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CALCULATION OF THE QUANTUM FINE STRUCTURE CONSTANT VIA THE STEFAN BOLTZMANN RADIATION LAW

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Abstract

We make the direct derivation of the SB (Stefan-Boltzmann) T^4 radiation law by a series of successive approximations, based on classical models of radiation emission and scattering from thermally agitated electrons, perturbed slightly by collective effects, with the constraint of electric charge quantization, then also modified by an effective ensemble emissivity, due to quantum suppression of radiation rates. This results in the expression the SB radiation law that includes α^{-1} explicitly $F_{SB} = ((3\pi/(80\alpha))^{4/3} ck^4 / (hc)^3) T^4$ and agrees with correct results to within 1 ppm. This agreement requires the highly accurate Wyler Formula to be operative: $\alpha^{-1} = (10/3\pi)(32\pi^5/15)^{3/4} = 137.0361$. In addition, this analysis strongly suggests the quantization of action, the founding concept of quantum mechanics, is actually emergent, being born from the principle of minimum action, many particle dynamics and the quantization of electric charge, in the Hadron Era of the Big Bang.

Keywords and phrases: Planck's constant, fine structure constant, Stefan Boltzmann law, big bang.

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1. Introduction

We attempt here to derive the Stefan-Boltzmann T^4 radiative power law based on classical models of ensemble systems at a condensed matter-vacuum interface, perturbed by thermal collective effects and a semi-classical effective emissivity representing quantum suppression of radiation rates by the ensemble.

It is expected, intuitively, that aside from the phenomena of quantum radiation suppression, that the other parts of the model will be basically classical versions of terms that differ only by factors due to collective effects that are close to one. That is, we expect that we can recover the SB radiation law from slightly perturbed isolated particle dynamics models, assuming a thermal distribution with quantum effects entering primarily as an effective emissivity factor applied to the whole ensemble.

We will approach this direct derivation of the SB radiation law by a series of successive approximations.

The calculation of alpha was first proposed by Wyler [1] who obtained the highly accurate formula:

$$\alpha^{-1} = \frac{10}{3\pi}\left(\frac{32}{15}\pi^{5..}\right)^{3/4} = 137.0361 \qquad (1)$$

which matches experimental measurements: $\alpha^{-1} = 137.03599$ to within 0.6 ppm.

However, attempts to derive this formula from physical models have not been previously found. However, the author has noted that the $32/15\,\pi^5$ portion of the Wyler formula is very similar to the dimensionless portion of the SB (Stefan-Boltzmann) constant, suggesting that semi-classical physical derivation might exist. Such a path of derivation by physical models may have been found, and is here reported.

The author realizes that calculation of the inverse of α using Black Body physics should require complex mathematic models involving distributions over frequency and many particle interactions in a partially ionized condensed matter environment. Yet, in its limiting ideal case, of a perfect Black body, the resulting radiated power per unit area of a vacuum interface is independent of the density and detailed composition of the condensed matter. This law also requires no relativistic effects to be in action since it is part of everyday life. This suggests that a simple physical modeling path can exist where errors in one part of the model cancel errors in other parts, with the ideal radiation law emerging. Similarly, the existence of a compact expression for α^{-1}, that is highly accurate, suggests that a modeling pathway exists where errors in various simple models cancel. It is such a path that we will attempt to find.

We will approach this direct derivation of the SB radiation law by a series of successive approximations. We first begin with a simple conceptual model based on merely identifying classical and quantum-collective factors, then advance to approximate estimates of such factors, then finally to best approximations models where hopefully, errors in one part of the model will cancel errors in other parts leading to highly accurate expressions. Such an approach, mirrors, in microcosm, the entire process of theoretical physics in human history.

2. Conceptual Model Formula

The derivation begins, with a physical model of photon-electron scattering, the *Zitterbewegung* where the Heisenberg uncertainty for an electron is modeled as a series of absorption and emission events occurring as it sits in a sea of ZPF radiation. An electron absorption time is considered to be $\tau_{abs} = r_e / c$, where $r_e = e^2 / m_e c^2$ is the electron classical radius. Following this absorption, the electron heads off on a new trajectory, but then after a mean free path distance $\lambda = \alpha^{-1} r_e$, a Compton radius, it emits a photon and again changes its trajectory. We can therefore, define a quantum emissivity of a condensed matter-vacuum interface: $\varepsilon' \cong \alpha$ (see Appendix 1).

Defining an emissivity as a ratio of photon emission rate per unit area divided by the absorption rate per area at a thermal equilibrium we can, based on the *Zitterbewegung* model, consider α to be an effective emissivity for an electron. This is opposed to a classical emissivity of unity based on Thompson scattering where an electron simply emits radiation as soon as it absorbs it. We will term this effect the "quantum-suppression" of radiation by electrons, as in the extreme case of the ground-state hydrogen atom.

We would then expect the SB law for radiated power per unit area of a hot condensed matter material P_{SB} at an interface with a vacuum to be approximately,

$$F_{SB} \cong 2\varepsilon' P_L' n' \delta' \cong \frac{2}{15} \pi^5 c \frac{(kT)^4}{(hc)^3}, \tag{2}$$

where the factor of two represents the inclusion of both polarizations, P_L' is Larmor classical radiated power per thermally agitated electron, perturbed by collective effects due the fact that the electrons are surrounded by other perturbed electrons, which also move when one electron moves. In the physical model we will adopt, the effective radiating density n' represents the density of electrons free to move, absorb energy in electron-electron collective interactions and then radiate it, in classical manner, being an overlapping population of free and loosely bound electrons in the condensed matter. In contrast the effective skin thickness δ' represents the layer thickness of the condensed matter where radiation from the interface with a vacuum occurs, due to a slightly different density of electrons, which we will call n'', which participate in the radiation near the interface, by quantum electron-photon events. We will consider the electron-electron radiation active density n' to be approximately similar to the density quantum active density n'' of electrons in this model so that $n' \approx n''$ even though their dynamics are different in this model.

The electrons move more sluggishly in condensed matter than in isolation, leading to less radiated power per electron. This effect will be treated in our modeling by assigning an effective mass, $m_{eff} \cong m_e$ to the electrons in their dynamics. As will be seen, this effect can be included as a factor on the Larmor classical radiated power per electron, which scales as the acceleration of the electrons squared, which will be reduced by a factor $C_{eff} > 1$ due to a larger effective mass of the electrons $m_{eff} > m_e$.

$$P'_L = \frac{P_L}{C_{eff}}. \quad (3)$$

We also have n' as the effective radiative free electron density and δ' is the effective radiation power mean free path length. This emissivity must include effects of higher order scattering of photons. The factor of approximately $1/136$ (not α) is then the effective emissivity, ε', (see Appendix 1) indicating quantum-suppression of radiation rates, and quantum higher order scattering , whereby

$$F_{SB} \cong 2\varepsilon' P'_L n' \delta' \cong \frac{2}{15} \pi^5 c \frac{(kT)^4}{(hc)^3}, \quad (4)$$

where we can write δ' using our model assumption $n' \approx n''$

$$\delta' \cong \frac{1}{n'\sigma'}, \quad (5)$$

where σ' is the effective cross section of electron-photon scattering, which we would expect to be the Thompson cross section with a small modification factor due to collective effects. Leading to the desired approximate expression, which is independent of density:

$$F_{SB} \cong \frac{2\varepsilon' P_L}{\sigma' C_{eff}}. \tag{6}$$

Let us now, attempt to arrive at more exact formula based on simple physical and mathematical models consisting of single particle dynamics modified by both collective and quantum effects:

Assumption 1. The radiation can be considered as due entirely due to electron-electron and photon-electron dynamics in a fixed positive background. That is, the electrons do the radiating, because they move and belong to two overlapping populations, one is involved in electron-electron interactions and another, in primarily electron-photon scattering. The radiating electron population is both free and bound electrons. It must also be remembered that the electrons that are "most free" will do most of the radiating and scattering of radiation.

Assumption 2. The next fundamental assumption guiding this model analysis is the assumption of quantization of dynamics, so that particles interact as discrete entities in groups based on the integers of charge Z unit cells of interaction scaling as Z^2. That is, quantization of charge, and quantization of electrodynamics are intimately connected. Thus factors of dynamic interaction will have the form Z^2 / m_{eff} where Z must be an integer, even if m_{eff} is not a mass quantum. That is, Z is a globally conserved quanta even if effective mass in not.

Assumption 3. The infinite range of electric forces means that every electron effects and is affected by every other electron in a condensed matter environment, regardless of whether the electrons are "bound" or "free", similarly quantum physics means also that particles are both discrete and yet simultaneously part of a continuous quantum field. That is, we make the assumption in our modeling that no clear boundary exists between classical collective and quantum electrodynamics in condensed matter.

With these fundamental assumptions in mind we will now proceed to derive the SB radiation law based on models of radiation and radiation scattering. As mentioned before we will approach this by successive approximations. The goal of the first approximations, is to giving approximate numbers, for ε', σ' and C_{eff}.

We begin with the Larmor expression for radiated power for isolated electrons, which we assume are thermally agitated.

$$P_L = \frac{2}{3}\frac{e^2}{c^3}a^2 = \frac{2}{3}\frac{e^2}{c^4}ca^2, \tag{7}$$

where we assume the acceleration a is due to thermal collisions in a sea of other electrons. We have for isolated electrons

$$a = \frac{e^2}{m_e r_{th}^2}, \qquad r_{th} = \frac{e^2}{kT}. \tag{8}$$

This model leads immediately to the T^4 dependence of the radiation rate:

$$a^2 = \frac{(kT)^4}{e^{8.}}\frac{e^4}{m_e^2}. \tag{9}$$

We now consider the effect of nearby electrons, leading to $m_e \to m_{eff}$

$$a'^{\,2} = \frac{(kT)^4}{e^{8.}}\frac{e^4}{m_{eff}^2}. \tag{10}$$

We can write this in terms of an effective radiating cross section

$$c^4\,\frac{(kT)^4}{e^{8.}}\frac{e^4}{m_{eff}^2c^4} = c^2\,\frac{(kT)^4}{e^{8.}}\frac{r_e^2}{C_{eff}}, \tag{11}$$

where $C_{eff} = (m_{eff}/m_e)^2$ is the factor due to the heavier effective mass of the electrons due to collective effects.

We then define for an effective radiating cross section

$$r_{eff}^2 \cong \frac{r_e^2}{C_{eff}}. \tag{12}$$

We therefore have a model of effective radiated power per electron due to electron-electron interactions.

We have then for our conceptual radiation power formula, where a density of thermally agitated electrons in a skin layer δ' radiates to create the Stefan Boltzmann radiation law

$$F_{SB} = 2\varepsilon' P_L' n\delta', \tag{13}$$

$$\delta' \cong \frac{1}{\sigma' n}, \tag{14}$$

$$n'\delta' \cong \frac{1}{\sigma'}, \tag{15}$$

where we would expect that the effective cross section σ' to have a value of approximately the Thompson cross section

$$\sigma' \cong \frac{8\pi}{3} r_e^2. \tag{16}$$

This then should reduce to an expression for the SB Law where we have density of Larmor radiating electrons, whose rate of radiation is reduced by collective effects. This gives the electrons a slightly higher effective mass, leading to factor $C_{eff} > 1$ and also an effective emissivity $\varepsilon' < 1$ due to the quantum suppression of radiation rates. So we have a conceptual formula for the radiation rate per unit area at a vacuum interface.

$$F_{SB} = \frac{2\varepsilon'}{\sigma'}\frac{P_L}{C_{eff}}. \tag{17}$$

3. First Approximation Model

We now will try to determine, to first approximation, values for the factors ε' and C_{eff}. We assume here that since the EM force has infinite range charged particles will interact as clusters in unit cells rather than directly as a density.

Here we adopt the physical model of electrons moving in a fixed positive background, with electron interacting as pairs but with three body and four body interactions also contributing and adding to the effective mass of the moving electrons. Thermally agitated electron-electron interactions with collective effects, independent of density, and since are assuming the electrons are interacting as pairs, we assume a reduced mass for electron-electron collisions $m_{e\text{-}e} \to m_e / 2$. Using a planar wave collective electrostatic behavior of electrons in a plasma

$$\frac{e^2}{m_{eff}} \cong \frac{\omega^2_{pe\text{-e}}}{(2\pi)^2 n_e}. = \frac{4\pi e^2 n_e}{(2\pi)^2 (m_e /2) n_e}. \tag{18}$$

The effective mass for electron-electron thermal scattering model is thus approximately

$$\frac{e_2}{m_{eff}} \cong \frac{4\pi e^2 n_e}{(2\pi)^2 (m_e /2) n_e} = \frac{2e^2}{\pi m_e}. \tag{19}$$

$m_{eff} = (\pi/2) m_e$, takes into account collective behavior of the electrons and the ion background. This is the “many body problem” which has no known exact solutions, only model calculations. In the dynamics model we will use, it is the electrons that move, not the ions. This leads to an increase in the effective dynamic mass of the electrons, due to the motion of the other electrons in concert with the original electron, and also pairs of other electrons and triples and so on. This leads to a reduction of the value of the scattering cross section of approximately $r_{eff}^2 \cong (2/\pi)^2 r_e^2$, therefore we assume an r_{eff}^2. This leads to a value for C_{eff}, in the first approximation due to electron effective mass effects,

$$r_{eff}^2 = \frac{r_e^2}{C_{eff}} \cong \frac{r_e^2}{(\pi/2)^2}. \quad (20)$$

We have then also the effective emissivity ε' (see Appendix 1)

$$\varepsilon' \cong 1/136. \quad (21)$$

So we have, in first approximation, again:

$$F_{SB} \cong \frac{2\varepsilon'}{\sigma'} \frac{P_L}{C_{eff}}, \quad (22)$$

where, in this first approximation, we will have $\varepsilon' \cong 1/136$ and $C_{eff} \cong (\pi/2)^2$ and we assume for now that σ' is approximately the Thompson cross section. Therefore, in this first approximation model, and with the exception of the factor due to quantum suppression of radiation rates, the SB law is a result of an ensemble of thermally agitated electrons in a positive background with dynamics only slightly perturbed by collective effects.

4. A More Precise Model

Stefan Boltzmann Law F_{SB} requires a radiation pressure at the radiating interface, with the constraint that it, like the SB radiation law itself, must be independent of density. We add the collective and quantum effects with a Guant factor g_q' much less than one. It is here we shall add the effect of quantum suppression of radiation rates by means of a physical mechanism, knowing that it will appear explicitly in the final approximate expression.

$$P'_L = \frac{2}{3} c \frac{e^2}{c^4} \frac{(kT)^4}{e^8} \frac{e^4}{m_e^2} g'_q = \frac{2}{3} c \frac{(kT)^4}{e^6} \frac{e^4}{m_e^2 c^4} g'_q, \tag{23}$$

$$P_L = \frac{2}{3} c \frac{(kT)^4}{e^6} \frac{e^4}{(\pi/2)^2 m_e^2 c^4}. \tag{24}$$

We now use the identity

$$e^2 = \frac{\alpha hc}{2\pi} \tag{25}$$

and obtain, a close approximation to the exactly known law

$$F_{SB} = \frac{2}{15} \pi^5 c \frac{(kT)^4}{(hc)^3}. \tag{26}$$

Force balance, since it cannot depend on particle density at the level of radiation pressure, is interpreted to mean that radiation pressure equals an adiabatic thermodynamic pressure due to quantum uncertainty. This means this part of the equilibrium at the vacuum interface effect must be independent of density and temperature and so must depend on quantum effects affecting each photon-electron interactions. Therefore, in this model the effect of quantum mechanics enters explicitly, aside from charge quantization, in the requirement of an adiabatic pressure balance between the electron population and the radiation pressure due to emitted radiation at a vacuum-to matter interface.

We have then an adiabatic quantum pressure equilibrium, where the $5/3$ exponent is consistent with an adiabatic pressure equilibrium with the radiation pressure: $P \propto n^{5/3}$

$$\frac{\varepsilon' P'}{Ac} = 2 \frac{2}{3} 8\pi^3 \alpha^{-3} \frac{(kT)^4}{(hc)^3} r_e^2 K^{-4/3} \left(\frac{\alpha}{C}\right)^{5/3} = \frac{P_{SB}}{c}. \tag{27}$$

We therefore choose a Gaunt factor:

$$g'_q = K^{-4/3}\left(\frac{\alpha}{C}\right)^{5/3} \tag{28}$$

which contains both quantum and collective electrodynamic effects, and two undetermined constants C, and K. We have then our model, with the expanded terms

$$P'_L = \frac{4}{3} 8\pi^3\alpha^{-3}c\,\frac{(kT)^4}{(hc)^3}\,\frac{r_e^2}{C^{1/3}}\,\frac{\alpha^{5/3}}{C^{4/3}}\,K^{-4/3}. \tag{29}$$

For this physical model, where only electron-electron dynamics contributes to radiation, only electrons interacting in groups of three, can produce radiation, single electrons and electron-electron dynamics not producing dipole EM radiation. This gives us (see Appendix 2) in order to obtain the same approximate number C_{eff} as before, we must have $Z^2 = C = 16$, meaning $Z = 4$ in a unit cell of interaction in order to agree with our model first approximate estimate for C_{eff} and so $C^{1/3} \cong 2.52$ which closely approximates $(\pi/2)^2 = 2.47$ but it is an algebraic number as required. Therefore, we have identified the value of C for this model, based on its dependence of Z^2, where Z is the charge quantum for a unit interaction. We will assume that the mass of electrons for photon-electron scattering is just m_e and thus $r_{eff} = r_e$. We then identify, in our conceptual formula:

$$\frac{\sigma'}{\varepsilon'} = \frac{8\pi}{3}\left(\frac{4\pi}{3}\,\frac{2}{3}\,r_e^3\right)^{2/3} K^{4/3}C^{4/3} \tag{30}$$

the effective scattering cross section for photon-electron scattering is the Thompson cross section times the density per unit area of the electrons $n_e^{-2/3}$ per unit cell. (See Appendix 3).

The effective electron-photon scattering cross section σ' is due to a different effective density n'' than that causing dynamic radiating density n'. The unit cell of each electron involved in electro-photon scattering is of volume $4\pi/3\, r_e'^3$ where $r_e'^3 = Z'^2 r_e^3$, a geometric mean of r_e^2 and $Z'^2 r_e$ where Z' is an effective Z value for the electrons in this electron-photon scattering environment (see Appendix 3). The effective Z' will assume in this model that Z' is integer, and enters as Z'^2 as did Z^2, earlier. This electron-photon scattering model we are assuming in this model that the electrons always act like charge quantum states.

We obtain then a parallel expression for the ratio

$$\frac{8\pi}{3}\left(\frac{8\pi}{9}\right)^{\frac{2}{3}} Z'^{\frac{4}{3}} r_e^2 = \frac{8\pi}{3}\left(\frac{4\pi}{3}\,\frac{2}{3}\,r_e^3\right)^{2/3} K^{4/3} C^{4/3}. \tag{31}$$

Requiring

$$K^{4/3} C^{4/3} = Z'^{\,4/3}. \tag{32}$$

Accordingly, the quantum radiation pressure term appears as an effective integral charge quantum state in the unit cell of the electron density for photon-electron density.

We have then for our best approximation of the SB radiation law under the assumptions of this model

$$F_{SB} = \frac{\frac{4}{3}\,8\pi^3 r_{eff}^2 \alpha^{-3} c\,\frac{(kT)^4}{(hc)^3}\,\alpha^{5/3}}{\frac{8\pi}{3}\,4\pi^{\frac{2}{3}}\left(\frac{1}{3}\right)^{4/3} Z'^{\,4/3} r_e^2}, \tag{33}$$

where $r_{eff}^2 = r_e^2 / C_{eff}$ so we have for $C_{eff} = 2^{4/3}$

$$F_{SB} = \frac{\frac{4}{3}8\pi^3 r_e^2 \alpha^{-3} c \frac{(kT)^4}{(hc)^3} \alpha^{5/3}}{\frac{8\pi}{3} 4\pi^{\frac{2}{3}} \left(\frac{1}{3}\right)^{4/3} r_e^2 C_{eff} Z'^{\,4/3}}. \tag{34}$$

We then identify

$$\frac{8\pi}{3} 4\pi^{\frac{2}{3}} \left(\frac{1}{3}\right)^{4/3} r_e^2 = \sigma', \tag{35}$$

where this quantity is approximately twice the Thompson Cross section σ_{th}

$$\sigma' \cong 2\frac{8\pi}{3} r_e^2. \tag{36}$$

We then simplify the expression using $\alpha^{-3} = \alpha^{-9/3}$

$$F_{SB} = \frac{\alpha^{-4/3}(3\pi)^{4/3} c \frac{(kT)^4}{(hc)^3}}{C_{eff} Z'^{\,4/3}}. \tag{37}$$

We thus have for $C = 16$ and the requirement that Z' is an integer, where it will be seen we are estimating the value of $1/\varepsilon' \cong 136 \cong (40)^{4/3}$. Allows us to make the identification

$$1/\varepsilon' = Z'^{\,4/3}, \tag{38}$$

$$K^{4/3} C^{4/3} = Z'^{\,4/3}. \tag{39}$$

We find the integer value of Z' that best yields our approximate value of $1/\varepsilon'$. We have already determined the value of C in this model as 16. So we have, in order to obtain approximate agreement with our provisional value of $1/\varepsilon' = 136$ we must have

$$16K = Z' = 40. \tag{40}$$

This determines the value of K

$$K = \frac{5}{2}. \tag{41}$$

So that we recover

$$Z'^{\,4/3} = (40)^{4/3} = 136.798. \tag{42}$$

Thus, we have for our best approximation of F_{SB}, following the form of Eq. 22, with the values $1/\varepsilon' = Z'^{\,4/3} = 136.798$ and $C_{eff} = 2^{4/3} = 2.52 = (1.59)^2$ and finally $\sigma' \cong 1.98\sigma_{th}$, all of which is consistent with initial assumptions concerning the direct derivation of the SB T^4 radiation law.

We have then the Stefan Boltzmann law of radiation flux from the surface of a hot body in our second approximation:

$$F_{SB} = \left(\left(\frac{3\pi}{80\alpha}\right)^{4/3} c\,\frac{(k)^4}{(hc)^3}\right)T^4. \tag{43}$$

This can be written, defining the Stefan Boltzmann constant, σ_{SB}, as $F_{SB} = \sigma_{SB}T^4$ where $\sigma_{SB} = 5.67037e - 08\ \mathrm{Wm^{-2}K^{-4}}$ (CODATA).

Our physical model calculation gives the value $\sigma_{SB} = 5.6703698e - 08\ \mathrm{Wm^{-2}K^{-4}}$. So this is within $1\,\mathrm{PPM}$ of the CODATA value.

Therefore, based on a physical model of classical thermally agitated electron-electron radiative collisions, perturbed by collective effects with the constraint of quantized charge, and assuming quantum suppression of radiation from the classical ensemble in condensed matter we can obtain the SB radiation law to high precision.

This highly accurate agreement between the model calculation and the CODATA value of σ_{SB} requires that the Wyler formula for alpha be operative:

$$\left(\frac{3\pi}{80\alpha}\right)^{4/3} = \frac{2}{15}\pi^5, \tag{44}$$

$$\frac{3\pi\alpha^{-1}}{80} = \left(\frac{2}{15}\pi^5\right)^{3/4}. \tag{45}$$

We have then, in this physically reasonable model the requirement:

$$\alpha^{-1} = \frac{10}{3\pi}\left(\frac{32}{15}\pi^5\right)^{3/4}. \tag{46}$$

5. Physics Interpretation

The fact that we can derive a highly accurate expression for the Stefan-Boltzmann radiation law based on simple but physical models, which are themselves based on physically reasonable assumptions is profoundly meaningful. It suggests that quantum electrodynamics and collective classical electrodynamics, i.e., plasma physics, are intimately connected and cannot be separated. It also suggests that quantization of action is intimately connected with quantization of electric charge. Accordingly, since it is possible to write $\alpha^{-1} = hc/(2\pi e^2)$

$$h = \frac{20}{3}\left(\frac{32}{15}\pi^5\right)^{3/4}\frac{e^2}{c} \tag{47}$$

suggesting that electric charge quantization and EM interactions may underlie quantization of action through spacetime geometry since the charge quantum can be detected independently of the quantum of action.

Finally since in the GEM theory [4, 5] a central number is $\sigma = (m_p / m_e)^{1/2} = 42.85$ with $m_p / m_e = 6\pi^5$ [1] we can write

$$\hbar = \frac{80}{3\pi}\left(\frac{6}{45}\pi^5\right)^{3/4}\frac{e^2}{c}. \tag{48}$$

We can achieve a complete of time variables from geometric factors and electric charge

$$\hbar c = 8\frac{10}{3\pi}\left(\frac{\sigma^2}{45}\right)^{3/4} e^2. \tag{49}$$

This yields

$$\hbar c = 8\frac{10}{3\pi}\left(\frac{\sigma}{3\sqrt{5}}\right)^{3/2} e^2 \tag{50}$$

or slightly less accurately:

$$\hbar c \cong 8\sigma^{3/4} e^2. \tag{51}$$

This suggests the interpretation that $\hbar c$ is a geometric projection of charge quantization via the "unfolding" of a tesseract (4-cube) into 8 3-cubes. This expression raises the interesting question of whether charge quantization, $\pm e$, a consequence of the compact nature of the 5th Kaluza-Kline dimension in the GEM theory, [4, 5] is more fundamental than the quantization of action $\hbar$. In an early Big Bang universe, we would expect the principle of minimum action to be obeyed, with Maxwell's Equations, leading to the selection the smaller "Electric Action" e^2/c. Quantum Mechanics could thus be considered an "emergent" phenomena stemming from the primordial quantization of charges, both electron and quarks, from a GEM-like Big Bang 5-D cosmology, and their subsequent collective dynamics. The latter phenomenon defining the local "arrow of time" via the Second Law of Thermodynamics [6]. The appearance of the proton-electron mass ratio, $6\pi^5$ in the formula for α, can be considered as a signature of the emergence of the larger Action Quantum $\hbar$ in the Hadron Era of the Big Bang, with the coincident appearance of the proton. Thus, the emergence of the larger Action Quantum mirrors the appearance of the larger stable mass quantum than the electron, which is the proton.

Appendix 1

We are making the assumption that quantum effects cause an effective emissive factor ε' to be applied to our semi-classical calculations of radiation rates.

$$F_{SB} \cong \frac{\varepsilon'}{\sigma'} \frac{P_L}{C_{eff}}, \tag{A1.1}$$

where we assume, for the purposes of this calculation that C_{eff} is the effective mass coefficient due to electron-electron collective effects. Let us assume that the quantum suppression of radiation rate can be described by the QED-consistent series

$$F_{SB} \cong (\alpha + a_1\alpha^2 \ \ldots) \frac{P_L}{C_{eff}\sigma'}, \tag{A1.2}$$

where we assume that second order process of absorption and emission contribute to the radiation rate from the surface. We can then write

$$F_{SB} \cong \alpha(1 + a_1\alpha \ \ldots) \frac{P_L}{C_{eff}\sigma'} \tag{A1.3}$$

which we can approximate to first order, assuming second order absorption emission is weaker at the interface than first order so $a_1 \cong 1$

$$F_{SB} \cong \frac{\alpha}{(1 - a_1\alpha)} \frac{P_L}{C_{eff}\sigma'}, \tag{A1.4}$$

$$F_{SB} \cong \frac{1}{137(1 - a_1/137)} \frac{P_L}{C_{eff}\sigma'}. \tag{A1.5}$$

Therefore, to first order we expect, for $a_1 \cong 1$

$$F_{SB} \cong \frac{1}{(137 - a_1)} \frac{P_L}{\sigma'}. \tag{A1.6}$$

So we would expect the quantum suppression factor ε' to be approximately

$$\varepsilon' \cong \frac{1}{136}. \tag{A1.7}$$

Appendix 2

We are making the physical assumption that electrons both free and loosely bound to atoms are radiating power as they are accelerated. The fact that neighboring electrons must participate in these dynamics can accounted for assigning the radiating electrons an effective mass greater than one due to the dynamics of neighboring electrons. We can model this in a similar way to a viral expansion in terms of contributions from single particle, particle pairs and particle triples at a constant density and temperature.

$$m_{eff} \cong m_e\left(A_1\left(\frac{n}{n_0}\right) + A_2\left(\frac{n}{n_0}\right)^2 + A_3\left(\frac{n}{n_0}\right)^3\right). \tag{A2.1}$$

Higher order contributions may also be present but since we are modeling all accelerations as due to electron-electron interaction in a massive immovable ion background, we will consider that electron scattering clusters of more than three are very unlikely and make negligible contributions. Further we can consider that we can write thus model this as a third order polynomial: single electrons cannot radiate, so we set $A_1 = 0.$ Also pairs of interacting electrons cannot radiate except as quadrupole or higher at much reduced rates compared to dipole radiation so we also set $A_2 = 0.$ This leaves then the effective mass of the radiating electrons to only be due to three body effects, so for model $A_3 = 1,$ where we consider $n = \ell^{-3},$ ℓ is an inter-electron distance for radiating particles where this means $n^3 = \ell^{-9}$

$$1 = A_3\left(\frac{n}{n_0}\right)^3, \tag{A2.2}$$

$$m_{eff} = A_3 m_e. \tag{A2.3}$$

Based on quantization of charge Z in this model we assume this coefficient A_3 should be integer Z to the $2/3$ power, so we have

$$m_{eff}{}^2 = Z^{4/3} m_e^2. \tag{A2.4}$$

And thus

$$C_{eff} = Z^{4/3}. \tag{A2.5}$$

The value of integer Z that gives a number closest to the first approximation value for $C_{eff} \cong 2.46$ is $Z = 2$. Therefore,

$$m_{eff}{}^2 = 2^{4/3} m_e^2 \tag{A2.6}$$

leading to

$$m_{eff}{}^2 \cong 2.52 m_e^2. \tag{A2.7}$$

Appendix 3

We model the cross section for electron-photon scattering as the Thompson scattering cross section times the mean scattering electron separation distance ℓ_e'' squared, where we define this as an inverse scattering electron density to the $2/3$ power.

$$\sigma' = \frac{8\pi}{3} n_e''^{-2/3}. \tag{A3.1}$$

Using the Wigner-Sietz Model [2], where density is defined as the inverse of the volume of unit spherical cells, with the expression for the most probable radius due to Chandrasekar [3] $r_{Ch} = (2/3)^{1/3} r_e$ and the ionization factor Z'^2 for each unit cell we obtain. Charge state Z being independent of direction must appear along the two lines of rotational symmetry as a quadratic. That is, a spherical unit cell must have a cross section proportional to Z'^2 where we mandate that Z' must be an integer to satisfy the quantization of charge. Z' can have two physical meanings in this model: as an ionization state of the positive background atoms or simply a number of free electrons per unit cell of volume. We have then

$$n_e''^{-1} = \frac{4\pi}{3}\frac{2}{3} Z'^2 r_e^3. \tag{A3.2}$$

This leads to the expression

$$\sigma'/\varepsilon' = \frac{8\pi}{3}\left(\frac{4\pi}{3}\frac{2}{3} Z'^2 r_e^3\right)^{\frac{2}{3}}. \tag{A3.3}$$

And therefore our scattering cross section in this model differs only slightly from the Thompson cross section for isolated electrons.

$$\sigma' = \frac{8\pi}{3} r_e^2 \left(\frac{8\pi}{9}\right)^{\frac{2}{3}}. \tag{A3.4}$$

That is, in agreement with our assumption that, aside from quantum suppression of radiation rates, the SB law can be derived by mild perturbations of classical, isolated particle dynamics by collective effects, we have, in this model:

$$\sigma' \cong 1.98 \frac{8\pi}{3} r_e^2. \tag{A3.5}$$

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