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TIME-VARYING AND AVALANCHINE MEMORY EFFECT IN ANOMALOUS DIFFUSIONS

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Abstract

We extend the simple non-Markovian walk model by memory enhanced with time which describes underlying mechanism of anomalous diffusions. In the extended models, we consider the competitions between positive and negative memory by controlling a parameter α and avalanche memory effect for various avalanche size. The models show well the anomalous behavior of diffusion including both superdiffusion and subdiffusion like the memory enhanced model, while for small α the steps are much more correlated than for the memory enhanced model. The avalanche memory does not affect the temporal correlation between steps with the same Hurst exponent as the case without avalanches in long time region, while it induces oscillatory temporal correlation in short time region.

1. Introduction

Random walks [1, 2] which were proposed to stochastically formulate transport and diffusion phenomena, have played a key role in statistical physics for over a century. The key quantity characterizing the random walks is the mean squared

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displacement (MSD) $\langle x^2(t)\rangle$ which grows linearly with time. Hurst, however, found the persistence of hydrologic time series indicating that the MSD behaves in nonlinear way [3-5] and in recent, such phenomena have been observed in many different systems such as chaotic [6], biophysical [7-11], economic systems[12, 13], and etc. The nonlinear behavior is recognized as anomalous diffusions compared with the linear behavior that is regarded as normal diffusion, and is characterized in terms of the MSD

$$\langle x^2(t)\rangle \sim t^{2H}. \tag{1}$$

Here $\langle\ldots\rangle$ means average over independent realizations, i.e., ensemble average, in general, in non-equilibrium. H is called as the anomalous diffusion or the Hurst exponent which classifies superdiffusion $(H > 1/2)$ in which the past and future random variables are positively correlated and thus persistence is exhibited, and subdiffusion $(0 < H < 1/2)$ which behaves in the opposite way, showing antipersistence.

A variety of models to describe the mechanism have been proposed [14-18] but they do not give any a universal mechanism but rather suggest very distinct origins, separately. The representative models among them are the fractional Brownian motion (fBM) [14], the Lévy fights [16, 19, 21, 22], and the continuous time random walks (CTRW) [15, 20, 22]. In the fBM, long-ranged temporal correlations between steps is given so that MSD scales like Eq. (1) within the range of $0 < H < 1$, and thus fBM describes both subdiffusion and superdiffusion however, its correlation is mathematically constructed and it shows stationary behaviors unlike nonstationary nature shown in real experiments and systems. Meanwhile other two models mimic further specific systems and describe only one region of anomalous diffusions, respectively. In Lévy fights, step-length distribution follows the power-law asymptotic behavior, so that the average distance per a step is infinite, which invokes superdiffusions. In CTRW model a time interval between two consecutive steps is a continuous random variable which is drawn according to the waiting time distribution (WTD). For the WTD possessing the finite average of waiting time the MSD is linearly dependent on time, that is, the normal diffusive behavior is shown, while for the cases where the WTD behaves asymptotically as power-laws and thus possesses infinite average of waiting time, subdiffusive behaviors are induced.

Recently, various microscopic non-Markovian models with memory effect which may be a novel key origin were proposed. In [23], a walker jumps persistently or antipersistently according to prior steps with a probability parameter and below the critical value of the control parameter, the model shows normal diffusive behaviors while above it, superdiffusive behaviors. Due to its simpleness, the microscopic memory effect, was easily applied to other models, among which Cressoni et. al. suggested that the loss of recent memory rather than the distant past can induce persistence, which is relate to the repetitive behaviors, psychological symptoms of Alzheimer disease [24]. In [25], it was shown that by adding a possibility that a walker does not move at all in the model of [23], diffusive, superdiffusive, and subdiffusive behaviors can exhibit in different parameter regimes. It has advantage to describe the anomalous diffusion within a single model just by changing the parameters, however, in this case, the subdiffusive property may be caused by the staying behavior rather than the memory effect and thus superdiffusion and subdiffusion are not induced by a single origin.

Meanwhile, we proposed the models with time-varying memory and the competition between Markovian and non-Markovian processes which describe anomalous diffusions including super and sub diffusions [26]. In the first model, non-Markovian processes induced by the full memory of entire history and Markovian processes constructed by the original random walk are competed by a probability parameter. In the second model, non-Markovian processes are induced by the latest memory rather than full memory and its realizations vary with time. From these models we found that in the regime where non Markovian nature pre-vails, superdiffusion is induced by the perfect memory, while the latest memory enhanced with time cause subdiffusions as well as superdiffusions. In this paper, we consider another perspective, the competition between persistent and antipersistent behavior and avalanche memory effect, in anomalous diffusions by proposing two novel models.

2. Models and Results

The following non-Markovian stochastic model is proposed, where for $t > 1$, σ_{t+1} is given by

$$\sigma_{t+1} = \begin{cases} -\sigma_t, \text{ with probability } 1/t^{\alpha}, \\ \sigma_t, \text{ with probability } 1 - 1/t^{\alpha} \end{cases} \tag{2}$$

and the walker starts at origin and moves to the right or left with equal probability at time $t = 1.$ Over time, the probability of taking the opposite direction with the latest step decreases and the probability of taking the same direction with the latest step increases. The larger value of parameter α is, the much faster the probability varies with time. The competition between the antipersistence and persistence with the previous step varies with time and its degree is controlled by the parameter $\alpha.$ We shall refer to this model as model A. Meanwhile in Eq. (2) when the probability of persistence and antipersistence are exchanged, the antipersistence will be dominant in the long time limit. We call it model *B*

Figure 1 shows the plot of the MSD versus time *t* for various α and in the inset $2H$ versus α is shown for the model A. The solid lines are the fitting lines of the MSDs for $\alpha = 0.1$ and $\alpha = 0.9$ whose slopes are 1.21 and 1.92, respectively. The line in the inset follows the relation $2H = (1 + \alpha)$ which is the result of the model in ref. [26] where the antipersistent behavior is replaced a random process, i.e., $+1$ or -1 is chosen equally with probability $1/t^{\alpha}.$ Except for $\alpha = 0.1$ the data is good agreement with the line. It indicates that the antipersistence may make the same effect as the random process for large $\alpha,$ but for small α antipersistence strengths the persistent tendency rather than random process.

For the model B, the MSDs are shown in Figure 2 and although they follow the persistent behavior in the early time, they eventually show the antipersistent behavior with growing time. The fitting lines represent $2H \simeq 0.78$ and $2H \simeq 0.17$ for $\alpha = 0.1$ and $0.9,$ respectively. The inset shows the plot of $2H$ versus α and the solid line represents that $2H = 1 - \alpha,$ which the results for the model in [26]. the data deviates from the guide line, that is, the persistent probability affects the antipersistent behavior, on the whole.

We next consider an another aspect of anomalous diffusion by suggesting avalanche memory which mimics successive steps into one direction during a certain interval. It is embodied as an avalanche memory model in which the rule of above proposed model is generalized, the time *t* in the probability of Eq. (2) is replaced by

τ, the moment happening avalanche step of which size s is fixed and thus real time t becomes $t = s * \tau$. That is, during s time interval, a walker successively steps into one direction which is decided by Eq. (2). Such successive steps into one direction would be regarded as successive memory for the previous step in a time interval.

Figure 3 shows the plot of the MSD versus time t with $s = 100$ for the avalanche model A. It shows quite different picture from the case of $s = 1$, the model A. The initial avalanche induces the ballistic diffusion in the range of $0 < t < s$ and the oscillatory behavior is shown after the initial avalanche within a certain period of time, which is invoked by the avalanche steps. Specially, for $\alpha = 0.1$ the oscillatory behavior is vivid, that is, the slower varying probability with time induces the clear oscillation due to the anti-correlation between step avalanches. The inset represents $2H = 1 + \alpha$, which is the same as the result of model A, deviating slightly for just small α from the line.

For the avalanche model B, the MSDs are shown in Figure 4 which also show the ballistic diffusion by the initial avalanche in early time region and small oscillations after ballistic behavior. In the long time limit, it also shows the subdiffusion behavior but the Hurst exponents are not linear for the parameter α and deviated from the solid guide line, $2H = 1 - \alpha$.

To confirm more precisely the relation between the Hurst exponent and the parameter α and the avalanche step effect, we ran the extra simulations where 50 ensembles including the 1000 independent runnings were built for $s = 1, 10, 100$ and 1000, respectively and the 50 values of Hurst exponent obtained from each ensembles are averaged and its standard deviation was calculated. The results were summarized in Table 1 which indicates that the Hurst exponent does not depend on the size of avalanche.

3. Conclusion

In conclusion, we have studied how do the competition between persistent and antipersistent nature and the avalanche step affect on the anomalous diffusion behavior by introducing microscopic non Markovian models. For growing persistence with time, mixing antipersistent and persistent steps with probability invokes random process in part and thus there is no difference between the diffusion by the random process and persistence in certain range of the parameter, however, for the case in which antipersistent behavior hold on somewhat even in the long time limit, the antipersistent plays role of strenghtening the persistence rather than random process by giving the larger Hurst exponent. Meanwhile, for growing antipersistence with time, the persistence nature makes different affections unlike the random process on the diffusive behaviors on the whole. The avalanche step have shown the novel features in anomalous diffusion, the ballistic, oscillatory, and sub or superdiffusion coexist, but the long time limit behavior is the same as that of the case without avalanche.

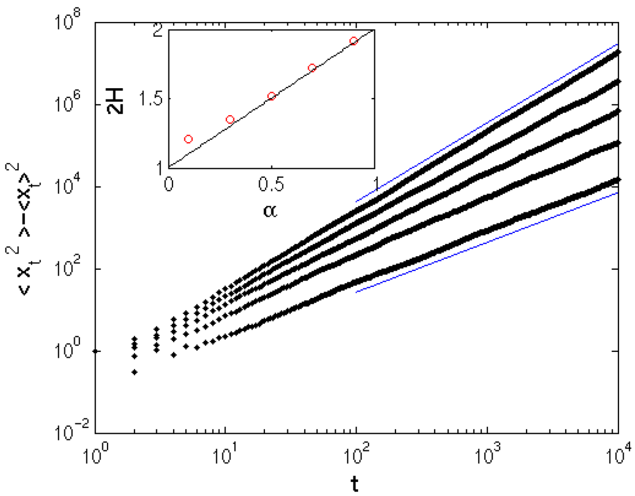

Figure 1. The plot of the MSD $\langle x_t^2 \rangle - \langle x_t \rangle^2$ versus time t for the model A with $\alpha = 0.1, 0.3, 0.5, 0.7,$ and 0.9 from the bottom to the top. The data were measured with the initial condition where a walker moves to the right or left with equal probability $1/2$ and 10^3 independent realizations. The inset shows the plots of the Hurst exponent for various α. The solid line represents $2H = 1 + \alpha$.

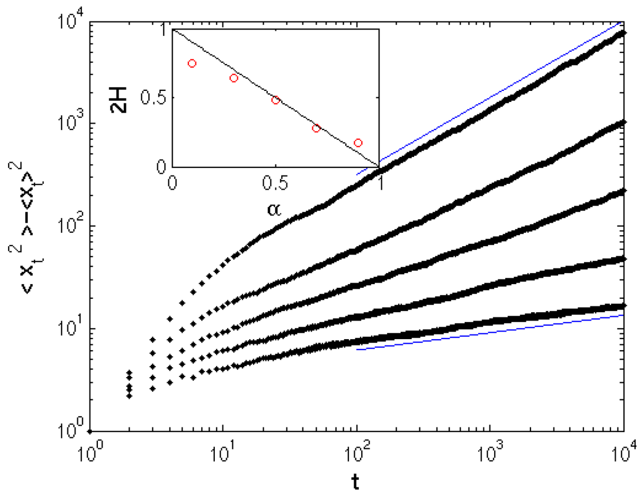

Figure 2. The plot shows the MSD as a function of time for the model B with $\alpha = 0.1,\ 0.3,\ 0.5,\ 0.7,$ and 0.9 from the top to the bottom. The inset shows the Hurst exponent versus the parameter α and the solid line is $2H = 1 - \alpha$.

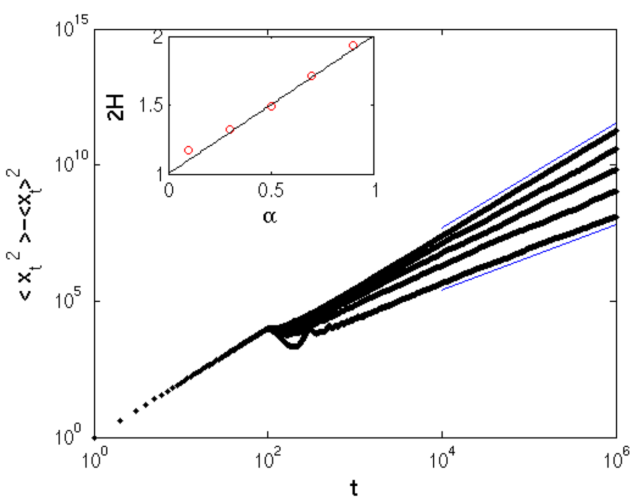

Figure 3. The plot of the MSD $\langle x_t^2 \rangle - \langle x_t \rangle^2$ versus time t for the avalanche model A with $s = 100$ and $\alpha = 0.1,\ 0.3,\ 0.5,\ 0.7,$ and 0.9 from the bottom to the top. The inset shows the plots of the Husrt exponent for various α. The solid line represents $2H = 1 + \alpha$.

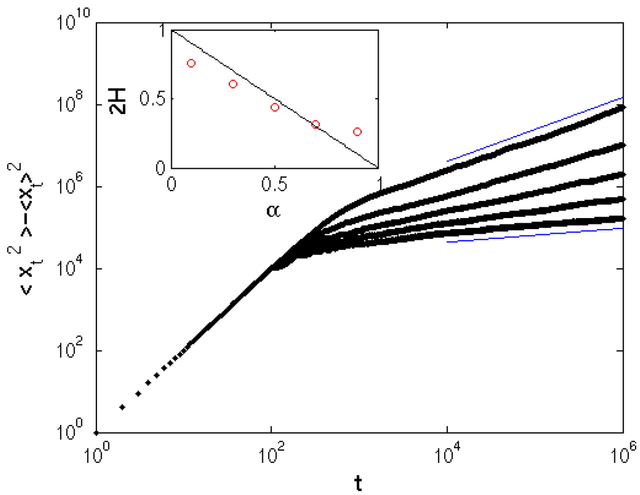

Figure 4. The plot shows the MSD as a function of time for the avalanche model B with $s = 100$ and $\alpha = 0.1, 0.3, 0.5, 0.7,$ and 0.9 from the top to the bottom. The inset shows the Hurst exponent versus the parameter α and the solid line is $2H = 1 - \alpha$.

Table 1. The values of the Hurst exponent for the above mentioned models with the various parameter α

		α				
model	size	0.1	0.3	0.5	0.5	0.9
A	1	1.21(0.022)	1.35(0.020)	1.52(0.019)	1.72(0.021)	1.92(0.014)
	10	1.21(0.019)	1.35(0.020)	1.52(0.017)	1.72(0.018)	1.92(0.016)
	100	1.21(0.018)	1.35(0.019)	1.52(0.021)	1.72(0.021)	1.92(0.016)
	1000	1.21(0.018)	1.35(0.020)	1.52(0.020)	1.72(0.020)	1.92(0.014)
B	1	0.78(0.021)	0.64(0.019)	0.46(0.017)	0.30(0.016)	0.17(0.013)
	10	0.78(0.019)	0.64(0.017)	0.47(0.015)	0.31(0.013)	0.17(0.013)
	100	0.78(0.015)	0.64(0.019)	0.47(0.016)	0.30(0.016)	0.17(0.014)
	1000	0.78(0.018)	0.64(0.017)	0.47(0.019)	0.31(0.015)	0.17(0.010)

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