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GALACTIC GEONS: REVISITING THE DARK MATTER PARADIGM

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Abstract

This study presents the mathematical derivation of geons, gravitationally stable spacetime structures, as an alternative to dark matter. Assuming galaxies are embedded in galactic-sized geons, the observed flat galaxy rotation curves can be explained without requiring the standard dark matter halo. This concept has been applied to the case of the Milky Way and profiles of density, pressure and rotation velocity have been derived, demonstrating a close correspondence with the observations. The geon's Gaussian density profile naturally explains the flat central density cores observed in dwarf galaxies, providing a compelling solution to the core-cusp problem. Furthermore, the early formation of geons shortly after the Big Bang offers a framework for understanding the rapid emergence of massive galaxies, addressing the challenges posed by recent James Webb Space Telescope observations. These findings suggest that geons could serve as both the gravitational scaffolding for galaxy formation and a replacement for cold dark matter,

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unifying multiple cosmological phenomena under a single theoretical framework.

1. Introduction

The enigmatic nature of galaxy rotation curves, first observed in the 1970s, remains one of the most compelling puzzles in astrophysics. Measurements of rotational velocities in galaxies exhibit a surprising flatness at large radii, deviating from the expected Keplerian decline. This phenomenon has traditionally been attributed to the presence of unseen mass, commonly referred to as dark matter. However, despite decades of dedicated research, direct detection of dark matter particles has proven elusive [1], motivating alternative explanations rooted in the fundamental physics of gravitation.

In this context, we revisit the concept of gravitational geons, self-sustaining field configurations governed purely by Einstein's Field Equations (EFE) [2]. Originally introduced by John Archibald Wheeler in 1955, geons "(gravitational electromagnetic entity)" were envisioned as stable, localized energy constructs formed from gravitational and electromagnetic fields [3]. However, these geons were primarily considered in small-scale scenarios, often limited by their susceptibility to radiation leakage and other instabilities. Wheeler did not present explicit geon solutions to the vacuum Einstein field equation, a gap which was partially filled by Brill and Hartle in 1964 [4]. In their paper they applied their method to the case of a static spherically symmetric background geometry and found that gravitational waves can remain confined in a region for a time much longer than the region's light-crossing time. This so-called gravitational geon is generated by a large number of high frequency, small amplitude gravitational waves. The time average of the curvature due to these waves creates the background geometry of the geon, and this background geometry traps the waves for a long time in a region of space called the "active" region.

In this paper, we will investigate a geon type of solution which is very different in nature from previously investigated types of geons. Here we will focus on a spherically symmetric, stationary geon with a spatial extension similar to that of galaxies, or even larger. The existence of such geons was already conjectured in 1998 [5], although a mathematical proof was not provided. Strictly speaking, the only non-trivial, matter-free solutions to the Einstein Field Equations are the Schwarzschild and Kerr spacetimes. Nevertheless, we will here introduce a modification to the EFE in which the momentum-energy content within the vacuum is treated as an effective stress-energy tensor which has similarities to the cosmological term. In that case, we find exact, non-trivial, analytical solutions to the EFE. Imagining that galaxies are embedded within such geons, we will demonstrate that it is possible to explain the observed, relatively flat, galaxy rotation curves. Further, it will be argued that such galactic sized geons can play a role in galaxy formation, providing an explanation for the observation of galaxies in the early universe (300 million years after the Big Bang), which are difficult to explain within the standard ΛCDM model [6-8].

2. Mathematical Geon Description

The Einstein Field Equations (EFE) contain 10 independent equations due to the symmetric 4×4 tensor $G_{\mu\nu}$. However, under the assumptions of spherical symmetry, stationarity, and no rotation, the system simplifies significantly:

- Spherical Symmetry: Ensures that the metric components depend only on the radial coordinate r, reducing complexity.

- Stationarity: Implies time-independence, eliminating time derivatives.

- No Rotation: Eliminates cross-terms like $G_{t\theta}$.

This symmetry reduces the independent components of $G_{\mu\nu}$ to:

- The tt -component (energy density).

- The rr -component (radial pressure).

- The $\theta\theta$ - or $\phi\phi$ -component (angular pressure).

The conservation of energy-momentum, $\nabla_\nu T^{\mu\nu} = 0$, ensures consistency among the equations, leaving three independent equations.

We will use a stress-energy tensor of the form $T_{\mu\nu} = diag\left(\rho e^{2\phi},\ p_r e^{2\Lambda},\ p_t r^2,\ p_t r^2 \sin^2\theta\right)$.

The metric ansatz for a spherically symmetric, stationary spacetime is:

$$ds^2 = -e^{2\phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \quad (1)$$

The Einstein Field equations are given by

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \quad (2)$$

in which Λ is the cosmological constant. In the following derivations, Λ is initially not considered.

Since we want a non-trivial solution without singularity at the origin, we choose a profile for the energy density which has a finite value at the origin, for which the derivative at $r = 0$ is zero and which decays for larger values of r. Here we chose a Gaussian density profile for the geon:

$$\rho(r) = \rho_0 e^{-r^2/R^2}, \quad (3)$$

where ρ_0 is the central density and a determines the characteristic size.

The tt -equation is:

$$\frac{1}{r^2}\frac{d}{dr}\left[r\left(1-e^{-2\Lambda}\right)\right] = 8\pi\rho(r). \tag{4}$$

Substitute $\rho(r) = \rho_0 e^{-r^2/R^2}$:

$$\frac{1}{r^2}\frac{d}{dr}\left[r\left(1-e^{-2\Lambda}\right)\right] = 8\pi\rho_0 e^{-r^2/R^2}. \tag{5}$$

Define the mass function:

$$m(r) = \frac{1}{2}r\left(1-e^{-2\Lambda}\right) \tag{6}$$

which represents the enclosed mass at radius r. The equation becomes:

$$\frac{dm}{dr} = 4\pi r^2\rho(r). \tag{7}$$

Substituting $\rho(r)$:

$$\frac{dm}{dr} = 4\pi r^2\rho_0 e^{-r^2/R^2}. \tag{8}$$

Integrating:

$$m(r) = 4\pi\rho_0\int_0^r x^2 e^{-x^2/R^2}\,dx. \tag{9}$$

The integral has a known solution:

$$m(r) = 4\pi\rho_0\left[\frac{\sqrt{\pi}R^3}{4}\,erf\left(\frac{r}{R}\right) - \frac{R^3}{2}\,e^{-r^2/R^2}\right]. \tag{10}$$

Taking the integral from 0 to infinity, the integral becomes:

$$M_{geon} = \pi^{3/2}\rho_0 R^3. \tag{11}$$

Thus, $e^{-2\Lambda(r)}$ can be determined from:

$$e^{-2\Lambda} = 1 - \frac{2m(r)}{r}. \tag{12}$$

The rr-equation is:

$$\frac{2}{r}\phi'(r)e^{-2\Lambda} - \frac{1}{r^2}\left(1 - e^{-2\Lambda}\right) = 8\pi p_r(r). \tag{13}$$

Substitute equation (12)

$$\frac{2}{r}\phi'(r)\left(1 - \frac{2m(r)}{r}\right) - \frac{1}{r^2}\left(\frac{2m(r)}{r}\right) = 8\pi p_r(r). \tag{14}$$

Simplify:

$$\phi'(r) = \frac{r}{2\left(1 - \frac{2m(r)}{r}\right)}\left(8\pi p_r(r) + \frac{2m(r)}{r^3}\right). \tag{15}$$

The $\theta\theta$-equation is:

$$e^{-2\Lambda}\left(\phi'' + \phi'^2 - \phi'\Lambda' + \frac{\phi'}{r} - \frac{\Lambda'}{r}\right) = 8\pi p_t(r). \tag{16}$$

Using equation (12), computing derivatives of $\phi(r)$ and $\Lambda(r)$, this equation provides consistency between $p_t(r)$, $p_r(r)$, and $\rho(r)$.

In addition, one has to satisfy the conservation equation:

$$\nabla_\nu T^{\mu\nu} = 0. \tag{17}$$

For the radial component $\mu = r$, we get:

$$\frac{dp_r}{dr} = -(\rho + p_r)\frac{d\phi}{dr} + \frac{2}{r}(p_t - p_r). \tag{18}$$

Next, we need to introduce the equation of state, relating the energy density to the pressure. One could consider $p_r = +\rho$ or $p_r = -\rho$. After some algebra, one can show that both choices lead to a solution of the EFE. Since we are aiming at a geon type solution, we will use therefore the second option, namely

$$p_r = -\rho. \tag{19}$$

In this way, we are sure that we are not dealing with some ordinary gas or plasma.

A common misconception in gravitational physics is that a negative radial pressure, such as the equation of state (equation (19)), necessarily leads to instability. While negative pressure is often associated with repulsive effects in cosmology, in localized gravitational systems, it can contribute to a stable equilibrium when properly balanced. For instance, the Tolman-Oppenheimer-Volkoff (TOV) equation describes equilibrium in relativistic stars, allowing stable configurations with anisotropic pressures, including negative radial pressure in certain regimes [9, 10]. Additionally, various stable gravitational structures exhibit regions of negative pressure. A key example is the gravastar model [10], in which an interior de Sitter-like vacuum state $(P = -\rho)$ is enclosed by a thin shell, maintaining a stable structure without collapse. Similarly, traversable wormhole solutions require exotic matter with negative pressure to support their throats [11, 12], yet under appropriate conditions, such configurations can be dynamically stable. Furthermore, cosmological de Sitter space, which dominates the large-scale universe today due to dark energy $(P = -\rho)$, is itself a maximally symmetric and stable solution of Einstein's equations [13]. These examples demonstrate that negative pressure does not inherently lead to instability and can instead be a key ingredient in constructing equilibrium configurations in General Relativity.

Using this choice of equation of state, we can now work out the solution. Since both the density and the radial pressure are already defined, we only need to find the tangential pressure p_t. This can be obtained either from the $\theta\theta$-equation or from the pressure balance equation (18) which is in fact the generalized Tolman-Oppenheimer-Volkov equation [14]. This results in the following expression for p_t;

$$p_t = -\rho\left(1 - \frac{r^2}{R^2}\right). \tag{20}$$

When choosing $p_r = +\rho$ one obtains $p_t = +\rho(1 - r^2/R^2)$.

In fact, there are some higher order corrections to p_t containing terms like $\rho^2 r^2$, ρm, $m^2 r^4$ but when converting back from geometrized units to SI units, these are to be multiplied by very small factors such as G/c^2 or G/c^4. It is worth mentioning that these solutions were never derived before.

The energy-momentum tensor $T_{\mu\nu}$ might be interpreted as an effective vacuum energy plus an anisotropic correction:

$$T_{\mu\nu} = -\rho g_{\mu\nu} + T_{\mu\nu}^{corr}, \tag{21}$$

where the correction term $T_{\mu\nu}^{corr}$ accounts for the deviation in p_t:

$$T_{\mu\nu}^{corr} = diag\left(0,\ 0,\ \rho\,\frac{r^2}{R^2},\ \rho\,\frac{r^2}{R^2}\right). \tag{22}$$

In this interpretation, the term $-\rho g_{\mu\nu}$ behaves as a vacuum energy (proportional to the metric tensor) and thus consistent with the EFE including the cosmological constant (see equation (2)). However, in this case, ρ is not a constant but depends on r. In the scalar field theories [15], Λ is replaced by a dynamic scalar field whose energy density evolves over time and space and in $f(R)$ gravity theories [16], the cosmological constant emerges as a dynamic entity related to the form of $f(R)$. The correction $T_{\mu\nu}^{corr}$ represents anisotropic stresses or deviations from a pure cosmological constant. In many modified gravity theories, corrections to the Einstein equations involve higher-order curvature terms such as R^2 or $R_{\mu\nu}R^{\mu\nu}$. These terms introduce modifications to the stress-energy tensor that naturally depend on the radial coordinate r, often scaling as r^2. The anisotropic correction term $T_\theta^\theta \sim \rho \frac{r^2}{R^2}$ suggests a second-order geometric effect arising from such modifications, where curvature contributions feed back into the stress-energy distribution, leading to additional pressure anisotropies in the angular components.

It is interesting to note that we are still free in the choice of the energy density at the origin, as well as in the value of the scale length R in the Gaussian energy density profile. So, these solutions can describe very small entities (scale of atoms) to very large entities (scale of galaxies or larger). Also, it is very likely that solutions, similar to this one, can be found in which the geon rotates. Because of the higher complexity, this path was not explored further.

3. Application to the Milky Way

In the following we will imagine that our galaxy is embedded in a geon, extending beyond the galaxy. As an example, we will take the case of the Milky Way (MW).

There is still a significant uncertainty in the total mass of the MW, and here we will consider some reasonable values for total, baryonic and bulge mass only for the sake of providing an idea of the effect of the geon on the galaxy rotation curve. Our aim is thus not to provide a detailed modelling of the MW. On the other hand, there are recent rotation curve results by the Gaia DR3 measurements [17] which we will use as a guidance. These results show rotation velocities of the order of $230\ \mathrm{km/s}$, some decline beyond $15\ \mathrm{kpc}$ and a pronounced decline beyond $19\ \mathrm{kpc}$. In our very simplistic model, we will use here a uniform baryonic mass of $4.6 \times 10^{10}\ \mathrm{M}_\odot$ up to a radius of $4\ \mathrm{kpc}$. If we assume for instance a total mass of the MW of 2.06×10^{11} M$_\odot$ and a baryonic mass 4.6×10^{10} M$_\odot$ we will choose the mass of the geon to be the difference between the two, namely 1.6×10^{11} M$_\odot$. Thus, the geon mass takes the roll of "dark matter". Then we chose a suitable scale length (R) of the Gaussian energy density profile, and we adjusted the central density such as to obtain the required total mass of the geon (using equation (11)). The scale length in this case should be somewhere between 8 and 12 kpc and in what follows, we choose the value $R = 9\,\text{kpc}$. In fact, at this point, we use it as a fitting parameter to the MW rotation curve. However, this might also hint to a connection between the mass accumulated in the geon (see next section) and the properties of the geon. In the weak field approximation, we can safely assume that the gravitational force on a test mass is the sum of the forces due to the central (baryonic) mass and the force due to the geon. The force on a test mass can be obtained by using the equation for $\phi'(r)$ (equation (15)) in which we substitute $p_r = -\rho$ and where we use the approximation that $2m/r << 1$, resulting in

$$F = m_{test}\phi'(r) = m_{test}\left(-4\pi r\rho + \frac{m}{r^2}\right). \tag{23}$$

When reverting to SI units, the first term needs to be multiplied by G/c^2 and the second term by G. Then we find that the first term becomes negligible, resulting in the simple equation:

$$F = \frac{m_{test}m}{r^2}. \tag{24}$$

Thus, the force on test mass, due to the geon, is just the Newtonian gravitational force, induced by the enclosed mass.

The profiles of the density (Gaussian, with scale length 9 kpc) and the calculated enclosed mass are shown in Figure 1.

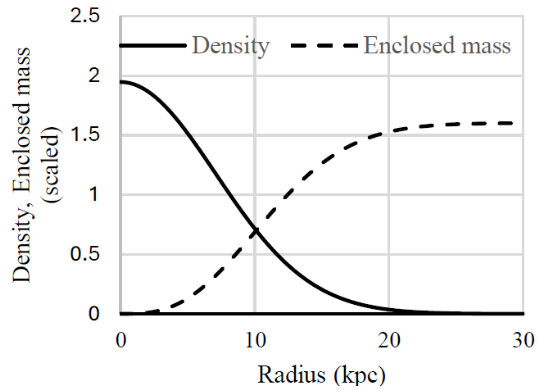

Figure 1. Density and enclosed mass of the geon. The units are scaled. The density $\left(\text{in kg/ m}^3\right)$ is obtained by multiplying the scaled values by 10^{-21}. The enclosed mass (in solar masses) is obtained by multiplying the scaled values by 10^{11}.

The calculated pressure profiles are shown in Figure 2. Note the region in which the tangential pressure changes sign.

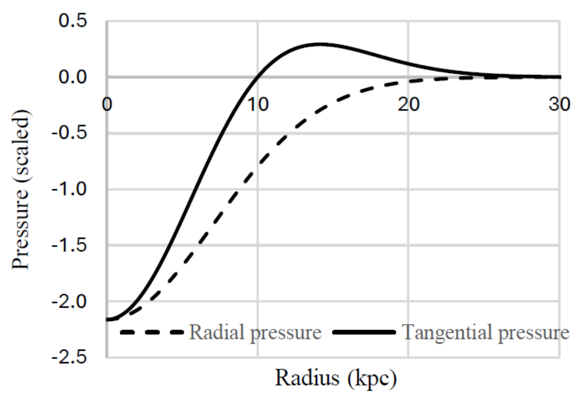

Figure 2. Radial and tangential pressure profiles of the geon. The units are scaled. The pressures (in Pa) are obtained by multiplying the scaled values by 10^{-38}.

Finally, the impact of the geon on the Milky Way rotation curve (simplistic model) is shown in Figure 3. The additional mass of the geon results in a relatively flat rotation curve and a decline in the region 15-20 kpc (in agreement with recent observations [17]).

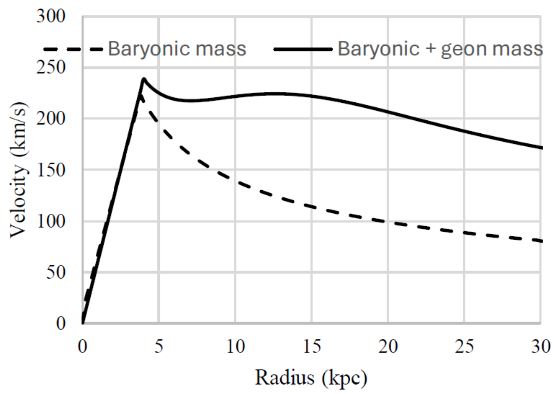

Figure 3. Calculated velocity profiles for the Milky Way (simplistic model), showing the Newtonian decay and the relatively flat profiles obtained by the additional force of the geon.

4. Early Galaxy Formation and the role of Geons

The ΛCDM model predicts a hierarchical structure formation process. Dark matter halos form first, through gravitational collapse, providing the scaffolding for baryonic matter to cool and condense into stars and galaxies. However, recent observations from the James Webb Space Telescope (JWST) of massive, mature galaxies at only 250-300 million years after the Big Bang seem to challenge this scenario [18].

The rate of star formation in these galaxies appears inconsistent with the cooling timescales of baryonic gas expected under the ΛCDM model. These galaxies show signs of well-developed morphologies, such as disk-like or compact structures, which are surprising given the chaotic environments expected during early galaxy formation.

Geons, hypothesized as localized configurations of spacetime, offer an alternative mechanism for structure formation. Unlike dark matter, geons are fundamentally gravitational phenomena and may have unique properties that make them ideal candidates for explaining early galaxy formation.

In this hypothesis, geons could form very early in the universe's history, shortly after the inflationary phase ended ($\approx 10^{-32}$ seconds). During this period, quantum fluctuations in the spacetime fabric - amplified by inflation - could generate regions of intense curvature. These regions might stabilize into geons through quantum gravitational effects. Importantly, geons could form seconds to minutes after the Big Bang, far earlier than the formation of large dark matter halos (between 100000 yrs to 300 Myrs after the Big Bang). Geons forming almost immediately after inflation, could begin clustering well before recombination. Once baryonic matter decoupled from radiation, geons would already have deep potential wells in place, potentially allowing them to attract baryonic matter more efficiently than dark matter halos. This earlier gravitational influence might explain how galaxies could form more rapidly and maturely than expected under the standard dark matter model.

The size and "depth" of a geon (related to its curvature profile and energy density) could determine the amount of baryonic matter it captures. This provides a physical basis for the observed relation between galaxy size and mass.

5. Geons as a Solution to the Cusp-core Problem

The Navarro-Frenk-White (NFW) profile [19], which describes the density of dark matter halos based on simulations, predicts a cuspy central density scaling as $1/r$. At small radii, this results in a steep rise in the density near the halo center. In contrast, observations of dwarf galaxies and low-surface-brightness galaxies often show a core, where the density flattens to a roughly constant value near the center [20]. This discrepancy challenges the validity of the standard cold dark matter model or suggests that additional physical processes (e.g., baryonic feedback, alternative dark matter models, or geons) are needed to explain the observations [21]. One of the most promising solutions to this cold dark matter issue is the stellar feedback mechanism but it seems to be only designed for gas-rich dwarfs, while the problem still remains for gas-poor dwarf spheroidal galaxies. Here, geons with a Gaussian density profile could provide a natural solution since the gradient of the density becomes zero at the centre.

6. Discussion

The geon solutions rely implicitly on assumptions about the quantum vacuum's properties. While the vacuum is often treated as homogeneous and isotropic, its complexity at microscopic scales suggests the potential for anisotropic effects, such as direction-dependent pressures. These effects could play a significant role in the stability and structure of geons and might have observable consequences in galaxy dynamics. Understanding the interplay between these effects and the broader cosmic environment remains an intriguing direction for future research.

Geons also offer testable predictions that could distinguish them from standard dark matter models. Their smooth density profiles may produce gravitational lensing patterns and rotation curves subtly different from those predicted by dark matter halos with Navarro-Frenk-White profiles. Additionally, their role in early galaxy formation could leave imprints on the large-scale structure of the universe or in the properties of high-redshift galaxies observed by telescopes like the James Webb Space Telescope.

Their behavior across different scales, ranging from dwarf galaxies to galaxy clusters, requires deeper analysis. Integrating geons into the broader cosmological framework, particularly in relation to dark energy and cosmic expansion, is also an open question.

7. Conclusions

It has been shown that a geon type solution exists to the Einstein Field Equations with a Gaussian density profile and for which the radial pressure is minus the energy density, being reminiscent of a vacuum equation of state. Real geon solutions could exist within the framework of modified or extended gravity theories. Such galactic sized geons can take over the role of dark matter, being distributed gravitational structures with a significant mass content. It has been shown that the observed relatively flat galaxy rotation curves can be explained when considering the galaxies to be embedded in large geons. In particular, the rotation curve for the Milky Way has been modelled using a simple baryonic mass profile embedded in a geon. Besides explaining the flat rotation curves, the geon concept can also offer an explanation to the observation of mature galaxies in the early universe, and it offers also a solution to the core-cusp problem in gas-poor dwarf spheroidal galaxies.

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