

## **ALGEBRAIC EXPRESSIONS FOR THE SIDES OF TRIANGLES - A SHORT COMMUNICATION**

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### **Abstract**

Algebraic triangle is an act or process of finding the sides of a triangle using algebraic expressions or forming algebraic expressions representing the sides of a triangle. It is a triangle calculator with the formation of algebraic expressions for the sides of a triangle. A triangle is a polygon that has three vertices. The sum of the lengths of any two sides of a triangle is always larger than the length of the third side. For right angled triangles Pythagorean theorem states that the square of the length of the hypotenuse equals to the sum of the squares of the lengths of the two other sides. On the basis of these two properties of triangles, I have developed seven types of algebraic expressions for the sides of scalene, isosceles and right-angled triangles.

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### **1. Introduction**

Geometry and architecture are two disciplines that are fundamentally linked. One of the most recognized geometric shapes is the triangle. The two most common triangular forms used in architecture are equilateral and isosceles. A common example of equilateral triangles used in architecture is the Pyramid complex of Giza in Egypt. Isosceles triangles are also found in architecture throughout the world, especially in modern pyramidal architecture. Isosceles triangles are used in the architecture of the East Building in the National Gallery of Art in Washington, D.C. Scalene triangles are not traditionally used in the structural characteristics of a building. They are, however, vital to the construction and design of the buildings. Right triangles are used to create perfect corners and straight lines. In churches, triangular windows are often featured as window frames or in the stained glass.

Truss Bridges have supporting structures constructed in triangular shapes. Because they evenly distribute the weight without changing the proportions. Triangles have applicability in such areas as astronomy, architecture, engineering, physics, navigation and surveying. Similar triangles can be used to measure how wide a river or lake is. Similar triangles are used in aerial photography to see the distance from the sky to the ground. Triangles can also be used in landscape photography to lead or point the viewer's eye.

Therefore, there are various standard methods for calculating the length of a side or the measure of an angle of a triangle. More complex methods may be required in different situations. So algebraic triangles are developed with algebraic expressions to meet the requirements of mathematics, science, engineering and architecture.

### **2. Main Text**

In ancient India Bhaskaracharya and Brahmagupta gave solutions of the sides of a right angled triangle. Here, I am giving solutions for the sides of scalene, isosceles and right-angled triangles.

Algebraic expressions are used to give solutions for the sides of a triangle.

For scalene triangles, algebraic expressions for the sides of the triangles are given below:

(i)  $m$ ,  $(mn + 1)$  and  $\min\{m(n - 1) + 2\}$  to  $\max\{m(n + 1)\}$  where  $m$  and  $n$  are two positive integers (for irrational or decimal numerical values of  $m$  and  $n$  it also holds true).

(ii)  $(m + n)$ ,  $\{(m + n) + 1\}$  and  $\{(m + n) + 2\}$ ,  $m$  and  $n \in N$ .

(iii)  $(m^a + n^a)$ ,  $(m^a + n^a + 1)$  and  $2(m^a + n^a)$ , where  $m$ ,  $n$  and  $a$  are positive integers.

(iv)  $(m^a + n^a)/(m^a - n^a)$ ,  $\{(m^a + n^a)/(m^a - n^a) + 1\}$  and  $2\{(m^a + n^a)/(m^a - n^a)\}$ .

(v) (An algebraic expression), (that algebraic expression + 1) and 2 (that algebraic expression) where the numerical value of the algebraic expression is positive rational or irrational number.

For isosceles triangles I give following solutions:

(vi)  $(m + 1)$ ,  $(m + n + 1)$  and  $(m + 1)$ , where  $n = -m, \dots, +m$ , i.e.,  $-m \leq n \leq +m$  and  $m$  and  $n$  are positive integers (for irrational values of  $m$  and  $n$  it also holds true).

Brahmagupta gave solutions for right angled triangles as:

(1)  $2mn$ ,  $m^2 - n^2$  and  $m^2 + n^2$  and

(2)  $\sqrt{m}$ ,  $\frac{1}{2}\left(\frac{m}{n} - n\right)$  and  $\frac{1}{2}\left(\frac{m}{n} + n\right)$ , where  $m$  and  $n$  are positives integers.

I give following solutions for right-angled triangle:

(vii)  $\sqrt{m/2}$ ,  $\sqrt{n/2}$  and  $\sqrt{m + n/2}$ , where  $m$  and  $n$  are +ve integers.

**3. Numerical Examples**

(i) Let  $m = 3$ ,  $n = 2$ ,

therefore, the sides are 3, 7 and 5 or 3, 7 and 9.

(ii) Let  $m = 4$ ,  $n = 3$ ,

therefore, the sides are 7, 8 and 9.

(iii) Let  $m = 5$ ,  $n = 3$  and  $a = 2$ ,

therefore, the sides are 34, 35 and 68.

(iv) Let  $m = 5$ ,  $n = 2$  and  $a = 2$ ,

therefore, the sides are  $\frac{29}{21}$ ,  $\frac{50}{21}$  and  $\frac{58}{21}$ .

(v) Let numerical value of an algebraic expression = 2.2,

therefore, the sides are 2.2, 3.2 and 4.4.

(vi) Let  $m = 5$ ,  $n = -5$ ,

therefore, the sides are 6, 1 and 6.

Let  $m = 5$ ,  $n = +5$ ,

therefore, the sides are 6, 11 and 6.

Let  $m = 5$ ,  $n = 4$ ,

therefore, the sides are 6, 10 and 6.

(viii) Let  $m = 18$ ,  $n = 32$ ,

therefore, the sides are 3, 4 and 5.

#### 4. Conclusion

“Solution of triangles” is the main trigonometric problem: to find missing characteristics of a triangle (three angles, the lengths of the three sides, etc.) when at least three of these characteristics are given. Therefore, when the three sides are known, the three angles can be calculated. The triangle can be located on a plane or on a sphere. This problem often occurs in various trigonometric applications, such as geodesy, astronomy, construction, navigation, etc. My algebraic expressions for three sides of a triangle give various triangles of different types. They can be used for various purposes.

#### References

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