}.$$

Hence the identity stated above.

Let us start with the expression:

$$\varphi_{ij} = \frac{1}{\sqrt{\chi}}(\mu_i u_j - \mu_j u_i). \tag{11}$$

After all calculations are done, we get:

$$\frac{\varphi^2}{4}\delta_i^j - \varphi_{ik}\varphi^{jk} = \frac{1}{\chi}\left(\frac{\mu^2}{2}\delta_i^j - \mu_i\mu^j - \mu^2 u_i u^j\right) = -T_i^j + 2\sigma c^2 u_i u^j.$$

Hence a new expression for the energy-momentum tensor of matter:

$$T_i^j = 2\sigma c^2 u_i u^j + \varphi_{ik}\varphi^{jk} - \frac{\varphi^2}{4}\delta_i^j. \tag{12}$$

Furthermore, taking into account (3a), (7b), and the orthogonality of the 4-vectors μ_i and u_i:

$$\frac{1}{2}(\varphi_{ij,\,k} + \varphi_{jk,\,i} + \varphi_{ki,\,j})\varphi^{jk}$$

$$= \frac{\mu^j u^k - \mu^k u^j}{2\chi}(\mu_{i,\,k} - \mu_{k,\,i})u_j + (\mu_{j,\,i} - \mu_{i,\,j})u_k + (\mu_{k,\,j} - \mu_{j,\,k})u_i$$

$$- (u_{i,\,k} - u_{k,\,i})\mu_j - (u_{j,\,i} - u_{i,\,j})\mu_k - (u_{k,\,j} - u_{j,\,k})\mu_i$$

$$= 0 + 0 + 0 - \gamma_i\mu^2 - \gamma_i\mu^2 + \gamma_j\mu^j\mu_i + \gamma_j\mu^j\mu_i = 0.$$

But, since (7a): $\gamma_j\mu^j\mu_i = \gamma_i\mu^2$.

Thus, the field satisfies the following nonlinear equations:

$$\left(\varphi_{ij,k} + \varphi_{jk,i} + \varphi_{ki,j}\right)\varphi^{jk} = 0. \tag{13}$$

Note that the constitutive equations of the Maxwell electromagnetic field, which are linear, satisfy these equations.

Therefore, φ_{ij} represents a generalized electromagnetic field and $\tilde{T}_i^{\,j} = \varphi_{ik}\varphi^{jk} - \frac{\varphi^2}{4}\delta_i^j$ is the associated energy-momentum tensor.

The equation of motion of matter in the form of a density can then be written, according to (10), (12) and (13) taking into account the no-divergence of the energy-momentum tensor $T_i^{\,j}$:

$$\left(2\sigma c^2 u_i u^j\right)_{,j} = \varphi^{jk}_{,k}\varphi_{ij}. \tag{14}$$

6. Internal Dynamics of an Isolated Particle

In the case of a free particle, to satisfy relation (9), we must set:

$$T_i^{\,j} = \sigma c^2 u_i u^j \Rightarrow \int T_i^{\,0} dV = \int \sigma c^2 u_i u^0 dV.$$

Consequently, according to (12):

$$\int \left(\sigma c^2 u_i u^0 + \widehat{\varphi}_{ik}\widehat{\varphi}^{0k} - \frac{\widehat{\varphi}^2}{4}\delta_i^0\right) dV = 0. \tag{15}$$

In the above relation, $\widehat{\varphi}_{ik}$ denotes the generalized electromagnetic field created by the particle itself.

Thus, $\sigma c^2 u_i u^j + \widehat{\varphi}_{ik}\widehat{\varphi}^{jk} - \frac{\widehat{\varphi}^2}{4}\delta_i^j = T_i^{\,j} - \sigma c^2 u_i u^j$ represents the energy-momentum density from an internal dynamics within the particle whose resultant is zero.

Indeed, equation (15) shows that the energy-pulse acting on half of the total mass 2σ is balanced by an energy-pulse of electromagnetic origin inside the particle. Therefore, the particle can only be driven in its global movement by an external field.

7. Motion of a Particle in an External Field

In this situation, it is necessary to distinguish the internal field $\hat{\varphi}_{ij}$ from the external field $\breve{\varphi}_{ij}$ of the particle and set $\varphi_{ij} = \hat{\varphi}_{ij} + \breve{\varphi}_{ij} = (\hat{\varphi} + \breve{\varphi})_{ij}$.

From (14), we obtain:

$$\left(2\sigma c^2 u_i u^j\right)_{,\,j} = (\hat{\varphi} + \breve{\varphi})^{jk}_{,\,k}(\hat{\varphi} + \breve{\varphi})_{ij}.$$

There is no doubt that the internal field is much greater than the external field and that the derivatives of the latter are negligible within the particle, we then obtain:

$$\left(\sigma c^2 u_i u^j\right)_{,\,j} - \hat{\varphi}^{jk}_{,\,k}\hat{\varphi}_{ij} + \left(\sigma c^2 u_i u^j\right)_{,\,j}$$

$$= \left(\sigma c^2 u_i u^j + \hat{\varphi}_{ik}\hat{\varphi}^{jk} - \frac{\hat{\varphi}^2}{4}\delta^j_i\right)_{,\,j} + \left(\sigma c^2 u_i u^j\right)_{,\,j} = \hat{\varphi}^{jk}_{,\,k}\breve{\varphi}_{ij}.$$

Taking into account (15) and the fact that σ is zero at the boundary of a spatial volume encompassing the particle, we find:

$$\int \left(\sigma c^2 u_i u^j\right)_{,\,j} dV \simeq \int \left(\sigma c^2 u_i u^0\right)_{,\,0} dV \simeq \int \hat{\varphi}^{jk}_{,\,k}\breve{\varphi}_{ij} dV.$$

That is:

$$\int u_{i,\,0} c^2 dm \simeq mc^2 u_{i,\,0} \simeq \breve{\varphi}_{ij}\int \hat{\varphi}^{jk}_{,\,k} dV. \quad \text{(Maxwell-Lorentz)} \qquad (16)$$

8. Composition of the Generalized Electromagnetic Field

Any antisymmetric tensor of order 2 can be defined from two no-divergent potential four-vectors:

$$\varphi_{ij} = A_{i,j} - A_{j,i} + \frac{1}{2}\varepsilon_{ijkl}\left(B^{k,l} - B^{l,k}\right).$$

This implies:

$$\varphi^{*}_{ij} = \frac{1}{2}\varepsilon_{ijkl}\varphi^{kl} = \frac{1}{2}\varepsilon_{ijkl}\left(A^{k,l} - A^{l,k}\right) + B_{i,j} - B_{j,i}.$$

Indeed, it is easy to show from these relations that: $\square A_i = \varphi_{ij}^{;j}$ and $\square B_i = \varphi_{ij}^{*;j}$.

Let:

$$\dot{\varphi}_{ij} = A_{i,j} - A_{j,i} \quad \text{and} \quad \ddot{\varphi}_{ij} = \frac{1}{2}\varepsilon_{ijkl}\left(B^{k,l} - B^{l,k}\right).$$

The energy-momentum tensor of the generalized electromagnetic field $\ddot{T}_i^{\cdot j}$, expressed as a function of $\dot{\varphi}_{ij}$ and $\ddot{\varphi}_{ij}$, is the sum of three tensors whose divergences correspond to three distinct forces:

$$\ddot{T}_i^{\cdot j} = \ddot{T}_i^{\cdot j}(\dot{\varphi}) + \ddot{T}_i^{\cdot j}(\dot{\varphi}, \ddot{\varphi}) + \ddot{T}_i^{\cdot j}(\ddot{\varphi}). \tag{17}$$

In particular, we recognize the energy-momentum of Maxwell's electromagnetic field in $\ddot{T}_i^{\cdot j}(\dot{\varphi}) = \dot{\varphi}_{ik}\dot{\varphi}^{jk} - \frac{\dot{\varphi}^2}{4}\delta_i^j$. Its divergence expresses the electromagnetic force described in classical electrodynamics.

The other two forces are not part of classical electrodynamics. It is possible that they have a limited range of action due to the non-linearity of the field $\ddot{\varphi}_{ij}$ according to (13).

8. Case of the Luminous Particle

In this case, $\sigma = 0$ and $T_{00} = \mu_0\mu_0$ with $\int \mu_0 dV = K$ according to (3b). The particle being extremely small, $\mu_0 \simeq K\delta$ where δ is the Dirac delta function and K a constant.

Consequently,

$$\int T_{00} dV = \int \mu_0\mu_0 dV = \int K\delta\mu_0 dV = K\bar{\mu}_0 = \chi h\nu. \quad \text{(Planck-Einstein)}$$

By postulating: $c\bar{\mu}_0 = \nu$, then

$$K = \chi hc. \tag{18}$$

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