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TOPOLOGICAL 1-SOLITON SOLUTIONS FOR THE GENERALIZED ROSENAU-KdV EQUATION

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Abstract

By using the solitary wave ansatz method, the exact topological 1-soliton solutions of the generalized Rosenau-KdV equation are obtained. The parameter regimes are identified for the existence of the above solutions.

1. Introduction

Nonlinear wave phenomena play a major role in various areas of scientific research such as fluid dynamics, optical fibers, nuclear physics, plasma physics, biology, solid state physics, chemical kinematics, chemical physics, and geochemistry. The problem of finding the exact and explicit solutions for the nonlinear evolution equations by using different methods [1-15] is of great importance for mathematicians, physicists and engineers; because these solutions provide physical information to understand the mechanisms of the physical models and help us to verify the numerical and analytic methods. The inverse scattering transform (IST) [1] was the pioneer dominant technique for finding the exact solutions of the nonlinear evolution equations. In recent years, some of the commonly used famous methods in the literature of Mathematical Physics for solving the nonlinear evolution equations are $\left(\frac{G'}{G}\right)$ expansion method [2], Exponential function

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method [3, 4], Homotopy Perturbation method [5], Reductive perturbation technique [6, 7], tanh method and sine-cosine method [8, 9], The tanh-coth method [10], Homotopy analysis method [11], First integral method [12], and Solitary wave ansatz method [13-15].

In this present article, we will use the solitary wave ansatz method to find the topological 1-soliton solutions of the well known generalized Rosenau-KdV equation [9] of the form:

$$u_t + ku_x + \alpha u_{xxx} + \beta u_{xxxxt} + \gamma\left(u^n\right)_x = 0, \tag{1}$$

where u is a real valued function and α, β, γ and k are real numbers. The exponent $n(> 1)$ is the power law nonlinearity parameter. It should be noted that the study of a new model which supports soliton-type solutions is very important and these solutions will be useful for future research work.

In 1988, Philip Rosenau [16] introduced the Rosenau equation of the form:

$$u_t + ku_x + \beta u_{xxxxt} + \gamma(u^2)_x = 0, \tag{2}$$

to describe the dynamics of dense discrete systems. In 2009, Zuo [9] introduced Rosenau-KdV equation (1) and obtained some solitons and periodic wave solutions of (1) using the sine-cosine and the tanh methods. Recently, Esfahani [15] studied the generalized Rosenau-KdV equation (1) by using sec-ansatze method and obtained solitary wave solutions.

2. Topological Soliton Solution

In this section, the solitary wave ansatz method is used to obtain topological 1-soliton solution of the generalized Rosenau-KdV equation (1). The starting hypothesis for a topological 1-soliton solution of equation (1) is

$$u(x, t) = A \tanh^p \tau = B(x - vt), \tag{3}$$

where A and B are called the free parameters for topological solitons and v is the velocity of the topological soliton. The unknown exponent $p(> 0)$ will be determined during the course of the derivation of the topological soliton solution of equation (1). Therefore, from (3), we get

$$u_t = ABpv\{\tanh^{p+1}\tau - \tanh^{p-1}\tau\}, \tag{4}$$

$$u_x = ABp\{\tanh^{p-1}\tau - \tanh^{p+1}\tau\}, \tag{5}$$

$$u_{xxx} = ApB^3\{(p-1)(p-2)\tanh^{p-3}\tau - [(p-1)(p-2)+2p^2]\tanh^{p-1}\tau$$
$$+ [(p+1)(p+2)+2p^2]\tanh^{p+1}\tau - (p+1)(p+2)\tanh^{p+3}\tau\}, \tag{6}$$

$$u_{xxxxt} = -AB^5 pv\{(p-1)(p-2)(p-3)(p-4)\tanh^{p-5}\tau$$
$$-(p-1)(p-2)[(p-3)(p-4)+2[(p-2)^2+p^2]]\tanh^{p-3}\tau$$
$$+\{2(p-1)(p-2)[(p-2)^2+p^2]+p(p-1)[(p-1)(p-2)+2p^2]$$
$$+p(p+1)[(p+1)(p+2)+2p^2]\}\tanh^{p-1}\tau$$
$$-\{2(p+1)(p+2)[(p+2)^2+p^2]+p(p-1)[(p-1)(p-2)+2p^2]$$
$$+p(p+1)[(p+1)(p+2)+2p^2]\}\tanh^{p+1}\tau$$
$$+(p+1)(p+2)[(p+3)(p+4)+2[(p+2)^2+p^2]]\tanh^{p+3}\tau$$
$$-(p+1)(p+2)(p+3)(p+4)\tanh^{p+5}\tau\}, \tag{7}$$

$$(u^n)_x = A^n Bnp\{\tanh^{np-1}\tau - \tanh^{np+1}\tau\}. \tag{8}$$

Substituting (4), (5), (6), (7) and (8) into (1), we get

$$ABpv\{\tanh^{p+1}\tau - \tanh^{p-1}\tau\} + ABkp\{\tanh^{p-1}\tau - \tanh^{p+1}\tau\}$$
$$+ ApB^3\alpha\{(p-1)(p-2)\tanh^{p-3}\tau - [(p-1)(p-2)+2p^2]\tanh^{p-1}\tau$$
$$+[(p+1)(p+2)+2p^2]\tanh^{p+1}\tau - (p+1)(p+2)\tanh^{p+3}\tau\}$$
$$-AB^5pv\beta\{(p-1)(p-2)(p-3)(p-4)\tanh^{p-5}\tau$$
$$-(p-1)(p-2)[(p-3)(p-4)+2[(p-2)^2+p^2]]\tanh^{p-3}\tau$$
$$+\{2(p-1)(p-2)[(p-2)^2+p^2]+p(p-1)[(p-1)(p-2)+2p^2]$$
$$+p(p-1)[(p+1)(p+2)+2p^2]\}\tanh^{p-1}\tau$$
$$-\{2(p+1)(p+2)[(p+2)^2+p^2]+p(p-1)[(p-1)(p-2)+2p^2]$$

$$+\ p(p+1)[(p+1)(p+2)+2p^2]\}\tanh^{p+1}\tau$$

$$+\ (p+1)(p+2)[(p+3)(p+4)+2[(p+2)^2+p^2]]\tanh^{p+3}\tau$$

$$-\ (p+1)(p+2)(p+3)(p+4)\tanh^{p+5}\tau\}$$

$$+\ \gamma A^n Bnp\{\tanh^{np-1}\tau-\tanh^{np+1}\tau\}=0. \tag{9}$$

Now from (9), equating the exponents $np+1$ and $p+5$ gives

$$np+1=p+4 \tag{10}$$

so that

$$p=\frac{4}{n-1}. \tag{11}$$

It is clear that the same value of p can be obtained if the exponents $np-1$ and $p+3$ are equated. Therefore, from (9), the linearly independent functions are $\tanh^{p+j}\tau$ for $j=\pm 1, \pm 3$ and ± 5. Now setting the coefficients of the linearly independent functions to zero, we get the following system of equations:

$$AB^5 pv\beta(p-1)(p-2)(p-3)(p-4)=0, \tag{12}$$

$$AB^3 p(p-1)(p-2)\{\alpha+\beta^2 v\beta[(p-3)(p-4)+2[(p-2)^2+p^2]]\}=0, \tag{13}$$

$$ABp(v-k)+AB^3 p\{(p-1)(p-2)+2p^2\}[\alpha+B^2 v\beta p(p-1)]$$

$$+\ AB^5 vp\beta\{2(p-1)(p-2)[(p-2)^2+p^2]$$

$$+\ p(p+1)[(p+1)(p+2)+2p^2]\}=0, \tag{14}$$

$$ABp(v-k)+AB^3 p\{(p+1)(p+2)+2p^2\}[\alpha+B^2 v\beta p(p+1)]$$

$$+\ AB^5 vp\beta\{2(p+1)(p+2)[(p+2)^2+p^2]$$

$$+\ p(p-1)[(p-1)(p-2)+2p^2]\}=0, \tag{15}$$

$$AB^3 p(p+1)(p+2)\{\alpha+B^2 v\beta[(p+3)(p+4)$$

$$+\ 2[(p+2)^2+p^2]]\}-\gamma A^n Bnp=0, \tag{16}$$

$$AB^5 pv\beta(p+1)(p+2)(p+3)(p+4) - \gamma A^n Bnp = 0. \tag{17}$$

From equations (12) and (13), we get $p = 1$ and $p = 2$.

From (11), it is possible to find that

if $p = 1$, $n = 5$, (18)

while if $p = 2$, $n = 3$. (19)

Hence, topological 1-soliton solutions of the generalized Rosenau-KdV equation will exist for $n = 3$ and $n = 5$. There is no other values of the power law nonlinearity parameter n for which the topological solitons will exist. To the best of our knowledge, this is a very important observation and is reported for the first time in the literature of the generalized Rosenau-KdV equation.

2.1. Case 1. $p = 1$, $n = 5$.

In this case equation (1) becomes

$$u_t + ku_x + \alpha u_{xxx} + \beta u_{xxxxt} + \gamma(u^5)_x = 0. \tag{20}$$

Substituting $p = 1$, and $n = 5$ into (14)-(17) gives

$$v = -\frac{\alpha}{20B^2\beta}, \qquad \beta \neq 0, \tag{21}$$

and

$$B = A^2\sqrt{-\frac{5\gamma}{6\alpha}}, \qquad \alpha\gamma < 0. \tag{22}$$

Thus for $p = 1$, and $n = 5$, the topological 1-soliton solution of Rosenau-KdV equation (20) is given by

$$u(x, t) = A \tanh B(x - vt), \text{ provided } \alpha\gamma < 0, \beta \neq 0, \tag{23}$$

where equation (22) gives the relation between the free parameters A and B and equation (21) gives the velocity of the soliton.

2.2. Case 2. $p = 2$, $n = 3$.

In this case equation (1) becomes

$$u_t + ku_x + \alpha u_{xxx} + \beta u_{xxxxt} + \gamma(u^3)_x = 0. \tag{24}$$

Substituting $p = 2$ and $n = 3$ into (14)-(17) gives

$$v = -\frac{\alpha}{40B^2\beta}, \qquad \beta \neq 0, \tag{25}$$

and

$$A = B\sqrt{-\frac{3\alpha}{\gamma}}, \qquad \alpha\gamma < 0. \tag{26}$$

Thus for $p = 2$ and $n = 3$, the topological 1-soliton solution of Rosenau-KdV equation (24) is given by

$$u(x, t) = A \tanh^2 B(x - vt), \text{ provided } \alpha\gamma < 0, \beta \neq 0, \tag{27}$$

where equation (26) gives the relation between the free parameters A and B and equation (25) gives the velocity of the soliton.

3. Conclusions

In this paper, we have used the Solitary wave ansatz method to obtain the topological 1-soliton solution of the generalized Rosenau-KdV equation. It has been proved that topological solitons exist only for two special forms of Rosenau-KdV equation when $n = 2$ and $n = 3$. Also, the parametric conditions for the existence of the topological soliton solutions are given. It should be noted that solitary wave ansatz method is a powerful efficient method to obtain exact topological and nontopological soliton solutions for nonlinear evolution equations.

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