

ANY ISOLATED NUMBER BELONGS TO A CERTAIN BINOMIAL SERIES

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Abstract

A binomial is an algebraic expression with two dissimilar terms connected by +ve or -ve sign. Binomial Theorem is a way of expanding a binomial expression (that are raised to) large powers. According to Newton's binomial theorem power of any algebraic expression can be expanded in many terms. But so far, it has not been tried to generate a whole binomial theorem or Binomial expression from an isolated single numbers. The reverse is done by me with two pairs of mathematical formulas, i.e., two pairs of formulas have been developed to generate Binomial series/expression from any isolated single given number. Before using these formulas it is needed to arrange the isolated number as a

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binomial term with single variable or two variables. These generated Binomials have many applications in permutation and combination, probability, Matrices and Mathematical Induction also. So, existing given Binomials and here generated Binomials are two different processes, but both are same and have same applications and importance in algebra.

1. Introduction

Special cases of the binomial theorem were known since at least the 4th Century B.C. when Greek mathematician Euclid mentioned the special case of the binomial theorem for exponent 2. There is evidence that the binomial theorem for cubes was known by the 6th Century AD in India.

Binomial coefficients, as combinatorial quantities expressing the number of ways of quantities expressing the number of ways of selecting r objects out of n without replacement, were of interest to ancient Indian Mathematicians. However, the pattern of members was already known to the European mathematicians of the late Renaissance, including Stifel, Niccolo Fontane Tartaglia and Simon Stevin. Issac Newton is generally credited with the generalized binomial theorem, valid for any rational exponent.

Binomial has many applications in mathematical model, statistics, probability, etc. The binomial distribution is used in genetics. To able to come up with realistic predictions, binomial theorem is used also. It has many uses in Distribution of IP Address, National Economic Predictions, architecture, Weather forecasting, etc. So, obtaining binomial series from any isolated number will give benefit in every field.

The binomial theorem is used heavily in statistical and probability Analyses. It is used in Weather Forecast Services, ranking up candidates, estimating cost in engineering projects, etc.

My two formulas are generating algebraic expressions for binomials from any isolated single number. Thus, any isolated single number can show all binomial

properties when it is expressed in binomial theorem with my formulas.

2. Main Text

Binomial theorem, for all $n \geq 1$ and $a, b \in R$

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}.$$

There are multiple proofs of Binomial Theorem. But, my paper has a completely new concept of formation or generation of a Binomial Theorem from any isolated number.

Newton's Binomial Theorem expands the power of single variable and two variables in many terms. But the reverse is to form a Binomial Theorem from any isolated natural number which is made in this paper.

In order to do this back calculation, I arrange the given number as ${}^n C_r x^{n-r}$ (n and r are two positive integers and $n > 1$). Then all previous terms of this number will be obtained by my formula as $r \times \int {}^n C_r x^{n-r} dx$ and all following terms of this number will be obtained by my formula as $\frac{1}{r+1} \times \frac{d}{dx} {}^n C_r x^{n-r}$.

Thus a whole Binomial Series/expression will be obtained from the given number.

Again, if the given number is arranged with two variables x and y as ${}^n C_r x^{n-r} y^r$, then all previous terms of this number will be obtained by my formula as $\int {}^n C_r x^{n-r} dx \times \frac{d}{dy} y^r$ and all following terms of this given number will be obtained by formula $\frac{d}{dx} {}^n C_r x^{n-r} \int y^r dy$.

Thus, a Binomial Theorem is generated from an isolated single number.

3. Application

We know, $(a + b)^2 = a^2 + 2ab + b^2$. It is a Binomial Theorem.

Now let an isolated number $2ab$

Since $2ab = {}^2C_1 a^{2-1} b^1$.

$$\begin{aligned} \text{So, its previous term} &= \int {}^2C_1 a^{1-1} da \times \frac{d}{db} b^1 \\ &= 2 \int da \times \frac{d}{db} b \\ &= \frac{2a^2}{2} \times 1 = a^2. \end{aligned}$$

$$\begin{aligned} \text{And it's following term} &= \frac{d}{da} {}^2C_1 a^{2-1} \int b^1 db \\ &= 2 \frac{d}{da} a \int b db \\ &= \frac{2b^2}{2} = b^2. \end{aligned}$$

Since $a^2 + 2ab + b^2 = (a + b)^2$.

So, my formulas are proved to be true.

4. Numerical Examples

Let, an isolated number is 270.

So, $270 = {}^5C_2 \times 3^{5-2}$.

According to my formula the previous term

$$\begin{aligned}
 &= r \int {}^n C_r x^{n-r} dx \\
 &= 2 \int {}^5 C_2 \times x^{5-2} dx = 2 \times 10 \int x^3 dx \\
 &= 20 \frac{x^4}{4} = 5 \times 3^4 = 405.
 \end{aligned}$$

Again, $405 = {}^5 C_1 3^{5-1}$.

$$\begin{aligned}
 \text{So, it's previous term} &= 1 \int {}^5 C_1 x^{5-1} dx \\
 &= 5 \times \frac{3^5}{5} = 243.
 \end{aligned}$$

According to my formula following term of 270

$$\begin{aligned}
 &= \frac{1}{(r+1)} \frac{d}{dx} {}^n C_r x^{n-r} \\
 &= \frac{1}{(2+1)} \frac{d}{dx} {}^5 C_2 x^{5-2} \\
 &= \frac{1}{3} \times 10 \frac{d}{dx} x^3 \\
 &= \frac{1}{3} \times 10 \times 3x^2 = 10 \times 3^2 = 90.
 \end{aligned}$$

Again, $90 = {}^5 C_3 \times 3^{5-3}$.

$$\begin{aligned}
 \text{So its following term} &= \frac{1}{3+1} \frac{d}{dx} {}^5 C_3 x^{5-3} \\
 &= \frac{1}{4} \times 10 \times \frac{d}{dx} x^2
 \end{aligned}$$

$$= \frac{5}{2} \times 2x = 5x = 15.$$

Again, $15 = {}^5C_4 x^{5-4}$.

$$\text{So its following term} = \frac{1}{(4+1)} \frac{d}{dx} {}^5C_4 x^{5-4}$$

$$= \frac{1}{5} \times 5 \times \frac{d}{dx} x = 1.$$

So, the Binomial Theorem is as

$$\begin{aligned} & 243 + 405 + 270 + 90 + 15 + 1 \\ & = 1024 = (3 + 1)^5. \end{aligned}$$

Thus my formula generates Binomial Theorems/series.

4. Conclusion

There are many important topics of Binomial Theorem, viz, Binomial Theorem for positive Integral Index, Pascal's Triangle, General Term, Middle Term and properties and application of Binomial Theorem. But algebraic generation of Binomial Theorem from isolated number with my two formulas paper is a completely new topic introduced by me in the realm of Binomial Theorem. Actually, we have developed the method of algebraic generation of Binomial Theorem from isolated number with my formulas. This will help to design mathematical models. It will generate much new binomial expressions from isolated numbers to identify the characteristic properties of the numbers. This is equally applied to statistical and Probability Analyses and all other fields where Binomial Theorem is applied. As the Binomial Theorem has played a crucial role in the development of mathematics, algebraic or analytic, pure or applied, so this generation of binomial theorem plays important role. Since 4th Century B.C there have been many evolutions of Binomial Theorem. This generation of Binomial Theorem is a new contribution. It is a new approach to Binomial Theorem.

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