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A NEW MECHANICS OF CORPUSCULAR-WAVE TRANSPORT OF MOMENTUM AND ENERGY INSIDE NEGATIVE INDEXED MATERIAL

SHANTANU DAS[1,*], SOUGATA CHATERJEE[2], AMITESH KUMAR[2], PAULAMI SARKAR[2], ARIJIT MAZUMDER[2], ANANTA LAL DAS[2] and SUBAL KAR[3]

[1]Reactor Control Division
Bhabha Atomic Research Centre (BARC)
Mumbai, India
e-mail: shantanu@barc.gov.in

[2]Society for Applied Microwave Electronics Engineering
and Research (SAMEER)
Kolkata, India

[3]Institute of Radio Physics and Electronics (IRPE)
University of Calcutta
Kolkata, India

Abstract

A century has passed regarding wave-particle duality, well an electromagnetic (EM) radiation in dispersionless free space vacuum is represented by a photon, with corpuscular and wave nature. The discussions from the past century aimed at the nature of photon inside a media having dispersion in the refraction property, other than free space.

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*Corresponding author

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We call mechanical momentum, wave-momentum, and try to match our 'thought experiments' with intriguing property of this 'photon' or pulse carrying EM energy packet, and more so we try to find its property energy, momentum inside a media, a positive refractive media. Well if the media show a negative refractive index behavior, then these queries are profound, and suitable explanations to these classical concepts of corpuscular-wave nature of photon inside these media are quest for the scientists dealing with these meta-materials. Here some of this counterintuitive nature of corpuscular-wave nature of photon inside negative indexed material is brought out, with possible 'new definition' of its 'wave-momentum', the concept of 'reactive energy' inside negative indexed material, along with possible 'new wave equation'. These definitions and expressions of 'wave-momentum' and 'reactive energy' pertaining to negative indexed material are new and discussed and derived by classical means.

1. Introduction

We have demonstrated negative refractive index 'meta-material' plasmonic structures in Ka-band. In our experimental investigation, we have made these plasmonic meta-material prisms of 45, 30 and 15 degrees to get enhanced transmittance of more than 15 dB from background; at negative angles indicating a refractive index of about –1.8, [20-26]. This paper is not aimed for this experimental design, where the meta-material realized by us is based on simple wire-array and labyrinth resonators, [20-26], but to focus on possible theory of the wave mechanics coupled to particle nature of the EM radiation, energy and momentum transport anomalies, a possible new momentum energy description. Also in our repeated observations on numerical experiments we get, as to if a pulse of EM radiation is launched inside a negative refractive index material (NRM), gets squeezed sharpened, [20-26] (refer Figure 1) similarly a spherical wave front in positive indexed media gets flattened as it propagates inside the NRM [20-26]. Though several approaches to explain these counterintuitive phenomena have been evolved, yet it is interesting if in the meta-material parlance particle-wave theory be founded! Here we give possible classical explanations to these counterintuitive phenomena and also new explanations regarding energy momentum, wave equation if applied to this negative indexed material: how shall they look, vis-à-vis positive indexed systems. This problem is a topic subject of investigation in modern optics also. We propose the concept of reactive energy and expression for new wave-momentum for pulse of electromagnetic energy inside a medium (negative refractive indexed), with suitable derivation along with new wave equation. The research papers [7-10, 12, 27] discussed momentum and energy of this reversed electrodynamics in other contexts. Herein, we are deriving the similar concepts with different approach, limiting to propagation of EM pulse inside NRM.

2. Phase and Group Refractive Index in Negative Refractive Index Material

Let us demarcate the two refractive indices, [1, 2], and [3, 11, 19] and this demarcation is essential in explaining the NRM theory. Take the refractive index dispersive that is a function of frequency call it $n_p(\omega)$, phase refractive index. This is basic refractive index by which the velocity of phases of travelling gets modulated inside a dispersive media. We call it phase index n_p. Similarly velocity of a group of frequency travelling wave gets modulated in the media that gives group refractive index n_g. In case of NRM, the phase refractive index if it were $n_p(\omega_0) = -1$ at a particular frequency ω_0, it would imply that in that media the phases would be travelling with speed of light but in opposite direction. There is a backward wave inside NRM [1-3, 11, 13-18, 25, 26]. Refer Figure 1C; where it is demonstrated that phase gets reversed while inside NRM compared to the free space propagation. Now if there is no change in the refractive index for phases with respect to frequency, meaning that $\{dn_p(\omega)/d\omega\} = 0$, we call it dispersionless medium. In that case the phase velocity $v_p(\omega)$ of the wave and group velocity $v_g(\omega)$ of the wave are same. In the free space (refer Figure 1A) both group of frequencies and the crests and troughs of phases are travelling with $v_p(\omega_0) = v_g(\omega_0) = c$. In the free space, we have same modulation for the phases of the signal and group of frequencies at a particular frequency and thus we say phase and group index are same $n_p(\omega_0) = n_g(\omega_0) = 1$. If the media were dispersive, we take phase refractive index as an ‘analytic’ function of the frequency, that is, $n_p = f_{\text{analytic}}(\omega)$ at a particular frequency ω_0. Expansion of Taylor [1-3, 11, 13-18, 25, 26], series (1) for this dispersive phase refractive index; taking the origin at $\omega = \omega_0$, that is frequency of NRM behavior, (only to its first derivative term at the frequency ω_0 near electric plasma and magnetic plasma resonance, where $\varepsilon_r < 0$ and $\mu_r < 0$ for NRM), is defined as group refractive index, which needs be positive. Meaning that

$$n_g(\omega) = n_p(\omega) + \omega \frac{dn_p(\omega)}{d\omega} \cong \frac{d}{d\omega}\{\omega n_p(\omega)\} \quad \text{at } \omega = \omega_0 \quad \text{and } n_g(\omega_0) > 0. \quad (1)$$

This demarcation of phase and group refractive index is very important in understating the behavior of NRM. NRM have unusual properties and in particular Snell's law predicts that the refracted ray of EM signal on entering such a medium would be refracted on the same side of normal to the surface of the incident beam. The wave number that is $k = n_p\,\omega/c$ has the opposite sign to its value in positive indexed media, since $n_p < 0$ at $\omega = \omega_0$. It is shown [1-3, 11, 13-18, 25, 26], that however that Poynting vector and flow of energy points in opposite direction to the wave vector, hence in the expected direction of the propagation of the EM wave, (see Figure 1C). The existence of negative values of ε_r and μ_r tends to suggest 'negative energy density'; but that is not the case when dispersion is taken into consideration. Indeed NRM can only exist if the NRM media is dispersive. Moreover causality (in form of Kramer-Kronigs relation) requires that group refractive index defined in (1) $n_g(\omega_0) > 0$ and group velocity $v_g(\omega_0) > 0$ are always positive [1-3, 11, 13-18, 25, 26].

In the introduction, we have made a statement of our prism experiment showing a negative value of refractive index of –1.8. We clarify that the observed negative refraction is for 'phase-refractive-index' as $n_p(\omega_0) \cong -1.8,$ at $\omega_0/2\pi \cong 33\,\text{GHz},$ with region of NRM as $\Delta\omega \cong 0.85\,\text{GHz},$ whereas the group refractive index $n_g(\omega_0) > 0,$ as this gives positive group velocity. We thus can say that we can observe a negative phase refractive index but the group refractive index shall always be positive. Equation (1) should be read at a particular frequency ω_0 of interest, where we are observing a negative refractive index, in our experimental case it were around 33 GHz, [25].

We can emulate and model by a simplest model as in (2). An NRM (phase refractive index), by a function such that ω_0 is a frequency below which the phase refractive index is negative and above which the phase refractive index is positive. [1-3, 11, 13-18, 25, 26], as (2)

$$n_p(\omega) = 1 - \frac{\omega_0^2}{\omega^2}. \tag{2}$$

This (2) is simplest form of model where one gets ENG (Epsilon Negative) and MNG (Mu Negative) material representation as $\varepsilon_r(\omega) = 1 - (\omega_{ep}{}^2 / \omega^2)$ and $\mu_r(\omega) = 1 - (\omega_{mp}{}^2 / \omega^2)$, where ω_{ep} and ω_{mp} are, respectively, electric and magnetic frequencies below which the permittivity and permeability are, respectively, negative. In (2), ω_0 is chosen in the region where $\varepsilon_r(\omega_0)$ and $\mu_r(\omega_0)$ both are negative so that $n_p(\omega_0) < 0$. This is design issue dealt in [1-3, 11, 13-18, 25, 26], to realize artificially NRM.

From (2) the differentiation with respect to ω gives $dn_p(\omega)/d\omega = 2(\omega_0^2)/\omega^3$, putting this and (2) in (1), we get

$$n_g(\omega) = \left(1 - \frac{\omega_0^2}{\omega^2}\right) + \omega\left(2\frac{\omega_0^2}{\omega^3}\right) = 1 + \frac{\omega_0^2}{\omega^2}. \tag{3}$$

We call ε_{r-} and μ_{r-} explicitly to distinguish NRM, for ENG and MNG with negative permittivity and negative permeability, respectively. For plasmonic system to achieve NRM, we need $\varepsilon_{r-} < 0$ and $\mu_{r-} < 0$, and for ideal case for $n_p = -1$, we need $\varepsilon_{r-} = -1$, and $\mu_{r-} = -1$, [13-18, 25, 26]. Well one can have electric plasma and magnetic plasma frequency overlapped, as $\omega_{ep} = \omega_{mp} = \omega_0$ below which the ε_{r-} and μ_{r-} are negatives, so we get NRM as (2). At the surface plasmon polariton resonance frequency $\omega = \omega_{ep}/\sqrt{2} = 0.7\omega_0$, the value of $\varepsilon_{r-} = -1$, [4-6, 25, 26]; thereby, giving the value of phase refractive index as, $n_p(\omega) = -1$. Also from (2), we find that $n_p(\omega) = -1$, when $\omega_0^2/\omega^2 = 2$. Putting this value of frequency, we obtain that $n_g = 3$ when $n_p = -1$ at the frequency of operation surface mode resonances, [4-6, 25, 26]. Thus we say that the phase refractive index is negative for NRM and the group refractive index is positive for NRM.

3. Electromagnetic Pulse Sharpening inside NRM

Well a wave with crest and trough moving and carrying a Gaussian pulse a 'packet' of energy, in free space travelling with speed of light c, (refer Figure 1A) when entering the NRM with $n_p = -1$, will retard the wave-packets speed to c/n_g, in this case $c/3$, (refer Figures 1B and 1C) though the direction of travel of wave-packet, energy will be in same direction as was in free space; but the phases crests and troughs will here start travelling in opposite to free-space with velocity $-c$. This is implication of the phase and group refractive index in NRM. The implication at NRM boundary of these opposite phases meeting will form a 'cusp' which will be oscillating at the junction of NRM to the free space (refer Figure 1B) [1-3, 4-6, 11, 25, 26]. This phenomenon of retardation of the wave-packet envelope and change of direction of travel of crest and trough the phase, inside NRM gives the 'pulse-sharpening' effect, and flattening of wave-front effect, what we have been observing in our experiments [25], also [4, 5, 6] (refer Figure 1C).

The cusps at the NRM boundary are due to counter propagation of the 'phases' of the waves inside and outside the NRM, they are surface charges, and at the boundary electric field at this cusp oscillates, [4-6, 13-18, 25, 26]; as two sets of impinging wave fronts meet at the interface with ENG (Epsilon Negative Material $\varepsilon_{r-} < 0$). The same cusp will be obtained for the MNG, (Mu Negative Material $\mu_{r-} < 0$) and it may be argued that 'surface' currents in that case for TE polarized incidence, will be at the boundary and magnetic field at the cusp then will oscillate, [4-6, 13-18, 25, 26]. However, these points are valid when the wave hits a slab with ENG and MNG, i.e., NRM here however there will be propagating modes inside NRM-from evanescent [4-6, 13-18, 25, 26]. In the case of double negative slab (NRM) there will be cusp formation at the boundary too. The formation of surface states or excitation of surface plasmon polariton is altogether different field in modern optics, where matching of wave vectors and phase velocities are mandatory, we will not deal with this subject here; however this is important.

4. Pulse of Electromagnetic Energy Travelling in free Space and inside Medium its Transmission and Reflection at the Interface Boundary

The discussion on this section is from classical electrodynamics principles [19]. Let us take following example, a pulse of EM energy travelling in free-space at a particular frequency ω_0, thus carrying an energy packet of $\hbar\omega_0 \text{eV}$. This packet of EM radiation may be represented as a Gaussian pulse; that will strike a medium (other than free-space) located at $z = 0$, by (4), this is derived in (5), [12, 19].

$$E = E_0 \sigma \sqrt{\pi} \{ e^{+iz\omega_0/c} e^{-i\omega_0 t} \} \exp\left[-\frac{\sigma^2}{4} (t - z/c)^2 \right]. \tag{4}$$

The field incident at $z = 0$ is adequately represented by complex electric field as

$$E^{\text{in}} = E_0 \int d\omega \exp\left[-(\omega - \omega_0)^2 / \sigma^2 \right] \exp[i(kz - \omega t)]$$

$$= E_0 \sqrt{\pi} \sigma \exp[-i\omega_0 (t - z/c)] \exp\left[-\frac{\sigma^2}{4} (t - z/c)^2 \right]. \tag{5}$$

The expression (4) is for travelling electric field that has two parts. The phase part given inside the {} brackets, and multiplied by Gaussian travelling envelope in free space as $\exp[-\sigma^2 (t - z/c)^2 / 4]$, having variance σ^2, i.e., the width of the packet (Full Width Half Maxima FWHM). The packet is travelling from left to right thus phases (crest and trough are translating in $+z$ -direction) with a phase velocity $v_p = c$, and the group, i.e., the envelope carrying the information/energy is travelling with group velocity $v_g = c$ in the same direction of $+z$ in free space having $n_p = n_g = +1$, [19]. Refer Figure 1A, (4) is depicted there travelling towards right with envelope as dashed and phases as solid lines.

We investigate what happens when this (4), (5) incident Gaussian electromagnetic pulse enters a medium. This Gaussian pulse is centered at angular frequency ω_0 and we assume that this energy beam is weakly focused so we take spatial spread in only one dimension. The reflection and refraction of electromagnetic waves at an interface are described by Fresnel law. For normal incident [19], we have reflection coefficient $\rho(\omega)$ and transmission coefficient $\tau(\omega)$ described as (6), [19]; both being function of frequency since impedance of media is dispersive.

$$\rho(\omega) = \frac{Z_0 - Z}{Z_0 + Z}, \qquad \tau(\omega) = \frac{2Z}{Z_0 + Z}, \tag{6}$$

where $Z = \sqrt{\mu/\varepsilon}$ is impedance of medium and Z_0 is free space impedance. Note for a NRM with $\mu_r = \varepsilon_r = -1$, the $Z = Z_0$, the incident beam suffers no reflection and is 100% transmitted. The forms of reflected and transmitted waves follow from the spectrum of the incidence pulse (5) as (7) and (8), [19].

$$E^{\text{ref}} = E_0 \int d\omega \exp[-(\omega - \omega_0)^2/\sigma^2]\rho(\omega)\exp[i\omega(t + z/c)], \tag{7}$$

$$E^{\text{trans}} = E_0 \int d\omega \exp[-(\omega - \omega_0)^2/\sigma^2]\tau(\omega)\exp[-i\omega(t - n_p\{\omega\} z/c)]. \tag{8}$$

It suffices for our purpose to assume that spectrum is narrow so that we can approximate $\rho(\omega)$ and $\tau(\omega)$ by their values at ω_0 and $n_p(\omega)$ by first two terms of Taylor series expansion (1). This leads to simple Gaussian forms for (7) and (8) as (9) and (10).

$$E^{\text{ref}} = \rho(\omega_0)E_0\sqrt{\pi}\sigma\exp[-i\omega_0(t + z/c)]\exp\left[-\frac{\sigma^2}{4}(t + z/c)^2\right], \tag{9}$$

$$E^{\text{trans}} = \tau(\omega_0)E_0\sqrt{\pi}\sigma\exp[-i\omega_0(t - n_p\ z/c)]\exp\left[-\frac{\sigma^2}{4}(t - n_g\ z/c)^2\right]. \tag{10}$$

For 100% transmission when $Z = Z_0$ say for NRM when $\varepsilon_r = \mu_r = -1$, with $n_p(\omega_0) = -1$ and $n_g(\omega_0) = 3$, we get $E^{\text{ref}} = 0$ since $\rho(\omega_0) = 0$, $\tau(\omega_0) = 1$ and transmitted field inside NRM is thus given below (11).

$$E^{\text{trans}} = E_0\sqrt{\pi}\sigma\exp[-i\omega_0(t + n_p z/c)]\exp\left[-\frac{\sigma^2}{4}(t - 3z/c)^2\right]. \tag{11}$$

5. Energy Momentum of Gaussian Electromagnetic Pulse

To this Gaussian pulse, there is a packet of energy $\hbar\omega_0$; we can associate momentum $\hbar\omega_0/c$ with this pulse. The research papers [7-10, 12, 27] discuss momentum of energy of this reversed electrodynamics, in different context but our approach and discussions are differently oriented. Inside a medium, we can have scenario where the momentum can have different interpretation if we say $p = n_p\ \hbar\omega_0/c$ as phase ‘wave’ momentum inside medium, then if the media has $n_p = -1$, we get confused by this negative momentum indicating a decrease in pressure for radiation of electromagnetic wave, when it strikes a boundary. Well call this momentum $n_p \hbar\omega_0 / c$ as 'wave' momentum, to distinguish from 'mechanical' momentum (12), (13) (containing group velocity and group index) as, Minkowski [7], or Abraham [8];

$$p_{m1} = n_p^2 \hbar\omega_0 / n_g c = v_g n_p^2 \hbar\omega_0 / c^2 , \tag{12}$$

$$p_{m2} = \hbar\omega_0 / n_g c = v_g \hbar\omega_0 / c^2 . \tag{13}$$

These definitions of mechanical momentum ensure that they are positive, inside NRM as well. Well these mechanical momentum definitions (12) and (13) give us non-confusing thought that even with $n_p < 0$ still there is positive electromagnetic pressure, as against definition of 'wave' momentum $p = n_p \hbar\omega_0 / c$, where we let believe if the electromagnetic pressure be negative in case of NRM! Well, only for phase reversal, we make use of wave-momentum, and for energy transport and electromagnetic energy pressure we shall make use of mechanical momentum. The confusion is arising because of dual nature of radiation, particle as well as wave nature.

Now when the Gaussian pulse or this electromagnetic energy enters a slab with $n_p \neq 1$ and $n_g \neq 1$, assuming 100% transmission into that slab, we have different electric field as from (11)

$$E = E_0 \sigma\sqrt{\pi}\{e^{in_p \omega_0 z/c} e^{-i\omega_0 t}\} \exp\left[-\frac{\sigma^2}{4}(t - n_g z/c)^2\right]. \tag{14}$$

Now if we state that $n_p = -1$, and $n_g = 3$, then we will observe that the Gaussian pulse envelope will compress itself and keep propagating inside NRM block in the same direction of $+z$, with group velocity $c/3$, but the phases will keep now translating in space in opposite direction but with phase velocity $-c$, refer Figure 1C. The meeting of the two opposite phases, (refer Figure 1B) at the NRM boundary gives rise to cusps-owing to surface modes, which travel and oscillate in direction perpendicular to propagation direction and along the surface of the interface, [4-6, 13-18, 25, 26].

Well we pose a query that is if (15) can be called a photon as it has now become inside NRM of our choice as

$$E_N^{\text{photon}} = E_0\sigma\sqrt{\pi}\{e^{-iz\omega_0/c}e^{-i\omega_0 t}\}\exp\left[-\frac{\sigma^2}{4}(t-3z/c)^2\right] \tag{15}$$

is different from original (4), that is $E_P^{\text{phton}} = E_0\sigma\sqrt{\pi}\{e^{+iz\omega_0/c}e^{-i\omega_0 t}\}e^{-\frac{\sigma^2}{4}(t-z/c)^2}$ in the free space. Equation (15) seems to suggest that the pulse envelope and the phases travel are in opposite direction, this packet need not be thus called a photon packet rather 'negative' photon packet! (refer Figure 1C).

Here we are visualizing that electromagnetic pulse (4) is a 'photon'. Well this is how scientist describes a 'packet of wave' as unit energy 'photon'. Equation (4) when we talk in limiting case with $\sigma \to 0$ becomes singular and ideally representing 'single-photon' with frequency ω_0. Well who has really seen a photon a mathematical abstraction and can thus well be approximated as in (4). Our argument of 'negative' photon stems from the fact that had there be 100% reflection to (4), $\rho(\omega_0) = 1$, then we get, a packet of original photon as in (16), from (9) where the envelope and phases are travelling in $-z$ direction after hitting the boundary at $z = 0$, thus retaining the character of original photon.

$$E^{\text{ref}} = \rho(\omega_0)E_0\sqrt{\pi}\sigma\exp[-i\omega_0(t+z/c)]\exp\left[-\frac{\sigma^2}{4}(t+z/c)^2\right]$$

$$= E_P^{\text{photon}} = E_0\sigma\sqrt{\pi}\{e^{-iz\omega_0/c}e^{-i\omega_0 t}\}e^{-\frac{\sigma^2}{4}(t+z/c)^2}. \tag{16}$$

Reflected photon is original photon as incident photon, while transmitted photon inside NRM is 'negative' photon.

6. Photon Momentum Transfer to the Medium

Taking clue from the above discussion let us define phase momentum, or wave-momentum of a photon packet as (17); this choice will be clear as we proceed for proof below.

$$p_c \overset{\text{def}}{=\!=} \frac{\operatorname{sgn}(n_p)}{\sqrt{|n_p n_g|}}\frac{\hbar\omega_0}{c} = N\frac{\hbar\omega_0}{c}, \tag{17}$$

where $n_p > 0$, $\operatorname{sgn} n_p = +1$, $n_p < 0$, $\operatorname{sgn} n_p = -1$.

Well if the photon is in free space, then (17) would be $p_c = \hbar\omega_0/c$ or if it were in our chosen NRM with $n_p = -1$ and $n_g = 3$, then inside NRM this 'negative' photon has wave-momentum as $p_c = -(1/\sqrt{3})\hbar\omega_0/c$. Well we could have chosen (17) to be as $p_c = (n_p/n_g)\hbar\omega_0/c$ too, but the chosen square root for n_g will be explained in the next section, by total energy balance formulation.

We start our discussion of effect of our single photon entering the medium from region of free space. If the photon is totally reflected, then because of the momentum conservation it transfers $2\hbar\omega_0/c$ momentum to the medium. If the photon passes into the medium in that case momentum will be transferred to the medium at the interface surface where there will be reflection and transmission, the momentum transferred to surface is given as

$$p^{\text{media}} = (1+R)\frac{\hbar\omega_0}{c} - Tp, \tag{18}$$

where the reflection probability R and transmission probability T [19] with respect to free-space impedance Z_0 and impedance of medium Z are defined as

$$R = \rho^2 = \left(\frac{Z_0 - Z}{Z_0 + Z}\right)^2, \qquad T = \tau^2 = \frac{4Z_0 Z}{(Z_0 + Z)^2}. \tag{19}$$

Putting (19) in (18) and using p of (17), we get the following algebraic manipulations

$$\begin{aligned} p^{\text{media}} &= \frac{\hbar\omega_0}{c} + \left(\frac{Z_0 - Z}{Z_0 + Z}\right)^2 \frac{\hbar\omega_0}{c} - \frac{4Z_0 Z}{(Z_0 + Z)^2} N \frac{\hbar\omega_0}{c} \\ &= \frac{2\hbar\omega_0}{c} + \left(\frac{Z_0 - Z}{Z_0 + Z}\right)^2 \frac{\hbar\omega_0}{c} - \frac{4Z_0 Z}{(Z_0 + Z)^2} N \frac{\hbar\omega_0}{c} - \frac{\hbar\omega_0}{c} \\ &= \frac{2\hbar\omega_0}{c} - \frac{\hbar\omega_0}{c}\left[1 + \frac{4Z_0 Z}{(Z_0 + Z)^2} N - \frac{(Z_0 - Z)^2}{(Z_0 + Z)^2}\right] \\ &= \frac{2\hbar\omega_0}{c} - \frac{\hbar\omega_0}{c}(1+N)\frac{4Z_0 Z}{(Z_0 + Z)^2} = \frac{2\hbar\omega_0}{c} - \frac{\hbar\omega_0}{c}(1+N)T. \end{aligned} \tag{20}$$

Therefore with the definition of wave-momentum as in (17), we get momentum transferred to the media, at the surface as

$$p^{\text{media}} = \frac{2\hbar\omega_0}{c} - \frac{\hbar\omega_0}{c}\left(1 + \frac{\operatorname{sgn}(n_p)}{\sqrt{|n_p n_g|}}\right)T. \tag{21}$$

Using the mechanical momentum definitions of (12) and (13), and doing the same algebraic manipulations, we get the mechanical momentums transferred to the medium at the surface as

$$p_{m1}^{\text{media}} = \frac{2\hbar\omega_0}{c} - \frac{\hbar\omega_0}{c}\left(1 + \frac{n_p^2 v_g}{c}\right)T,$$

$$p_{m2}^{\text{media}} = \frac{2\hbar\omega_0}{c} - \frac{\hbar\omega_0}{c}\left(1 + \frac{v_g}{c}\right)T. \tag{22}$$

Well, all these momentums transferred to medium at the surface of all types (21) and (22) reduce to $2\hbar\omega_0/c$ for a perfectly reflecting surface when $T = 0$, corresponding to change in momentum due to reflection. It is also clear that mechanical momentum transferred to medium by definition of p_{m2} will always be positive as $v_g < c$, however the definition of p_{m1}, and p_c (17), when used the momentum transfer to the medium at surface can be positive or negative depending on the property of media.

Let us take an example of ideal case whence $R = 0$ and $T = 1$, zero reflection and 100% transmission, for NRM with $n_p = -1$, $n_g = 3$, and $v_g = c/3$. The condition for this is $\varepsilon_{r-} = \mu_{r-} = -1$, gives $Z = Z_0$, thus $R = 0$. Here the photon passes into NRM with 100% probability $(T = 1)$. For this NRM condition the momentum transfer associated with mechanical momentums are identical, corresponding to $(1 - v_g/c)$, that is, $2/3$ of the original photon mechanical momentum transferred to the media. The mechanical momentum retained by photon is $(1/3)$ the original photon momentum. This process is depicted in Figure 1. Whereas the wave-momentum transferred (21) for these values is $(\sqrt{3} + 1)/\sqrt{3} = 1.577$ of the original momentum. The wave-momentum retained by ‘negative’ photon is $(-1/\sqrt{3})$ times the original momentum, pointing in opposite direction to wave-momentum of original photon. This also factually matches that inside NRM phase velocity is opposite to the energy flow or group velocity [4-6, 13-18, 25, 26].

The case where $n_p = -1$, $n_g = 1$, (hypothetically if it exists) the wave momentum transferred (21) to the medium is twice the original wave-momentum, and no mechanical momentum gets transferred to the media, well this is case of total internal reflection. For a medium $n_p = 1$ and $n_g = 1$, the wave and mechanical momentum transferred to the medium is zero, that is, all the momentum is retained by photon.

This contradiction is embedded in principle of theory of 'wave-particle' duality. Really if we consider photon as particle that its linear momentum will be $p = mv$ and this mechanical momentum is proportional to velocity. But while considering photon or radiation as wave then its linear momentum is $p = h/\lambda = \hbar k = \hbar\omega/v$ here, is inversely proportional to velocity. This contradiction is unessential if the medium is free space dispersionless vacuum (where $v = c$), but brings about certain problem if the photon is inside a media (positive refractive indexed or negative refractive indexed). We can confess therefore that value of linear momentum of photon carrying radiation energy packet at present is far from being established concept.

7. Derivation of Expression for Wave-momentum of Electromagnetic Pulse 'a Photon'

This section will elucidate the choice of our definition of wave momentum for photon as in (17). Let a photon pulse be travelling in free space. Observer sitting on the crest and another observer sitting on the envelope, travelling in free space they will find themselves at rest with respect to each other, while the packet enters the NRM, the two observers will find that they are moving away from each other. This is this nature of wave-momentum that is generator of infinitesimal translations, and the infinitesimal translations of the 'waves' correspond to motion of its crests and troughs, and in NRM 'opposes' the direction of motion of radiation. It is for this reason the wave-momentum points in the opposite direction to the mechanical momentum inside NRM. Perhaps due to this reason one may state that photon is transformed to 'negative' photon inside NRM, its characteristics is different than that of original photon.

Consider photon travelling in free space with mechanical energy E_m, that is, energy associated with its corpuscular part, and with phase or wave-momentum as $p = \hbar\omega_0/c$ having wave energy as pc, thus total energy is E, having relation as (23) below, [19].

$$E^2 = p^2c^2 + m^2c^4. \tag{23}$$

Call v_p as phase velocity and v_g as group velocity of monochromatic EM signal travelling in the region $0 < z < (d/2)$, where the $n_p(\omega) = +1$, with relative permeability $\mu_{r+} = 1$, and relative permittivity as $\varepsilon_{r+} = 1$. Conventionally, we can write for the dispersionless ideal region $(z < (d/2))$ that

$$v_p v_g = c^2. \tag{24}$$

This we are assuming that $v_p = (\omega/k) = c$; $(d\omega/dk) = c$ in a vacuum where EM waves are travelling is ideal condition. Now we pose ourselves a question as to how we are writing (24), that is square of velocity of light equal to product of the phase velocity and group velocity. The answer to that we addressed in following description.

If a space between radiator and receiver is filled by vacuum that carrying between them electromagnetic radiation with energy E and to that we assign a linear momentum as $p = E/c$, is also accompanying by a mass $m = E/c^2$. Really radiator after emitting wave-packet recoils with velocity $v_{\text{recoil}} = p/M = E/Mc$, where M is the mass of radiator. The wave packet reaches receiver sitting at distance Z after time $t = Z/c$, and the radiator moves a distance $\Delta z = tv = (Z/c)(E/Mc)$ $= ZE / mc^2$. The requirement of stillness of inertia of entire system gives moment balance as $(\Delta z)M = ZE / c^2$. This description could be interpreted as, when energy E is transported from radiator to receiver, the mass of radiator gets decreased, but the mass of receiver gets increased by m equal to E / c^2! The question is for the multiplier as c^2, which is numerically equal to square of velocity of light in vacuum, which is used to justify the dimensions of the energy mass equation that is $m = E / c^2$! Well can this multiplier have different physical meaning?

Let us associate c_g as group velocity of the wave-packet, then in above paragraph the expression for time will be $t = Z / c_g$. Let the phase velocity associated to crest and trough be identified as wave-velocity as c_p, then wave momentum correlation will be $p = E / c_p$, this makes the accompanying mass as $m = E / c_p c_g$. This validates our choice of multiplier $c^2 = v_p v_g$, and this could be physical interpretation also.

Now for negative indexed material NRM (lossless and ideal case, with $n_p = -1)$, we can write an approximate relation (25), for region $((d/2) < z < (3d/2))$, where we have assumed perfect condition as $\mu_{r-} = \varepsilon_{r-} = 1$; with refractive index as $n_p(\omega_0) = -1$, and $n_g(\omega_0) \cong +1$. This enables the propagating modes inside the LHM slab, with (25). In (25), we assume $v_g \cong c$, inside LHM, (though not possible in actual practice).

$$v_p v_g \cong -c^2. \tag{25}$$

This negative sign in right hand side represents that group velocity and phase velocity are 180^0 apart from each other, magnitude being *c*. Energy mass momentum expression for particle at speed of light in relativistic approach is (23), and substituting (24), we get

$$E^2 = p^2 c^2 + m^2 c^4 = p^2 (v_p v_g) + m^2 (v_p v_g)^2, \tag{26}$$

where *E* is total energy, *p* is momentum of the particle (wave) which is present inside the meta-material, *m* is (rest) mass of the particle carrying the energy packet. Well the rest mass of photon is zero, but we can always associate a mass $m = E_m / c^2$ for the electromagnetic energy carrying mechanical (corpuscular) energy E_m. This mechanical energy is responsible for radiation positive radiation pressure. While the other part of energy we may associate to phase wave-momentum energy due to the wave nature associated with photon-movement or translation of phases ‘crests’ and ‘trough’s’ motion, in the media.

Manipulating (26), we get as follows:

$$E^2 = p^2(v_p v_g) + m^2(v_p v_g)^2$$

$$= m^2 v_p v_g \left[v_p v_g + \frac{p^2}{m^2} \right]. \tag{27}$$

Equation (27) is for free-space, medium with positive phase and group velocity and both equal to c. That is $v_p = v_g = c$.

Now we use (27), for NRM medium and manipulate as below:

$$E^2 = p^2 c^2 + m^2 c^4$$

$$= p^2(-v_p v_g) + m^2(-v_p v_g)^2 = m^2(v_p v_g)^2 - p^2(v_p v_g)$$

$$= m^2 |v_p v_g| \left[|v_p v_g| - \frac{p^2}{m^2} \right]. \tag{28}$$

Putting $|v_p v_g| \cong c^2$ in equation (28), we get

$$E^2 = m^2 c^2 \left[(c^2) - \left(\frac{p}{m} \right)^2 \right] = (m^2 c^2) \left[c^2 - \frac{p^2}{m^2} \right] = m^2 c^4 + (-p^2 c^2). \tag{29}$$

We split (29) into two parts, the mechanical (corpuscular) energy part $(m^2 c^4)$ and the energy transport by wave-momentum part $(-p^2 c^2)$.

Equation (29) shows that particle energy is retained itself by the particle, inside NRM where the phase velocity is opposite to group velocity. In this case no (mechanical-corpuscular) energy is transferred to the NRM medium. This we derive from the part of rest mass-energy that is the first part of expression mc^2; meaning that corpuscular energy by photon is retained.

But the intriguing question is the energy due to wave-momentum part is imaginary, inside NRM! That is equal to ipc (considering the positive root). We can ascribe to this imaginary 'negative'- photon' a wave-momentum a value $-\hbar\omega_0/c$.

Now we retard the group velocity to $c/3$, and have phase reversal with phase velocity inside NRM as $-c$, then $|v_p v_g| = c^2/3$, and put the same in (26) to get

$$E^2 = m^2\left(\frac{c^2}{3}\right)\left[\frac{c^2}{3} - \frac{p^2}{m^2}\right]$$

$$= \frac{1}{9}(m^2c^4 - 3p^2c^2) = \frac{1}{9}m^2c^4 + \frac{1}{3}(-p^2c^2). \tag{27a}$$

Here the particle inside the NRM has less total corpuscular energy; the difference of energy has been absorbed by the media itself. Expression (27a) suggests one third of the corpuscular energy $(1/3)mc^2$ is retained by the 'photon' inside the NRM slab, and the two thirds of its corpuscular energy are given to the slab!!

Well the energy due wave momentum of the photon manifests as imaginary energy in this case as $i(1/\sqrt{3})pc$, (again retaining the positive root). We can ascribe to this imaginary 'negative'-photon' a wave-momentum, a value $-(1/\sqrt{3})\hbar\omega_0/c$.

The momentum transfer cases we have discussed in earlier section also and maps correctly with the total energy argument cases as described here.

Well let us consider the length of NRM slab, as $(3d/2) - (d/2) = Z$, with $n_p = -1$, and $n_g = 3$. The photon is retarded in comparison to its position in absence of medium by distance z, which is

$$z = (c - v_g)\frac{Z}{v_g} = (n_g - 1)Z, \tag{28a}$$

where Z is the thickness of medium. The relativistic form of Newton's first law of motion requires that the centre-of-mass energy of a system not subjected to any external force should be stationary or in uniform motion. Our medium is isolated from such external influence, then the relevant total energy is sum of photon energy $\hbar\omega_0$ and the rest mass energy of the medium Mc^2, where M is mass of medium. The fact that photon has been retarded by the medium means the centre-of-mass-energy can only have been in uniform motion if the medium has itself moved to the right by a distance Δz, then the moments are

$$(\Delta z)(Mc^2) = (z)(\hbar\omega_0). \tag{29a}$$

Substituting value of z from (28a), we get

$$\Delta z = \frac{\hbar\omega_0 Z}{Mc^2}(n_g - 1). \tag{30}$$

This motion can only take place if energy transfer takes place from photon whilst inside the medium. The required velocity of the medium is $v_g\,(\Delta z)/Z$, from which we can readily obtain momentum

$$p^{\text{medium}} = Mv_g\,\frac{\Delta z}{Z} = \frac{\hbar\omega_0}{c}\left(1 - \frac{v_g}{c}\right) = \frac{2}{3}\,p_0, \tag{31}$$

where the p_0 is the initial momentum of the photon in free space. Momentum conservation suggests that we ascribe the difference between the initial momentum and this medium momentum to the photon momentum inside the medium. From previous section the mechanical momentum of photon in this NRM would be

$$p^{\text{NRM}}{}_{m1} = n_p^2\,\hbar\omega_0 / n_g c = v_g n_p^2\,\hbar\omega_0 / c^2 = \frac{1}{3}\frac{\hbar\omega_0}{c} = \frac{1}{3}\,p_0, \tag{32}$$

$$p^{\text{NRM}}{}_{m2} = \hbar\omega_0 / n_g c = v_g\,\hbar\omega_0 / c^2 = \frac{1}{3}\frac{\hbar\omega_0}{c} = \frac{1}{3}\,p_0. \tag{33}$$

The wave momentum of photon inside this NRM slab is

$$p^{\text{NRM}}{}_{c} = \frac{\operatorname{sgn}(n_p)}{\sqrt{|n_p n_g|}}\frac{\hbar\omega_0}{c} = -\frac{1}{\sqrt{3}}\,p_0. \tag{34}$$

Equations (32) and (33) state that $(1/3)$ of the mechanical momentum is retained by the ‘photon’ inside this NRM. This is well equating as if 1/3 of ‘particular’ photon corpuscular energy is retained by photon inside NRM, whereas the wave-momentum retained by photon inside NRM (34) is $-(1/\sqrt{3})$ times the original wave momentum.

8. Imaginary ‘Reactive Energy’ and ‘Wave-momentum’ inside Medium

In the previous section, we could balance the retardation effect stating that the corpuscular energy that comprising of mechanical photon momentum is transferred to the medium thereby inside NRM the retardation of photon takes place. What was intriguing was imaginary energy of the photon inside the NRM, what we termed as 'reactive' energy. This reactive energy of photon inside NRM is making the waves of phases travel backward inside NRM as contrary to positive indexed material.

Could we reframe the wave-momentum inside a media be it positive indexed or be it negative indexed as we have defined in (17); rewritten as in (35)? Well the discussion suggests yes why not!

$$p_c \stackrel{\text{def}}{=\!=} \frac{\operatorname{sgn}(n_p)}{\sqrt{|n_p n_g|}} \frac{\hbar\omega_0}{c} = \frac{\operatorname{sgn}(n_p)}{\sqrt{|n_p n_g|}} p_0. \tag{35}$$

A new way to define canonical momentum inside slab, be it positive refractive indexed or negative refractive indexed system, also this agrees with what we derived from total energy balance description in the previous section.

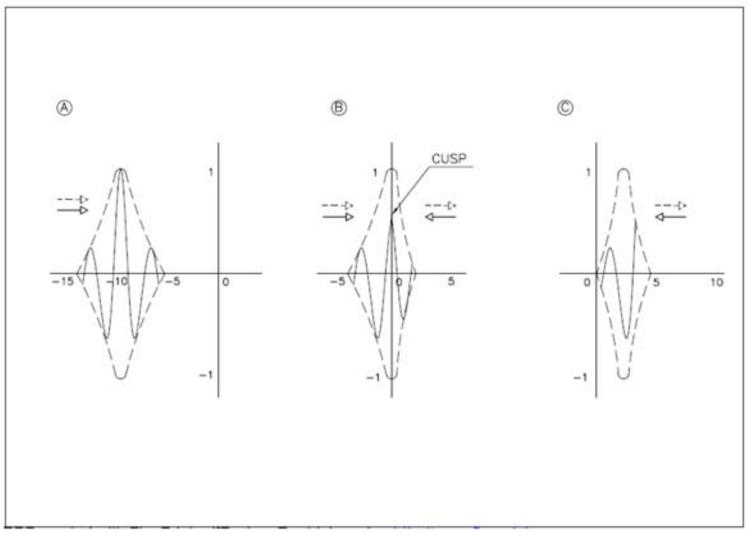

Figure 1. Propagation of electromagnetic pulse. **A.** Pulse propagating towards right in free space, having envelope (dashed) and phases (solid) travelling with velocity c in same direction. **B.** The same pulse touches the media with NRM with phase index as –1, and group index as +3; shows that at the boundary there is 'cusp' formation and envelope retards. Here the phases travel in opposite direction and the group (envelope) travels in same direction. This cusp oscillates at the surface of the NRM boundary. **C.** The pulse is travelling as envelope with squeezed envelope inside NRM towards the right direction with velocity $+c/3$ whereas the phases are travelling opposite to envelope, with velocity $-c$. The pulse is sharpened and

squeezed. This is ideal case of loss-less NRM while lossy structures will have attenuated pulse as it travels.

9. Wave Equation Explanation

We can identify the motion of the photon pulse with mechanical momentum but the wave momentum corresponds rather to motion of the phase fronts. The difference is analogous to that between phase and group velocities for a wave; the phase velocity is that at which the phase front propagate, while the pulse and its associated energy propagate at group velocity, thus the phase velocity does not appear in mechanical momentum expressions used above.

We now resort to classical wave as photon and see if we can distinguish between positive refractive indexed media and negative refractive indexed media, through wave equation.

(a) Classical quantum prescriptor and Schrodinger wave equation

Total energy of system is expressed as kinetic plus potential as

$$T + V = \frac{p^2}{2m} + V = E. \tag{36}$$

By putting standard Q prescriptors that is $p \to i\hbar\nabla$ and $E \to i\hbar(\partial/\partial t)$, and in addition asking these prescriptors to operate on wave function ψ, the standard Schrodinger wave equation is obtained as

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}. \tag{37}$$

The plane wave solution in vector form is $\psi = A\exp\left(-i\frac{1}{\hbar}p.r\right)$.

With $p = \hbar k$ as photon’s momentum vector linked with its wave vector, and $E = \hbar\omega$, without any potential the wave travels in straight line and we have $E = p^2/2m$ (as $V = 0$) and we obtain potential free wave equation as

$$\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + E\psi = 0. \tag{38}$$

This has two solutions

$$\psi(x) = Ae^{ix\sqrt{2mE/\hbar^2}} + Be^{-ix\sqrt{2mE/\hbar^2}} \tag{39}$$

case for positive E propagating case.

$$\psi(x) = Ae^{x\sqrt{2mE/\hbar^2}} + Be^{-x\sqrt{2mE/\hbar^2}} \tag{40}$$

case for negative E bounded case. This bounded case is for surface wave happens for ENG or MNG only.

(b) A new quantum prescriptor and Schrodinger wave equation

Let us take the Q prescriptors modified as

$$p \to -i\hbar \exp(-i\theta)\frac{\partial}{\partial x}, \qquad E \to \hbar\omega, \qquad p \to \hbar[k\exp(i\theta)].$$

Put them in potential free energy expression $E = p^2/2m$, when we operate this on wave function ψ, we get a new Schrodinger equation as

$$e^{-2i\theta}\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + E\psi = 0. \tag{41}$$

Well the solutions are for this wave equation then

$$\begin{aligned}\psi(x) &= Ae^{ix\sqrt{2mE/\hbar^2}\exp(i\theta)} + Be^{-ix\sqrt{2mE/\hbar^2}\exp(i\theta)}\\ &= Ae^{ixk\exp(i\theta)} + Be^{-ixk\exp(i\theta)}.\end{aligned} \tag{42}$$

(42) is case for propagating case.

$$\begin{aligned}\psi(x) &= Ae^{x\sqrt{2mE/\hbar^2}\exp(i\theta)} + Be^{-x\sqrt{2mE/\hbar^2}\exp(i\theta)}\\ &= Ae^{xk\exp(i\theta)} + Be^{-xk\exp(i\theta)}.\end{aligned} \tag{43}$$

(43) is case for bounded case.

A quick verification shall state that for $\theta = 0$, one gets wave equation for normal media where the Right Handed Media (RHM), while $\theta = \pi$ gives a wave propagation in Left Handed Media (LHM) with NRM. This also opens up a possibility of having a system in between RHM and LHM.

This gives a wave description of RHM and LHM where in the later case the phase is opposite the energy flow can be represented as different quantum prescriptors and different Schrodinger wave equations. At least mathematics hints so; well physical consequences are far from reality, at present for these new Q-prescriptors. The rotational component $\exp(i\theta)$ may be personified as demarcation between phase velocity and group velocity and their relation to the phase and group indices, a future work! The future work shall also relate the relation between this rotational component with that of $N = \operatorname{sgn} n_p \big/ \sqrt{|n_p n_g|}$ in new formulation of the canonical (wave) momentum.

10. Conclusion

Experimental realization of negative index of refraction has as a result raised important questions about the validity of this negative value in well-known formulas of physics. The question of corpuscular energy transport inside negative indexed material, formation of reactive (imaginary) energy inside the negative indexed substances, well the character of photon pulse especially its momentum (corpuscular and wave) is addressed along with duality of particle-wave nature of photon. Few new concepts regarding new wave-momentum inside slab and reactive energy inside negative indexed material, and new generalized wave equation is proposed to meet the future theoretical advances on these realized negative indexed materials.

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