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MATHEMATICAL FORMULATION OF NUCLEAR REACTION MECHANISMS

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Abstract

In this paper, we answer the following question "Which phenomena actually occur when two colliding nuclei interpenetrate?" This purpose was reached by building a mathematical model. Within this model, in one hand, were defined the studied system and its states and in other hand, were elaborated the concepts of path followed by the system, in its evolution, as a series of system states and a new view of mechanisms as operators acting on the states of the system. The paper was ended by giving many examples of known mechanisms written, according this model, as operators.

1. Introduction

The usual analysis of a system compound of two colliding bodies can be schemed as follows, we consider one body, generally the heavier, as the target that acts on the other body assumed as the projectile when it

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comes up to it. Then, we analyze experimentally the changes that affect, after interaction, the states of the projectile and the target. In theory, the physicist oriented by the verification of his predictions by the experience, for convenience, fits his model according to the same picture as of the experience. This type of analysis is frequently used not only in nuclear physics but in other topics of physics also. For example, in classical physics it is the case of the matrix optics [1]. This model gives a theoretical description very close of the experimental scheme, the emergent ray is mathematically presented as a result of the application of a square matrix that gives, due its construction, the action of the optical medium on the incoming ray. And in modern physics, precisely in quantum mechanics, another example is, for the scattering theory, the S-matrix [2] that in turn transposes the experimental scheme theoretically and thus links the asymptotic incoming and outgoing waves. Each knows that this manner of doing is natural and considered in its globality has done its proofs but whatever that it is adopted by the scientific community it is true that it stays a possibility among others that the physicist adopts before performing his analysis.

Thus, we can instead of choosing one of the bodies as acting on the other or vice versa choose the two bodies interacting each other and follow the steps of the method. Thus, we keep the method in whole and change only the choice of the system what is permissible. Here is a manner of doing that is certainly not habitual but provides some results (as the formula that describes the mechanisms taking place for each outcome of a nuclear collision, the dependency of mechanisms upon the impact parameter and others that will be published in right time) different from those obtained by the classical method exposed above but are certainly complementary. The considerations given above concerning the physical framework of this work permit me to go back to the goal of this paper. In this work, it is shown which considerations were made and developments done to reach the mathematical formulation of the reaction mechanisms.

2. Model

The collision phenomenon has been largely studied in the classical framework (Newton mechanics) and in the modern framework (relativity and quantum mechanics). The common thing for all these studies is that the system studied is either the projectile or the target but never them together. Conversely to these studies, in this work the system studied is composed of the two colliding nuclei taken together and the focus is made on the nucleonic structure of the colliding nuclei and the mechanisms underlying their changes during the evolution of the system formed by them.

The geometrical shape of the colliding nuclei has no interest in this model. Thus, they are considered as two virtual bags (Figure 1) that approach (state a), interpenetrate (stage b), and then separate (stat-e c). During the period $(t_2 - t_1)$ the system evolutes from its initial state (a) to its final state (c), and because of the interactions either or both of the colliding nuclei loses (lose) progressively its (their) initial nucleonic composition (compositions) and structure (structures) and then the new entities appear (state c).

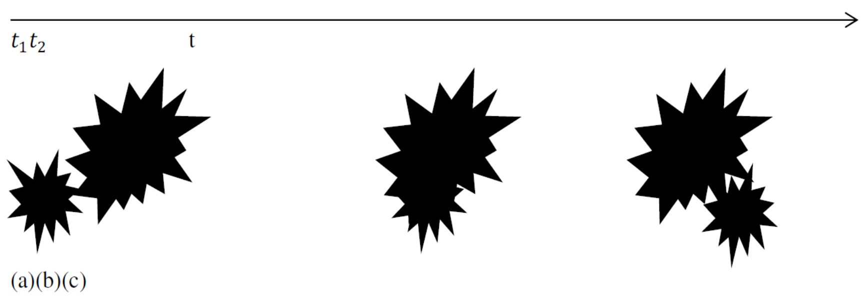

Figure 1. The system evolution in time that begins by its state (a) and finishes by the state (c). During the period $t_2 - t_1$ the colliding nuclei penetrate deeply and interact strongly inside their common space.

Mathematically the system studied is a couple of nuclei; (target, projectile) in which either of the nuclei will be represented by a column

$\begin{matrix} N \\ Z \end{matrix}$ where N and Z design, respectively, neutron and proton numbers. In other terms a nucleus will be represented by a vector of the plane $(\mathbf{O}, \mathbf{N}, \mathbf{Z})$ that the column $\begin{matrix} N \\ Z \end{matrix}$ is its components. Entrance and exit channels are noted, respectively, by E and S then represented by two columns boards $\begin{matrix} N_{1E} & N_{2E} \\ Z_{1E} & Z_{2E} \end{matrix}$ and $\begin{matrix} N_{1S} & N_{2S} \\ Z_{1S} & Z_{2S} \end{matrix}$. In the board (E) the columns are the vector components of the colliding nuclei and in the board (S) those of the issue of the collision.

The following convention takes for the totality of this work.

Convention. The left column of the board (E) represents the target and the right one the projectile. For the board (S) the left column corresponds to the residual nucleus and the right one to the emitted particle.

To reach S from E the system follows a path $M(M_1, M_2)$ formed by a series of couples in the plane $(\mathbf{O}, \mathbf{N}, \mathbf{Z})$; or a series of boards in $(\mathbf{O}, \mathbf{N}, \mathbf{Z}) \times (\mathbf{O}, \mathbf{N}, \mathbf{Z})$. Whatever the space considered each element of them is called system state and the ensemble of the system states is noted E_1 for the first ensemble and E_2 for the second. E_i (where $i = 1, 2$) is called system states ensemble. It is evident that the channels E and S are particular states of the system.

3. Mechanisms

The introduction of the concept "system state" will permit the introduction of nuclear collision mechanism concept. The question is then, how to do it?

To response this question let me give some numerical examples. Through these particular examples comes up the idea of mechanism that will be then generalized.

A. Numeric Examples

Suppose we have the nuclear collision $E \to S$ where $E = \begin{matrix} 1 & 3 \\ 1 & 3 \end{matrix}$ and $S = \begin{matrix} 2 & 2 \\ 1 & 3 \end{matrix}$. To reach S (if we use the matrix notation $\begin{matrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{matrix}$ for a given board) a possible transformation of the system is: $\begin{matrix} c_{11} \leftarrow & c_{12} \\ c_{21} & c_{22} \end{matrix}$. Let it be noted H_1^1 where the letter H means the transformation interests only the upper line, 1 in indices indicates a unit was taken from c_{12} (the case sides the base of the arrow) and 1 in powers for saying that it is added to c_{11} (the case sides the arrow head).

We can consider another exit channel of the system, say $S' = \begin{matrix} 1 & 4 \\ 1 & 2 \end{matrix}$. It is clear that the transformation consists in unit diminution of the case c_{22} and unit augmentation of the case c_{12}. Following the same way as above, the transformation can be schemed as $\begin{matrix} c_{11} & c_{12} \\ & \uparrow \\ c_{21} & c_{22} \end{matrix}$, let this transformation be noted D_1^1. By following the same processes in the two examples cited previously one arrives to B_1^1 and G_1^1, respectively, for the transformations gendering the exit channels $\begin{matrix} 1 & 3 \\ 0 & 4 \end{matrix}$ and $\begin{matrix} 1 & 3 \\ 0 & 4 \end{matrix}$.

B. Generalization

The previous discussion of particular cases permits to make the following assumption.

Assumption. Each final system state (E_{final}) can be seen as a result of application of an operator θ to the initial system state $(E_{initial})$. In other words: $E_{final} = \theta E_{initial}$. These formulas of θ written above are certainly particular cases of a more general formula of θ that we must find.

A first step to do that (generalize the writing of the operator θ) is to suppose that θ is a product of the four previous operators.

Additionally, the unit number in powers and indices will be replaced by relative numbers (elements of $\mathbb{Z}$). That means $\theta = G_g^g B_b^b D_d^d H_h^h$.

For a more general form of θ, we suppose that the indices and powers are different for the same particular operator, so then $\theta = G_g^{g'} B_b^{b'} D_d^{d'} H_h^{h'}$.

According to the discussion above the action of θ for any intermediate system state

$$x = \begin{pmatrix} n_1 & n_2 \\ z_1 & z_2 \end{pmatrix},$$

$$\theta x = \begin{pmatrix} n_1 - g + h' & n_2 + d' - h \\ z_1 + g' - b & z_2 + b' - d \end{pmatrix}. \tag{1}$$

C. Mechanisms Calculus

In this paragraph, we try to answer the question: How to calculate the mechanism corresponding to a given nuclear reaction?

For convenient reasons, in the following developments the matrix notation is adopted but the learner must know that the state of the system is not mathematically a matrix.

According to the assumption of the previous paragraph, for a system in its initial state $E = \begin{pmatrix} n_{e1} & n_{e2} \\ z_{e1} & z_{e2} \end{pmatrix}$ that evolves to its final state

$S = \begin{pmatrix} n_{s1} & n_{s2} \\ z_{s1} & z_{s2} \end{pmatrix}$:

$$S = \theta E, \tag{2}$$

where $\theta = G_g^{g'} B_b^{b'} D_d^{d'} H_h^{h'}$.

Explicitly (2) is written as:

$$\begin{pmatrix} n_{s1} & n_{s2} \\ z_{s1} & z_{s2} \end{pmatrix} = \begin{pmatrix} n_{e1} - g + h' & n_{e2} + d' - h \\ z_{e1} + g' - b & z_{e2} + b' - d \end{pmatrix}. \tag{3}$$

We can calculate the corresponding mechanism by solving the equation (4). In fact, if we equal the members of the equation (4) it comes:

$$\begin{cases} n_{s1} = n_{e1} + h' - g, \\ z_{s1} = z_{e1} + g' - b, \\ n_{s2} = n_{e2} + d' - h, \\ z_{s2} = z_{e2} + b' - d, \end{cases} \tag{4}$$

where n_{ei}, z_{ei}, n_{si}, z_{si} are given. We solve the fourth equations (4) according: g', b', d', h'.

Then it comes:

$$\begin{cases} h' = n_{s1} - n_{e1} + g, \\ g' = z_{s1} - z_{e1} + b, \\ d' = n_{s2} - n_{e2} + h, \\ b' = z_{s2} - z_{e2} + d. \end{cases} \tag{5}$$

θ becomes ones these quantities replaced by their values:

$$\theta = G_g^{z_{s1} - z_{e1} + b} B_b^{z_{s2} - z_{e2} + d} D_d^{n_{s2} - n_{e2} + h} H_h^{n_{s1} - n_{e1} + g}. \tag{6}$$

After simplification[1] of the right side of (6), θ takes the form:

$$\theta = G_0^{z_{s1}-z_{e1}} B_0^{z_{s2}-z_{e2}} D_0^{n_{s2}-n_{e2}} H_0^{n_{s1}-n_{e1}}. \tag{7}$$

For practical reasons, the introduction of an ensemble of new operators was necessary. In fact, each operator introduced above (Paragraph A: Numeric Examples) can be written as a product of two more fundamental operators (a creator and an annihilator). $G_1^1 = G_0^1 G_1^0$, $B_1^1 = B_0^1 B_1^0$, $D_1^1 = D_0^1 D_1^0$, $H_1^1 = H_0^1 H_1^0$ where (according to the convention of the paragraph titled: Model),

G_0^1 is a proton creator in the target and G_1^0 is a neutron annihilator in the target.

B_0^1, B_1^0 are successively the proton creator in the projectile and the proton annihilator in the target.

D_0^1, D_1^0 are successively the neutron creator in the projectile and the proton annihilator in the projectile.

H_0^1, H_1^0 are successively the neutron creator in the target and the neutron annihilator in the projectile.

Each one of these elementary transformations is called micro-mechanism. Any product of micro-mechanisms is called mechanism. Inversely a mechanism is a product of micro-mechanisms, for example $\theta = G_0^1 G_1^0 B_0^1$, $\theta' = D_0^1 D_1^0 H_1^0$, ...

4. Examples of Mechanisms

In this paragraph, among the known mechanisms were chosen those

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[1]See Appendix

who are widely cited in scientific literature to be written as operators.

The convention adopted is what of the first paragraph: within the system state in collision the left column is the target and the right column is the projectile.

Table 1. Examples of mechanisms, their reaction schemes and their writing under operator forms

Mechanism name	**Reaction scheme**	**Mechanism formula**
Pick up	Neutron case $\begin{pmatrix} n_{e1} & n_{e2} \\ z_{e1} & z_{e2} \end{pmatrix} \to \begin{pmatrix} n_{e1}-1 & n_{e2}+1 \\ z_{e1} & z_{e2} \end{pmatrix}$	$D_0^1 H_0^{-1}$
	Proton case $\begin{pmatrix} n_{e1} & n_{e2} \\ z_{e1} & z_{e2} \end{pmatrix} \to \begin{pmatrix} n_{e1} & n_{e2} \\ z_{e1}-1 & z_{e2}+1 \end{pmatrix}$	$G_0^{-1} B_0^1$
Stripping	Neutron case $\begin{pmatrix} n_{e1} & n_{e2} \\ z_{e1} & z_{e2} \end{pmatrix} \to \begin{pmatrix} n_{e1}-1 & n_{e2}+1 \\ z_{e1} & z_{e2} \end{pmatrix}$	$D_0^{-1} H_0^1$
	Proton case $\begin{pmatrix} n_{e1} & n_{e2} \\ z_{e1} & z_{e2} \end{pmatrix} \to \begin{pmatrix} n_{e1} & n_{e2} \\ z_{e1}-1 & z_{e2}+1 \end{pmatrix}$	$G_0^1 B_0^{-1}$
Fusion	$\begin{pmatrix} n_{e1} & n_{e2} \\ z_{e1} & z_{e2} \end{pmatrix} \to \begin{pmatrix} n_{e1}+n_{e2} & 0 \\ z_{e1}+z_{e2} & 0 \end{pmatrix}$	$G_0^{z_{e2}} B_0^{-z_{e2}} D_0^{-n_{e2}} H_0^{n_{e2}}$

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Appendix

$\theta = G_g^{g'} B_b^{b'} D_d^{d'} H_h^{h'}$ is a given operator and $\begin{pmatrix} n_1 & n_2 \\ z_1 & z_2 \end{pmatrix}$ a system state so that are:

$$\theta \begin{pmatrix} n_1 & n_2 \\ z_1 & z_2 \end{pmatrix} = \begin{pmatrix} n_1' & n_2' \\ z_1' & z_2' \end{pmatrix}. \tag{A1}$$

Explicitly, (A1) is presented as:

$$\begin{pmatrix} n_1 - g + h' & n_2 + d' - h \\ z_1 + g' - b & z_2 + b' - d \end{pmatrix} = \begin{pmatrix} n_1' & n_2' \\ z_1' & z_2' \end{pmatrix}. \tag{A2}$$

The left side of (A2) can be written as:

$$\theta = G_0^{g'-b} B_0^{b'-d} D_0^{d'-h} H_0^{h'-g} \begin{pmatrix} n_1 & n_2 \\ z_1 & z_2 \end{pmatrix}.$$

So, θ can be written at least under two forms. Additionally, if we suppose that there exists another operator θ', it is easy to demonstrate that $\theta = \theta'$.

Statement. For each couple of system states there exists one and only one operator that transforms the first state in the second state, and this operator has two writings. Therefore, the two writings of a given operator are considered equivalent.

References

[1] https://en.wikipedia.org/wiki/Ray_transfer_matrix_analysis

[2] http://kau.diva-portal.org/smash/get/diva2:1662818/FULLTEXT01.pdf