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DYNAMICS: FROM ARCHITECTONICS TO GEOMETRY

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Abstract

In the light of the architectonical approach developed in a previous article, we derive the three rational points of view developed progressively in the history of physics. They are obtained in two steps that reveal a certain hierarchy between them. The architectonical framework is first decoupled, providing thus the geometrical point of view which in turn leads, naturally and simultaneously, to two other points of view corresponding to the variational and group theoretical formulations.

1. Introduction

A first article [1] relative to dynamics was devoted to the development

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of a unifying architectonical Leibnizian formulation that includes the quantitative solutions usually derived by use of analytical principles (variational, geometrical, group theoretical …), each revealing one point of view. A second article [2] went further, showing the possibility of deriving, on an equal footing, the analytical structures that lead to these solutions. Thus, the formal structures that correspond to different analytical principles become theorems (i.e., deduced simultaneously instead of being postulated separately as usually done).

In this third article, a different strategy is adopted where a certain hierarchy is revealed in the passage from the architectonical approach to the analytical formulations. Indeed, by decoupling the architectonical formulation, appears a dynamical point of view which turns out to be rich enough to derive the space-time metrical structure and to allow deducing, in a natural way, the two points of view corresponding to the variational and group theoretical formulations. This point of view, obtained by the decoupling procedure, will give rise to the geometrical formulation expressible in two different (vector and scalar) versions.

In Refs. [1, 2], dynamics is obtained, in $(1 + 1)$ dimensions, through a constraint C imposed on the second-order operator O^2 $(O = Id/dx$ is a generator of conserved entities). One determines thus the two conserved entities (energy E and impulse p), required to get a well-posed physical problem $(C = O^2E$ with $p = OE)$. Similarly to a previous work, we start from the same initially coupled architectonical structure, derived in Ref. [2]:

$$C = E/c^2 = O^2E = Id/dx\,[\,IdE/dx\,] = I^2d^2E/dx^2 + I[\,dI/dx\,]dE/dx$$

$$\text{with} \quad p = OE = IdE/dx\,. \qquad (1)$$

Eqs. (1) are under-determinate [indeterminate as to the points of view $(O = Id/dx$, I being an arbitrary function of $x)$ and determinate for the worlds: here Einstein's world ($C = E/c^2$, the constraint C coinciding with the so-called "relativistic mass" $M = E/c^2$, as shown in [2])].

Thanks to a filtering procedure, the indeterminate points of view, expressed through the couple: (I, x) of non-conserved entities are eliminated in favor of the conserved entities represented by the couple: (E, p). This procedure leads to the determinate (easily integrable) structure:

$$C = E/c^2 = pdp/dE \Rightarrow E = (E_0{}^2 + c^2p^2)^{1/2} \tag{2}$$

which is independent of any point of view. As to the architectonical under-determinate second-order differential (points of view dependent) structure, given in (1), it is formally cumbersome and mathematically complicated to solve and to integrate. It is possible, however, to simplify it, as shown in [2], by introducing two new complementary entities F and G, having the same dimension as E. The three identifications: $G = E$, $G = F$ and $F = E$ led to three well-determinate points of view that turned out to be structurally identical to those postulated by the variational, the geometrical and the group theoretical formulations, developed progressively in the history of physics.

While in Ref. [2], the three points of view has been put at the same "horizontal" level where they appear on an equal footing ($G = E$, $G = F$ and $F = E$), thanks to the introduction of the simplifying entities F and G, here the same under-determinate structure, given in Eqs. (1), is treated in a different way (with no additional simplifying entities). Appears then a certain verticality and hierarchy between the different points of view. Precisely, one benefits from the fact that the initial coupled under-determined structure (1) includes, among an infinity of potential points of view, a singular one that renders this structure decoupled. Thus, instead of resorting to new simplifying entities, introduced from outside, as in [2], the structure is examined from within to clear the pertinent articulations it reveals.

The point of view resulting from the decoupling procedure turns out to coincide with the geometrical point of view. One also realizes that this formulation which appears initially into a hybrid form (partly vector, partly scalar), can be expressed in a unified manner through a purely scalar formulation where the duality notion plays a central role. This scalar formulation presents different advantages, among which the capacity of deriving the space-time metrical structure and the possibility of deducing naturally and directly two other well-known points of view. Such a deductive passage from the geometrical point of view to two other rational ones, provides this point of view a certain priority and centrality: any inverse passage cannot be achieved without postulating additional hypotheses. We finally show how to extend this formulation from $(1 + 1)$ to $(1 + 3)$ dimensions.

In brief, unlike [2] where the three points of view are derived on an equal footing, here appears a certain hierarchy where the architectonical approach precedes and determines the geometrical method which in turn precedes and determines the variational and the group theoretical formulations.

2. Decoupling Procedure and Determination of the Corresponding point of view

The substitution of $p = OE$ into $C = O^2E$, both given explicitly in Eqs. (1), transforms these equations into:

$$C = E/c^2 = I\,dp/dx \qquad \text{with} \qquad p = I\,dE/dx \tag{3}$$

from which one deduces: $C/I = dp/dx$. By equating it to a constant m (identified with the mass concept), the dynamical structure becomes decoupled. Thus, the indeterminate couple (I, x) becomes determinate, noted by (D, u). This yields the differential structure:

$$C/D = dp/du = m \quad \text{with} \quad p = D\,dE/du \quad \text{and} \quad D = E/mc^2 \tag{4}$$

that may also be written as:

$$C = E/c^2 = mD = D\,dp/du \tag{5}$$

with

$$p = D\,dE/du \Leftrightarrow DdE - pdu = 0. \tag{6}$$

One deduces thus:

$$E = mc^2 D \quad \text{and} \quad p = mu, \tag{7}$$

where the constant, resulting from the integration of $dp/du = m$, vanishes, as a consequence of the limit condition: $u = 0,\ p = 0$. As to the expression of D, corresponding to: $D = (1 + u^2/c^2\ )^{1/2}$, it is obtained by combining (7) and (2), after having identified E_0 with mc^2 $(E_0 = mc^2)$.

In order to establish a link between the decoupled point of view and the geometrical one, let us note that the proportionality relation: $p = mu$ implies:

$$pdu = udp. \tag{8}$$

Its substitution into (6) leads to the symmetrical form:

$$DdE - udp = 0 \tag{9}$$

at the basis of the duality notion between (D, u) and (dE, dp), which can be written in a unified compact form: $\mathbf{u} \cdot \mathbf{dp} = 0$. This is a characteristic feature of the geometrical approach developed in the next two Sections 3 and 4, where one shows how the above decoupled point of view coincides with the geometrical formulation, presented in its vector and scalar versions.

3. Link of the Decoupled point of view to the Geometrical Formulation

One first notes that Eqs. (7) and (9) can be expressed in the unified compact form:

$$\mathbf{p} = m\mathbf{u} \qquad \text{such that} \qquad \mathbf{u} \cdot \mathbf{dp} = 0 \tag{10}$$

with the notations:

$$(E/c,\ p) = \mathbf{p} = \{p^{\alpha}\} \quad \text{and} \quad (cD,\ u) = \mathbf{u} = \{u^{\alpha}\} \quad \text{with} \quad \alpha = 0, 1, \tag{11}$$

where the scalar product $\mathbf{a} \cdot \mathbf{b} = a_{\alpha} b^{\alpha} = \eta_{\alpha\beta} a^{\beta} b^{\alpha}$ satisfies the Minkowskian signature: $\eta = (1, -1)$.

One deduces from (10) the expression: $\mathbf{u} \cdot \mathbf{du} = 0$, the integration of which yields: $\mathbf{u} \cdot \mathbf{u} = c^2$, where the constant of integration (noted c^2) has the dimension of a velocity squared. Eq. (10) may thus be written as:

$$\mathbf{p} = m\mathbf{u} \qquad \text{such that} \qquad \mathbf{u} \cdot \mathbf{u} = c^2. \tag{12}$$

One recognizes here the usual structure of Einsteinian dynamics, written in a compact geometrical form. Let us underline that the scalar product $\mathbf{u} \cdot \mathbf{u} = c^2$, usually based on the space-time metrical structure is, at this point, independent of space-time which will be derived from dynamics after having replaced the infinitesimal form: $\mathbf{u} \cdot \mathbf{dp} = 0$, derived in (10), by its finite counterpart: $\mathbf{u} \cdot \mathbf{F} = 0$ as shown below through Eqs. (16)-(20).

4. The Vector and Scalar Versions of the Geometrical Formulations

Let us deduce the dynamical structures corresponding to the vector and scalar versions of the geometrical formulation, intimately related to the Anglo-Saxon and Continental traditions, respectively represented by Newton and d'Alembert.

4.1. Fundamental principle of dynamics (vector Newtonian approach)

On setting:

$$\mathbf{F} = d\mathbf{p}/d\tau \qquad \text{and} \qquad \mathbf{a} = d\mathbf{u}/d\tau\,, \tag{13}$$

where τ is some parameter whose physical interpretation will be clarified later on through (18) and (19), one may rewrite Eqs. (10) or (12) as follows:

$$\mathbf{F} = m\mathbf{a} \qquad \text{such that} \qquad \mathbf{a}\cdot\mathbf{u} = 0. \tag{14}$$

The scalar product: $\mathbf{u}\cdot\mathbf{u} = c^2$, given in (12), transforms into the orthogonal relation $\mathbf{a}\cdot\mathbf{u} = 0$. The vector expression: $\mathbf{F} = m\mathbf{a}$ corresponds to the so-called "fundamental principle of dynamics", initiated by Newton through his vector point of view, but applied here to Einsteinian dynamics.

4.2. Principle of virtual power (scalar d'Alembertian approach)

The above vector $(\mathbf{F} = m\mathbf{a})$ and scalar $(\mathbf{a}\cdot\mathbf{u} = 0)$ expressions given in (14) can be unified into a scalar formulation, using the notion of a virtual field (here a virtual motion $\mathbf{u}^*$) that goes back to d'Alembert. Thus, the vector expression: $\mathbf{F} = m\mathbf{a}$ may be written in a scalar form: $(\mathbf{F} - m\mathbf{a})\cdot\mathbf{u}^* = 0$, provided one assumes its validity for any virtual motion $\mathbf{u}^*$. Thus, the structure corresponding to Eqs. (14) - partly vector partly scalar - transforms into a purely scalar structure:

$$(\mathbf{F} - m\mathbf{a}) \cdot \mathbf{u}^{*} = 0 \qquad \text{such that} \qquad \mathbf{F} \cdot \mathbf{u} = 0 \tag{15}$$

known as the principle of virtual power [based on the duality notion between kinematical and dynamical entities $\mathbf{u}$ and $\mathbf{F}$], largely used in the French schools of mathematics and theoretical mechanics, where the author received his initial scientific training and developed his first research in continuum physics [6, 7]. This scalar (also called energy) principle, usually postulated and dealt with in direct connection to space-time physics, is not postulated here: it is deduced from Leibniz's architectonical approach.

Brief recall. The duality between $\mathbf{u}$ and $\mathbf{F}$ (or its infinitesimal counterpart: $\mathbf{dp}$), postulated in the geometrical method through: $\mathbf{u} \cdot \mathbf{F} = \mathbf{0}$ (or its infinitesimal counterpart: $\mathbf{u} \cdot \mathbf{dp} = 0$), is now deduced. It takes its source in the general relation: $p = I\, dE/dx$ particularized by the decoupling procedure that leads to: $p = D\, dE/du$ with $pdu = udp$. It is the combination of these two relations that yields: $DdE - udp = 0$ or equivalently the compact form: $\mathbf{u} \cdot \mathbf{dp} = 0$, given in (10).

5. Emergence of the Space-Time Metrical Structure from Dynamics

While the space-time notions: (r, t) and their metrical structure are primary in the conventional (variational and geometrical) formulations, they emerge here from architectonical dynamics. The metrical structure will result from: $\mathbf{u} \cdot \mathbf{dp} = \mathbf{u} \cdot \mathbf{F}d\tau$, derived from (10) and (13), where one uses the commutativity property: $\mathbf{u} \cdot \mathbf{F} = \mathbf{F} \cdot \mathbf{u}$, getting thus:

$$\mathbf{u} \cdot \mathbf{dp} = \mathbf{u} \cdot \mathbf{F}d\tau = \mathbf{F} \cdot \mathbf{u}d\tau = \mathbf{F} \cdot \mathbf{dx}, \tag{16}$$

where we have set:

$$\mathbf{dx} = \mathbf{u}d\tau \qquad \text{or} \qquad \mathbf{u} = \mathbf{dx}/d\tau. \tag{17}$$

Accounting for $\mathbf{u} = \mathbf{dx}/d\tau$ and $\mathbf{u} \cdot \mathbf{u} = c^2$ given in (17) and (12), one gets the space-time metrical structure:

$$\mathbf{dx} \cdot \mathbf{dx} = c^2 d\tau^2 \tag{18}$$

written in a Minkowskian compact form or equivalently in an explicit Lorentzian form:

$$c^2dt^2 - dr^2 = c^2d\tau^2, \tag{19}$$

where we have set:

$$\mathbf{dx} = \{dx^\alpha) = (dx^0, dx^1) = (cdt, dr). \tag{20}$$

6. Opening to new Investigations

In order to pave the way for the application of the scalar version of the geometrical formulation to other possible worlds, one shows that Eqs. (15), should be extended as follows:

$$(\mathbf{F} - m\mathbf{a}) \cdot \mathbf{u}^* = 0 \qquad \text{such that} \qquad \mathbf{f} \cdot \mathbf{u} = 0 \tag{21}$$

with

$$\mathbf{f} = d\mathbf{p}/d\tau, \quad \mathbf{F} = d\mathbf{P}/d\tau \quad \text{and} \quad (E/c, p) = \mathbf{p} = \{p^\alpha\}$$

$$\text{and} \quad (cC, p) = \mathbf{P} = \{P^\alpha\}, \quad \alpha = 0, 1. \tag{22}$$

Obviously, when the Einsteinian world is considered: $C = E/c^2$, one recovers the structure given in (15) since one has then: $\mathbf{p} = \mathbf{P}$ and $\mathbf{f} = \mathbf{F}$, but if one considers other worlds, as those developed in [1] and reflected by the constraint C, these equalities are not valid anymore. As an exercise, one may derive the Newtonian dynamical world by setting:

$C = m$ (instead of $C = E/c^2$).

In addition to the fact that the architectonical framework constitutes a safe basis for the geometrical formulation, in its two versions (vector and scalar), let us emphasize that the scalar version is actually more general and universal than the vector version. It does not only lead naturally to two other points of view, as shown in Section 7, but it also turns out to be universal (applicable to other dynamical worlds). Indeed, unlike the vector version based on $\mathbf{F} = m\mathbf{a}$ with $\mathbf{a} \cdot \mathbf{u} = 0$ which is none other than the differential form of $\mathbf{u} \cdot \mathbf{u} = c^2$, corresponding to Einstein's space-time structure: $\mathbf{dx} \cdot \mathbf{dx} = c^2 d\tau^2$, the scalar version, based on the duality notion: $\mathbf{f} \cdot \mathbf{u} = 0$, given in (21), corresponds to $DdE - udp = 0$ from which one deduces, by setting: $v = u/D$, the first Hamilton canonical equation $v = dE/dp$ which is universal (applicable to any dynamical world).

7. Natural Emergence of two Additional points of view

Let us show how the above decoupled point of view, that led to the geometrical formulation, allows deriving, in a natural way, two other points of view, the structures of which turn out to be identical to those of the variational and group theoretical formulations.

Since the combination of: $p = D\, dE/du$ with $udp = pdu$ (valid only for the decoupled point of view) leads to the two equivalent forms:

$$DdE - pdu = 0 \qquad \text{and} \qquad DdE - udp = 0 \tag{23}$$

their division by D yields the two remarkable and singular properties:

$$dE - pdw = 0 \qquad \text{and} \qquad dE - vdp = 0 \tag{24}$$

with two new parameters w and u, defined by:

$$dw = du/D \quad \text{and} \quad v = u/D. \tag{25}$$

It is remarkable that the parameters w and v, coincide, respectively, with the rapidity and the velocity [1-5]. These are usually given by the two formulations based on group theory and the calculus of variations while they originate here from the geometrical formulation which in turn takes its source in the architectonical approach, through a decoupling procedure. Let us finally underline the structural richness of (23) that leads naturally and immediately to (24) while the inverse passage is not possible without recourse to additional hypotheses.

8. Quantitative Determination of the three points of view

By combining (2), satisfying $E_0 = mc^2$, with $p = mu$, $p = dE/dw$ and $v = dE/dp$ deduced from $(7)_2$, and (24), respectively and after some calculations and formal manipulations, one is left with:

$$p = mu, \qquad E = mc^2(1 + u^2/c^2)^{1/2}, \tag{26}$$

$$p = mc\sinh(w/c), \qquad E = mc^2\cosh(w/c), \tag{27}$$

$$p = mv/(1 - v^2/c^2)^{1/2}, \qquad E = mc^2/(1 - v^2/c^2)^{1/2}, \tag{28}$$

where the parameters u, w and v indicate the points of view attached to the celerity, the rapidity and the velocity, respectively. Let us recall that the three structures given in Eqs. (26), (27) and (28) are deduced here from the architectonical approach while they are usually derived by postulating three different physical principles, mathematically expressed by the geometrical, group theoretical and variational formulations, respectively.

9. From $(1+1)$ to $(1+3)$ Dimensions

This derivation of dynamics in $(1+1)$ dimensions - which generalizes the investigations performed by authors like Barbour, Landau, Sampanthar, Lévy-Leblond, Provost and Comte, recalled in Refs. [1-6] of our synthetic paper [1] - can be extended to $(1+3)$ dimensions.

Regarding the form of (2), obtained by use of the filtering procedure that led to the (points of view independent) structure: $E/c^2 = pdp/dE$, the expression: pdp should be replaced by: $p_i dp_i$ with $i = 1, 2, 3$ ($p_i dp_i = p_1 dp_1 + p_2 dp_2 + p_3 dp_3$). After integration, the expression of energy: $E = (E_0{}^2 + c^2 p^2)^{1/2}$ transforms into: $E = [E_0{}^2 + c^2 p_i p_i]^{1/2}$ with $p_i p_i = p_1{}^2 + p_2{}^2 + p_3{}^2$.

For the extension of the spatiotemporal points of view, one replaces u and v, by u_i and v_i where the definition: $v = u/D$, with $D = (1 + u^2/c^2)^{1/2}$ transforms into: $v_i = u_i/\Delta$ with $\Delta = (1 + u_i u_i/c^2)^{1/2}$.

As to the extension of Eqs. (26)-(28), after adopting the simplifying notation:

$$u_c = (u_i u_i)^{1/2}/c, \quad w_c = (w_i w_i)^{1/2}/c \quad \text{and} \quad v_c = (v_i v_i)^{1/2}/c \qquad (29)$$

one is left with:

$$p_i = mu_i, \qquad E = mc^2(1 + u_c{}^2)^{1/2}, \qquad (30)$$

$$p_i = mw_i[\sinh(w_c)/w_c], \qquad E = mc^2 \cosh(w_c), \qquad (31)$$

$$p_i = mv_i/(1 - v_c{}^2)^{1/2}, \qquad E = mc^2/(1 - v_c{}^2)^{1/2}. \qquad (32)$$

By eliminating u_i, w_i and v_i from (30)-(32), one recovers the basic relation: $E = mc^2(1 + p_i p_i / m^2c^2)^{1/2}$, linking together the conserved entities E and p.

One also verifies that: $p = D\, dE/du$ transforms into: $p_i = \Delta\, \partial E/\partial u_i$. This extension attached to the celerity also applies to the rapidity w (satisfying: $p = dE/dw$) and the velocity v (verifying: $p = (1 - v^2/c^2)\, dE/dv$ as shown in Eqs. (9)-(11) of Ref. [2]), leading thus to: $p_i = \partial E/\partial w_i$ and $p_i = (1 - v_i v_i / c^2)\, \partial E/\partial v_i$. These considerations will be developed extensively in a future work, devoted to a systematical study of architectonical dynamics in $(1 + 3)$ dimensions.

Thanks to this extension from $(1 + 1)$ to $(1 + 3)$ dimensions, the above results reveal that Eqs. (10)-(22) remain formally identical, except that $\alpha = 0, 1$ with the signature: $\eta = (1, -1)$ transform into: $\alpha = 0, 1, 2, 3$ with the signature: $\eta = (1, -1, -1, -1)$.

10. Conclusion

Besides its generality and unifying character, the architectonical approach violently contrasts with the analytical ones based on kinematics, corresponding to spatiotemporal definitions [attached to coordinate time for $v_i = dr_i/dt$ (variational method) and invariant time for $u_i = dr_i/d\tau$ (geometrical method)]. Here, dynamics is autonomous and does not require any space-time consideration like for the variational and geometrical points of view, usually founded on the spatiotemporal constraints imposed by the Lorentz invariance. On the contrary, as shown above, the space and time notions, the Lorentz invariance and the different points of view can be deduced from the Leibnizian architectonical approach instead of being postulated, leading thus to a better foundation of physics.

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