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CORRECTIONS AND MORE INSIGHTS FOR WEAKLY Po, T_0-IDENTIFICATION *P*, AND THEIR NEGATIONS

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Abstract

Within this paper, needed corrections are made for weakly *P*o spaces and properties, T_0-identification P properties, and their negations and recent results are applied that make the weakly *P*o process straightforward, quick, and easy.

1. Introduction and Preliminaries

T_0-identification spaces were introduced in 1936 [9].

Definition 1.1. Let (X, T) be a space, let R be the equivalence relation on X defined by xRy iff $Cl(\{x\}) = Cl(\{y\})$, let X_0 be the set of R equivalence classes of X, let $N : X \to X_0$ be the natural map, and let $Q(X, T)$ be the decomposition

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topology on X_0 determined by (X, T) and the map N. Then $(X_0, Q(X, Y))$ is the T_0-identification space of (X, T).

Within the 1936 paper [9], T_0-identification spaces were used to further characterize metrizable spaces.

Theorem 1.1. *A space* (X, T) *is pseudometrizable iff* $(X_0, Q(X, Q(X, T)))$ *is metrizable*.

In 1975 [8], T_0-identification spaces were used to further characterize Hausdorff spaces.

Theorem 1.2. *A space* (X, T) *is weakly Hausdorff iff* $(X_0, Q(X, T))$ *is Hausdorff* [9].

Within the paper [1], the metrizable and Hausdorff properties were generalized to weakly *P*o properties.

Definition 1.2. Let *P* be topological properties such that *P*o = (P and T_0) exists. Then a space (X, T) is weakly *P*o iff its T_0-identification space $(X_0, Q(X, T))$ has property *P*. A topological property *P*o for which weakly *P*o exists is called a weakly *P*o property [1].

In the 1936 paper [9], it was shown that for each space, its T_0-identification space has property T_0. Thus, for a topological property *P* for which *P*o exists, a space is weakly *P*o iff its T_0-identification space has property *P*o.

Within the paper [1], it was shown that for a weakly *P*o property *Q*o, a space is weakly *Q*o iff its T_0-identification space is weakly *Q*o, which led to the introduction and investigation of T_0-identification *P* properties [2].

Definition 1.3. Let *S* be a topological property. Then *S* is a T_0-identification *P* property iff both a space and its T_0-identification space simultaneously shares property *S*.

In the introductory weakly *P*o property paper [1], it was shown that weakly *P*o is neither T_0 nor "not-T_0", where "not-T_0" is the negation of T_0. The need and use of "not-T_0" revealed "not-T_0" as a useful topological property and tool, motivating the inclusion of the long-neglected properties "not-P", where P is a topological property for which "not-P" exists, as important properties for investigation and use in the study of topology. As a result, within a short time period, many new, important, fundamental, foundational, never before imagined properties have been discovered, expanding and changing the study of topology forever.

As an example, the existence of the least of all topological properties was never before even imagined in the study of topology, but in the paper [3], the use of T_0 and "not-T_0" revealed that $L = (T_0$ or "not-T_0") is the least of all topological properties. As another example, in the paper [4], topological properties P and "not-P", where both P and "not-P" exist, were used to quickly and easily prove there is no strongest topological property revealing yet another important, foundational, never before known property within the study of topology.

As is often the case, the existence of something never before imagined creates problems, as was the case in topology. The knowledge and investigation of the least of all topological properties *L* revealed needed changes in the definitions of product properties [5] and subspace properties [6] in order to preserve continuity in the study of each of those properties. Within those two papers, new properties and examples, never before imagined, for each of the two properties were given, not only making the study of each of those two properties more complete and accurate, but, also, expanding the knowledge and extending the frontier in the study of topology.

Below, recent discoveries [7] are used to correct past errors in the study of weakly *P*o and related properties and combined with other known properties to continue their study.

2. Weakly *P*o Corrections and Additional Insights

In the 1936 paper [9], a "special topological property *W*" was sought such that if a space has property *W*, then its T_0-identification space is metrizable, which then implies the initial space has property *W*. As given above, for metrizable, the "special topological property *W*" is pseudometrizable. The same is true for the characterization of T_2 given in the 1975 paper [8]. The same pattern continued in the study of weakly *P*o spaces and properties. However, the search for that "special topological property *W*" for a hopefully, correctly chosen weakly *P*o property *Q*o was uncertain and limited, making the process tedious, at best, and never ending. Thus, the question of whether there was a shortcut for the weakly *P*o space and property search process arose.

The continued work on weakly *P*o spaces and properties and the knowledge and insights gained from that work led to the following answers of the above question [7].

Answer 2.1. Let Q be a topological property for which both Qo and (Q and "not-T_0") exist. Then Q is a T_0-identification P property that is weakly *P*o and Q = weakly Qo = (Qo or (Q and "not-T_0")) [7].

Answer 2.2. $\{Q \mid Q$ is a T_0-identification P property$\} = \{Q$o$\mid Q$o is a weakly Po property$\} = \{Q$o $\mid Q$ is a topological property and Qo exists$\}$ [7].

Answer 2.3. $\{Q \mid Q$ is a T_0-identification P property$\} = \{Q \mid Q$ is a weakly Po$\} = \{Q \mid Q$ is a topological property and both Qo and (Q and "not-T_0") exist$\}$ [7].

Thus, for a topological property such as compact, which can be both T_0 and "not-T_0", compact is a T_0-identification P property that is weakly *P*o with compact = weakly (compact)o = ((compact)o or (compact and "not-T_0")) and, unlike the previous search process, the process is certain, and quickly and easily done.

Since for L = (T_0 or "not-T_0"), both (L and T_0) and (L and "not-T_0") exist, then L is a T_0-identification P property that is weakly *P*o and Lo = T_0 is a weakly *P*o property. Within the paper [3], it was thought that T_0 failed to be a weakly *P*o property, which was then used to conclude that L is not weakly *P*o. A second look at the work in the paper [3] revealed the use of faulty logic and, thus, the statement above concerning L is a needed correction in the study of weakly Po spaces and properties.

By the results above, if Q is a topological property for which $Q = Q$o, then $Q = Q$o is a weakly Po property, but $Q = Q$o is not a T_0-identification P property or weakly Po. Within the recent paper [7], a topological property W that can be both T_0 and "not-T_0" was given that is a T_0-identification P property that is weakly Po such that W = weakly Qo, again making the search process certain, and quick and easy.

Definition 2.1. Let Q be a topological property for which Qo exists. A space (X, T) has property QNO iff (X, T) is "not-T_0" and $(X_0, Q(X, T))$ has property Qo.

In the recent paper [7], it was shown that QNO exists and is a topological property, and $W = (Q$o or $QNO)$ is a T_0-identification P property that is weakly Po with W = weakly Qo. Applying the process for metrizable gives W = pseudometrizable = (metrizable or (pseudometrizable)NO) = (metrizable or (pseudometrizable and "not-T_0")) is a T_0-identification P property that is weakly Po such that pseudometrizable = weakly (pseudometrizable)o = weakly (metrizable). Thus, for a topological property Q for which Qo exists and $Q = Q$o, there is a certain, quick, and easy answer to the question, which can be made more understandable knowing the "special topological property W" for which W = weakly Qo. Within the literature, there are many known such W.

Since L is the least of all topological properties and L is a T_0-identification P property that is weakly Po, then $\{Q \mid Q$ is a T_0-identification P property$\} = \{Q \mid Q$ is weakly Po$\}$ has least element L.

3. A Needed Correction for the Negation of Weakly *P*o

As stated above, in the paper [3], it was thought that L was not weakly Po and, thus, for each topological property Q that is weakly Qo, "not-(weakly Qo)" exists, which now is known not to be true since "not-L" does not exist. Thus, to correct the statements in the paper [3] concerning "not-(weakly Qo)", L would have to be removed from consideration and the conclusion "weakly Qo $\neq L$" would have to be removed, as given below.

Theorem 3.1. *Let Q be a weakly Po different from L. Then "not-(weakly Qo)" exists and is a topological property, both (Q and T_0) and (Q and "not-T_0") exist, ("not-Q")o = (("not-(Qo)"o, weakly (("not-Q")o) exists, weakly ("not-Q")o = weakly (("not-(Qo)"o) = "not-(weakly Qo") $\neq$ weakly Qo, and ("not-Q)o $\neq$ Qo.*

Theorem 3.2. *Let Q be a topological property different from L. Then Q is weakly Po iff* "not-Q" *is weakly Po.*

In the result below, $\{L\}$ was added.

Theorem 3.3. $\mathcal{D} = \{L\} \cup \{(Q, \text{"not-}Q\text{"}) \mid Q$ *is weakly Po$\}$ is a decomposition of all topological properties that are weakly Po.*

If $\{Q \mid Q$ is weakly Po$\}$ had a strongest element S, then S would imply both pseudometrizable and "not-pseudometrizable", which is a contradiction. Thus $\{Q \mid Q$ is weakly Po$\}$ has no strongest element.

The results above are used to quickly, easily, and correctly give weakly ("not-Q")o for a topological property $Q \neq L$ that is weakly Po.

Theorem 3.4. *Let Q be a topological property that is weakly Po and different from L. Then weakly ("not-Q")o = (("not-Q")o or (("not-Q") and "not-T_0")).*

Proof. Since Q is not L, then "not-Q" exists and since weakly ("not-Q")o exists, then both ("not-Q")o and (("not-Q") and "not-T_0") exist. Thus weakly ("not-Q")o = (("not-Q")o or (("not-Q") and ("not-T_0"))).

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