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Abstract

The purpose of this work was to investigate the possibility of particles traveling faster than the speed of light *c* in a vacuum, without violating special relativity. This investigation led to the discovery of quantum universes where the possibility of particles traveling faster than the speed of light *c* in a vacuum was found to be true for some quantum universe quantum states. The value of physical constants like the mass of subatomic particles, Planck's constant, and the elementary charge were derived for quantum universes. Schrödinger equations along with physical constants of quantum universes were used to analytically determine wave functions and properties of the hydrogen atom of quantum universes. A new concept, atomic particle transitions, remotely similar to atomic electron transitions was determined and discussed. Properties of theoretical hydrogen photon engines that use powerful economic electromagnetic energy operating on the principle of

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atomic particle transitions were discussed. It was demonstrated that quantum universes have extremely different discrete atomic densities. Quantum universes were shown to exhibit discrete dilated or hastened time periods. Discretely different frequencies, wavelengths, and propagation velocities of electromagnetic waves of quantum universes were determined. Consequences of special relativity as related to quantum universes were discussed.

1. Introduction

Multiple universes have been proposed in philosophy, transpersonal psychology, religion, music, astronomy, physics, and all kinds of literature, particularly in science fiction, fantasy, and comic books. In this paper multiple universes are proposed that were named “quantum universes”. Multiple universes have been given many names including “parallel worlds”, “parallel universes”, “parallel realities”, “parallel dimensions”, “alternate realities”, “alternate timelines”, “alternate dimensions”, “dimensional planes”, and “quantum realities”. This is partial evidence, that considerable work has been done on multiple universes over the past 20-30 years. Multiple universes in past scientific literature have sometimes been labeled a “multiverse”. A multiverse or multiple universes have been classified as Brian Greene’s nine types, Max Tegmark’s four levels, M-theory, Cyclic theories, Anthropic principle, Occam’s razor, and Modal realism. “Quantum universes” is not related to a multiverse and does not fit any of these classifications. It is entirely new and original.

This paper is an extension of quantum mechanics and quantum theory to that of quantum universes. Quantum mechanics has provided the derivation of the properties and behavior of quantum systems of our quantum universe. This paper extends this derivation of the properties and behavior of quantum systems to all quantum universes. Quantum theory of our quantum universe is the theoretical basis that explains the nature and behavior of matter and energy on the atomic and subatomic level. Similarly, new quantum theory is used to explain quantum universes.

The motivation to determine if it was possible for particles to travel, in three-dimensional space, faster than the speed of light was strong. For if the results were positive, it might mean that “faster than light” interplanetary travel was a possibility. However, Albert Einstein published the theory of special relativity in 1905. He used the Lorentz factor to determine relativistic mass-energy equivalence in our quantum universe. This proved that the closer to the speed of light you get a particle, the more massive it becomes and the more energy is required to achieve its velocity. It proved that particles in our quantum universe can never reach or exceed the speed of light c in a vacuum. Thus, if a particle can travel faster than the speed of light, it was reasoned it must be in concert with Albert Einstein’s theory of special relativity. And, this reasoning motivated the new initiative.

This work was undertaken because there might be particles that could travel faster than the speed of light c in a vacuum, without violating the theory of special relativity. Thus, it was hypothesized that there could be particles in the three-dimensional vacuum of space, that could travel faster than the speed of light c in a vacuum, that do not violate the theory of special relativity. This work did indicate that particles of less massive quantum universe quantum states can travel faster than the speed of light c in a vacuum, without violating the theory of special relativity.

The search for particles that could travel faster than the speed of light in a vacuum led to subatomic particles of atoms of quantum-mechanical systems of quantum universes that had discretely, much greater mass and much less mass than those in our quantum universe. The subatomic particles led to atoms of quantum universes that were both discretely much larger and much smaller in size than those in our quantum universe. These atoms defined limitless multiple universes, that were both discretely much denser and much rarer than our quantum universe.

The new concept of quantum universes is introduced because it might explain some of the mysteries of modern-day physics. Our universe could be just one of limitless quantum universes.

The new concept of different discrete values for the physical constants of quantum universes, e.g., discrete masses of subatomic particles, discrete elementary charges of electrons or protons, discrete Bohr radii, discrete Planck's constants and discrete reduced Planck's constants are introduced so that physical laws and Schrödinger equations for quantum universes can be determined.

The new concept of different discrete quantum universe speeds of light in vacuum integrated with Einstein's special theory of relativity is introduced because it explains how particles can travel, limited only by its quantum universe speed of light.

The new concept of an atomic particle transition is introduced because it shows how the atoms of quantum universes are formed. An atomic particle transition is introduced because the concept shows how the size of subatomic particles and atoms can be discretely changed. It shows how the mass of subatomic particles can be discretely changed. It is introduced because the concept shows how a quantum universe quantum state of a quantum system can be transitioned into another quantum universe quantum state. It shows how a quantum system can transition from one quantum universe to another. It is introduced because the concept justifies how particles can move at "faster than light" speeds.

It was hypothesized that hydrogen atoms should be capable of transitioning from our quantum universe, to quantum universe number negative one, by multiple atomic particle transitions, so as to make the hydrogen atoms disappear from our quantum universe, by experiment in a high-tech laboratory. This hypothesis should be immediately testable in the framework of current knowledge. Irradiating hydrogen atoms with the exact appropriate electromagnetic frequency could accomplish this. This frequency (see Section 5) is approximately $3.30 \times 10^{15}\,\text{Hz}$ ($9.10 \times 10^{-8}\,\text{m}$ wavelength). That this could be done, is experimentally distinguishable from existing knowledge. Thus, it is that, hydrogen atoms transitioned to quantum universe number negative one, by multiple atomic particle transitions, so as to make hydrogen atoms disappear from our quantum universe, is experimentally distinguishable from existing knowledge.

This work is an extension of the work of Erwin Schrödinger. Schrödinger equations are shown to be capable of determining the discrete quantum states of quantum systems of quantum universes. Schrödinger equations are solvable in quantum universes. The discovery of the discrete masses of subatomic particles in atoms of quantum universes and the discovery of values of other physical constants, like the elementary charge e and the reduced Planck's constant $\hbar$ of quantum universes enable this.

The organization of the paper is as follows: Section 1 Introduction. Section 2 determines the physical constants required to develop the physical laws of quantum universes. Section 3 derives Schrödinger equations for quantum universes. Section 4 derives wave functions and properties for the hydrogen atom of quantum universes. Subsection 4.1 is a mathematical check of the wave functions of the hydrogen atom of quantum universes. Section 5 defines and discusses atomic particle transitions of quantum universes. Section 6 shows physical laws of quantum universes. Section 7 defines and discusses electromagnetic waves, photons, and a theoretical hydrogen photon engine of quantum universes. Section 8 shows some of the consequences in quantum universes resulting from Albert Einstein's special theory of relativity. Section 9 is Conclusion of the study.

The Gaussian system of units (Gs) is used for deriving equations rather than the International System of Units (SI). Use of the Gaussian system of units importantly ensures that mechanical and electromagnetic units can be unambiguously derived from the same three base units. This is very important, in that it is necessary to derive important physical laws of quantum universes, e.g., Schrödinger equations for quantum universes and their wave functions.

2. Physical Constants of Quantum Universes

It was hypothesized that quantum states of quantum systems of quantum universes were made up of the same chemical elements as our quantum universe. It was hypothesized that each quantum universe has its own universe number k. It was hypothesized that quantum states in quantum universes are described by quantum numbers; universe k, principal n, azimuthal l, and magnetic m. It was hypothesized that quantum universes, quantum universe quantum states, quantum universe electromagnetic waves or photons, and physical constants of quantum universes are described by universe quantum numbers. It was hypothesized that subatomic particles, atoms, or quantum systems of quantum universes are never described by more than one universe quantum number. It was hypothesized that if a quantum system changed its universe quantum number it also changed the quantum universe to which it belongs. It was hypothesized that the universe quantum number k was equal to zero for our quantum universe. It was hypothesized that universe quantum numbers k are all integers. It was hypothesized that greater quantum numbers were associated with upper quantum universes. It was hypothesized that lesser quantum numbers were associated with lower quantum universes.

It was hypothesized that quantum numbers, universe k, principal n, azimuthal l, and magnetic m describe values of conserved quantities in the dynamics of a quantum system in quantum universes. It was hypothesized that quantum numbers can be defined as the sets of numerical values which give acceptable solutions to Schrödinger equations of the hydrogen atom for quantum universes. Section 4 shows that quantum numbers, universe k, principal n, azimuthal l, and magnetic m meet this definition of quantum numbers in quantum universes.

The mass of subatomic particles m, the reduced Planck's constant $\hbar$, the elementary charge e, and the speed of light c in a vacuum are thought to be physical constants whose numerical values never change with time. In this work, it was hypothesized that these physical constants whose numerical values never change with time depend on the quantum universe.

It was hypothesized that physical constants, physical properties, and Gaussian units took on the following equation format in quantum universes:

$$\beta_k = \alpha^{\mu k}\beta, \tag{1}$$

where k was a universe quantum number that was an integer, β_k were β in the kth quantum universe, α was the fine structure constant, μ was the coefficient of k where μ was an integer or a fraction of integers, and β was a physical constant, physical property, or Gaussian unit in our quantum universe. If two different β had identical Gaussian units then the value of μ was the same. If β was dimensionless then μ was zero.

This work was based on the following three hypotheses: First, and most important, was that most everything, e.g., subatomic particles, atoms, and electromagnetic waves of quantum universes, were discrete scale models of themselves in quantum universes. This hypothesis was identical with the equation

$$\Delta x_k = \alpha^{-k}\Delta x, \tag{2}$$

where Δx was a displacement in our quantum universe, Δx_k was the same discrete displacement in the kth quantum universe, and μ equaled negative one. Thus, particles, atoms, and electromagnetic waves of quantum universes would be discrete exact scale models of each other but they would vary greatly in discrete size. A quantum system that would change its universe quantum number would modify the discrete displacements of the quantum system.

Second, velocities were

$$v_k = \alpha^{-2k}v \tag{3}$$

where v was a velocity in our quantum universe, v_k was the same velocity in the kth quantum universe, and μ was equal to negative two. A particle that would change its universe quantum number would discretely modify the velocity of the particle.

Third, it was proposed that momentum of mass was conserved in quantum universes. This yielded the equations in quantum universes,

$$p_k = m_k v_k = \alpha^{ak} m \alpha^{-2k} v = \alpha^{0k} p, \tag{4}$$

where p was momentum in our quantum universe, p_k was same momentum in the kth quantum universe, m was a mass in our quantum universe, m_k was the same discrete mass in the kth quantum universe, μ for momentum p was zero because momentum of mass was conserved between quantum universes, and μ for mass was a.

The concept of mass-energy equivalence allows mass to be converted into energy and vice-versa. Thus, matter can discretely change its mass due to this concept. The coefficient μ for a mass equaled two when a was solved for in Eq. (4) so that

$$m_k = \alpha^{2k} m, \tag{5}$$

where discrete masses of matter in quantum universes were m_k and a mass of matter in our quantum universe was m.

Masses of subatomic particles in quantum universes were the same as Eq. (5). Then

$$m_k = \alpha^{2k} m, \tag{6}$$

where discrete masses of subatomic particles in quantum universes were m_k, and the mass of a subatomic particle in our quantum universe was m. Subatomic particle masses in quantum universes would be discretely created by atomic particle transitions (remotely similar to atomic electron transitions, see Section 5).

Electrons were subatomic particles and had units of mass; hence discrete rest masses of electrons in quantum universes were

$$m_{ek} = \alpha^{2k} m_e, \tag{7}$$

where discrete rest masses of electrons in quantum universes were m_{ek}, and the rest mass of an electron in our quantum universe was m_e. Discrete rest masses of electrons in quantum universes would be discretely created by atomic particle transitions.

Protons were subatomic particles and had units of mass; hence discrete rest masses of protons in quantum universes were

$$m_{pk} = \alpha^{2k} m_p, \tag{8}$$

where discrete rest masses of protons in quantum universes were m_{pk}

and the rest mass of a proton in our quantum universe was m_p. Discrete masses of protons in quantum universes would be discretely created by atomic particle transitions.

The speed of light in vacuum for quantum universes had units of velocity, so that μ equaled negative two the same as Eq. (3). Thus, discrete speeds of light in vacuum were

$$c_k = \alpha^{-2k} c, \tag{9}$$

where discrete speeds of light in a vacuum in quantum universes were c_k and the speed of light in vacuum in our quantum universe was c.

Table 1 shows the speed of light c_k in vacuum for quantum universes.

Table 1. Calculated approximate values of the speed of light in a vacuum for quantum universes

Universe k	c_k	Speed of light (m/s)
4	$\alpha^{-8}c$	3.72×10^{25}
3	$\alpha^{-6}c$	1.98×10^{21}
2	$\alpha^{-4}c$	1.06×10^{17}
1	$\alpha^{-2}c$	5.63×10^{12}
0	$\alpha^{0}c$	3.00×10^{8}
−1	$\alpha^{2}c$	1.60×10^{4}
−2	$\alpha^{4}c$	8.52×10^{-1}
−3	$\alpha^{6}c$	4.54×10^{-5}
−4	$\alpha^{8}c$	2.42×10^{-9}

The large values of the speed of light c_k in vacuum for positive quantum universe numbers in Table 1 were noted. Using the equation for the speed of light in vacuum Eq. (9) and $c = d_0/\Delta t_0$ it was noted that the length of time for light to travel one light year in quantum universes was

$$\Delta t_k = \frac{d_0}{c_k} = \frac{d_0}{\alpha^{-2k} c} = \frac{d_0}{\alpha^{-2k} \frac{d_0}{\Delta t_0}} = \frac{\Delta t_0}{\alpha^{-2k}}, \tag{10}$$

where Δt_k was the time it takes for light to travel a light year in quantum universe number k, d_0 is the distance light travels in one light year in our quantum universe, and Δt_0 is the time light takes to travel one light year in our quantum universe. Δt_0 is one year or 525600 minutes. When k was one, α^{-2k} is approximately 137^2. Thus, the time it took for light of quantum universe number one to travel a light year was approximately twenty-eight minutes. Δt_0 is one year or 31536000 seconds. When k was two α^{-2k} is approximately 137^4. Thus, the time it took for light of quantum universe number two to travel a light year was approximately 89.5 milliseconds.

The "classical electron radius" can be found from classical mechanics by equating $E = m_e c^2$ to $E = e^2 / r_e$. It is

$$r_e = \frac{e^2}{m_e c^2}, \tag{11}$$

where the classical electron radius is r_e and elementary charge is e. This equation was written in quantum universes as

$$r_{ek} = e_k^2 / m_{ek} c_k^2 = (\alpha^{jk} e)^2 / \alpha^{2k} m_e (\alpha^{-2k} c)^2$$

$$= \alpha^{-k} r_e = \alpha^{-k} e^2 / m_e c^2, \tag{12}$$

where r_e was the classical electron radius in our quantum universe, r_{ek} was the discrete electron radius in the kth quantum universe, discrete elementary charges of electrons and protons in quantum universes were e_k, the elementary charge in our quantum universe was e, μ for r_e was negative one because r_e had units of a displacement, and μ for the elementary charge was j.

Discrete elementary charges of electrons and protons in quantum universes were

$$e_k = \alpha^{-\frac{3}{2}k} e, \tag{13}$$

where μ for e was equal to j that was equal to negative three halves when j was solved for in Eq. (12).

The Bohr radius [1] is

$$a_0 = \hbar^2 / m_e e^2, \tag{14}$$

where a_0 is the Bohr radius and $\hbar$ is the reduced Planck's constant. This same equation in quantum universes was

$$a_k = \hbar_k^2 / m_{ek} e_k^2 = (\alpha^{bk}\hbar)^2 / \alpha^{2k} m_e \left(\alpha^{-\frac{3}{2}k} e \right)^2$$
$$= \alpha^{-k} a_0 = \alpha^{-k} \hbar^2 / m_e e^2, \tag{15}$$

where a_k were the discrete Bohr radii in quantum universes, discrete reduced Planck's constants in quantum universes were $\hbar_k$, μ for a_0 was negative one because a_0 had units of a displacement, and μ for the reduced Planck's constant was b.

Then, reduced Planck's constants in quantum universes were

$$\hbar_k = \alpha^{-k}\hbar, \tag{16}$$

where μ for $\hbar$ was equal to b that was equal to negative one when b was solved for in Eq. (15). From Eq. (16) Planck's constants in quantum universes were

$$h_k = \alpha^{-k}h, \tag{17}$$

where discrete Planck's constants in quantum universes were h_k and Planck's constant in our quantum universe was h.

The fine structure constant α [2] is

$$\alpha = e^2 / \hbar c. \tag{18}$$

The fine structure constants in quantum universes were

$$\alpha_k = e_k^2 / \hbar_k c_k = \left(\alpha^{-\frac{3}{2}k} e\right)^2 \bigg/ \alpha^{-k}\hbar\alpha^{-2k}c = \alpha^{fk}\alpha, \tag{19}$$

where α_k were the fine structure constants in quantum universes and μ for α was equal to f that was equal to zero when f was solved for in Eq. (19). In other words, the dimensionless fine structure constant α would be of the same value in all quantum universes.

The physical constants; the discrete rest masses of electrons m_{ek} Eq. (7), the discrete speeds of light in vacuum c_k Eq. (9), the discrete elementary charges of electrons or protons e_k Eq. (13), the discrete Bohr radii a_k Eq. (15), the discrete reduced Planck's constants $\hbar_k$ Eq. (16), the discrete Planck's constants h_k Eq. (17), of quantum universes can replace physical constants of our quantum universe to form laws of quantum universes.

3. Schrödinger Equations for Quantum Universes

It was hypothesized that immutable laws of physics, including the Schrödinger equation, applied to quantum universes. Laws of physics for quantum universes were found by replacing the classical physical constants in physical laws of our quantum universe with the appropriate physical constants of quantum universes. In this manner the nonrelativistic Schrödinger equations for quantum universes were found by replacing the classical physical constants in the Schrödinger equation with physical constants of quantum universes. This method was used throughout the paper to derive other laws of physics that applied to quantum universes. It should be noted that when this was done for immutable physical laws the results were always consistent.

The time-dependent Schrödinger equation [3] is usually given as a postulate of quantum theory.

$$i\hbar \frac{\partial}{\partial t} \psi(r, t) = \hat{H}\psi(r, t), \tag{20}$$

where the imaginary unit is i, a partial derivative with respect to time is indicated by $\partial/\partial t$, the Hamiltonian is $\hat{H}$, the wave function is $\psi(r, t)$, the radius of a wave function is r, and time is t.

This time-dependent Schrödinger equation was modified for quantum universes by replacing $\hbar$, $\psi(r, t)$, $\hat{H}$ with $\hbar_k$, $\psi_k(r, t)$, $\hat{H}_k$. Thus,

$$i\hbar_k \frac{\partial}{\partial t} \psi_k(r, t) = \hat{H}_k\psi_k(r, t), \tag{21}$$

where reduced Planck's constants in quantum universes were $\hbar_k$, wave functions in quantum universes were $\psi_k(r, t)$, and Hamiltonians in quantum universes were $\hat{H}_k$. These Schrödinger equations are entirely general, and can be solved for quantum systems of quantum universes.

The preceding equations were expanded into three-dimensional time-dependent Schrödinger equations of quantum universes for one electron quantum systems.

$$i\hbar_k \frac{\partial}{\partial t}\psi_k(r, t) = -\frac{\hbar_k^2}{2m_{ek}}\nabla^2\psi_k(r, t) + V_k(r, t)\psi_k(r, t), \tag{22}$$

where the Laplacian was ∇^2, rest masses of electrons in quantum universes were Eq. (7), and electrostatic potential energies of one electron quantum systems in quantum universes were $V_k(r, t)$. They described how quantum universe quantum states evolved over time.

Physical constants Eqs. (7), (16) were inserted into Eq. (22) to put it in terms of physical constants of our quantum universe that gave

$$i\alpha^{-k}\hbar \frac{\partial}{\partial t}\psi_k(r, t) = -\alpha^{-4k}\frac{\hbar^2}{2m_e}\nabla^2\psi_k(r, t) + V_k(r, t)\psi_k(r, t). \tag{23}$$

These were three-dimensional time-dependent Schrödinger equations for quantum universes for one electron quantum systems in terms of classical physical constants.

Now $\psi_k(r, t)$ was written as the product of $\psi_k(r)$ and $f_k(t)$. Eq. (23) was written as

$$\psi_k(r)i\alpha^{-k}\hbar\frac{df_k(t)}{dt} = f_k(t)\left[-\alpha^{-4k}\frac{\hbar^2}{2m_e}\nabla^2 + V_k(r)\right]\psi_k(r), \tag{24}$$

where the functions of time in quantum universes were $f_k(t)$, time-independent wave functions in quantum universes were $\psi_k(r)$, and the time-independent electrostatic potential energies of a one electron quantum system in quantum universes were $V_k(r)$.

Then variables were separated that gave

$$\frac{i\alpha^{-k}\hbar}{f_k(t)}\frac{df_k(t)}{dt} = \frac{1}{\psi_k(r)}\left[-\alpha^{-4k}\frac{\hbar^2}{2m_e}\nabla^2 + V_k(r)\right]\psi_k(r). \tag{25}$$

The left-hand sides of Eq. (24) were only a function of time and the right-hand sides were only a function of r. This was because both sides were equal to the same constants. Because the right-hand sides of Eq. (25) had the units of energy, the constants (that were total energy eigenvalues) were designated E_k.

The right-hand sides of Eq. (25) now became ordinary differential equations.

$$-\alpha^{-4k}\frac{\hbar^2}{2m_e}\nabla^2\psi_k(r) + V_k(r)\psi_k(r) = E_k\psi_k(r), \tag{26}$$

where the total energy eigenvalues in quantum universes were E_k. The latter equations were time-independent Schrödinger equations of quantum universes, of one electron quantum systems, in terms of classical physical constants and rectangular coordinates.

4. Wave Functions and Properties of the Hydrogen Atom of Quantum Universes

Time-independent Schrödinger Eq. (26) for the hydrogen atom in quantum universes were converted into polar coordinates;

$$-\alpha^{-4k}\frac{\hbar^2}{2m_e}\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi_k(r)}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi_k(r)}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2\psi_k(r)}{\partial\phi^2}\right] + V_k(r)\psi_k(r) = E_k\psi_k(r), \tag{27}$$

where the polar coordinates were r, θ, ϕ. These Schrödinger equations for the hydrogen atom assumed that the protons were at fixed positions (the Born-Oppenheimer approximation [4]). Slightly more accurate solutions would result from the protons being taken into account where the rest masses of the protons were given by Eq. (8).

Electrostatic potential energies for the hydrogen atom in quantum universes were found by inserting elementary charges Eq. (13) into the formula for potential energies.

$$V_k(r) = -\frac{e_k^2}{r} = -\frac{\left(\alpha^{-\frac{3}{2}k} e\right)^2}{r}. \tag{28}$$

Then Eq. (28) were substituted into Eq. (27) that gave

$$-\alpha^{-4k}\frac{\hbar^2}{2m_e}\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi_k(r)}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi_k(r)}{\partial\theta}\right)\right.$$
$$\left.+\frac{1}{\sin^2\theta}\frac{\partial^2\psi_k(r)}{\partial\phi^2}\right]-\alpha^{-3k}\frac{e^2}{r}\psi_k(r) = E_k\psi_k(r). \tag{29}$$

These were nonrelativistic time-independent Schrödinger equations for the hydrogen atom in quantum universes, in terms of classical physical constants, that were solved to find the radial wave functions and spherical harmonics of the hydrogen atom in quantum universes.

Wave functions $\psi_k(r)$ were factored into $R(r)_{(k)nl}Y_l^m(\theta, \phi)$, where $R(r)_{(k)nl}$ were radial wave functions in quantum universes and $Y_l^m(\theta, \phi)$ were spherical harmonics.

Radial wave functions $R(r)_{(k)nl}$ then obeyed

$$-\alpha^{-4k}\frac{\hbar^2}{2m_e}\frac{1}{r^2}\left[\frac{d}{dr}\left(r^2\frac{dR(r)_{(k)nl}}{dr}\right)+l(l+1)R(r)_{(k)nl}\right]$$

$$- \alpha^{-3k} \frac{e^2}{r} R(r)_{(k)nl} = E_k R(r)_{(k)nl}. \quad (30)$$

The latter equations were radial type equations for the hydrogen atom for quantum universes in terms of classical physical constants.

The method of replacing a physical constant of our quantum universe with the physical constants of quantum universes was used to find the radial wave functions $R(r)_{(k)nl}$ of the hydrogen atom in quantum universes. The radial wave function $R(r)_{nl}$ [5] of the hydrogen atom in our quantum universe is

$$R(r)_{nl} = \left(\frac{2}{na_0}\right)^{\frac{3}{2}} \left(\frac{(n-l-1)!}{2n[(n+l)!]^3}\right)^{\frac{1}{2}} e^{-\frac{r}{na_0}} \left(\frac{2r}{na_0}\right)^{l} L_{n-l-1}^{2l+1}\left(\frac{2r}{na_0}\right), \quad (31)$$

where the Bohr radius in our quantum universe is Eq. (14) and the functions $L_{n-l-1}^{2l+1}(2r/na_0)$ are the associated Laguerre polynomials of degree $n - l - 1$. The radial wave functions $R(r)_{(k)nl}$ of the hydrogen atom in quantum universes were found by replacing the physical constant, the Bohr radius Eq. (14) of our quantum universe, with the physical constants, the Bohr radii of quantum universes. Then

$$R(r)_{(k)nl} = \left(\frac{2}{na_k}\right)^{\frac{3}{2}} \left(\frac{(n-l-1)!}{2n[(n+l)!]^3}\right)^{\frac{1}{2}} e^{-\frac{r}{na_k}} \left(\frac{2r}{na_k}\right)^{l} L_{n-l-1}^{2l+1}\left(\frac{2r}{na_k}\right), \quad (32)$$

where Bohr radii in quantum universes were Eq. (15). Eq. (32) were the radial wave functions of the hydrogen atom in quantum universes.

The associated Laguerre polynomials were obtained according to the following formula:

$$L_{n-l-1}^{2l+1}\left(\frac{2r}{na_k}\right) = \sum_{i=0}^{n-l-1} \frac{(-1)^i [(n+l)!]^2 \left(\frac{2r}{na_k}\right)^i}{i!\,(n-l-1-i)!\,(2l+1+i)!}. \quad (33)$$

The total energy eigenvalue E [6] of the hydrogen atom is

$$E = -\frac{m_e e^4}{2\hbar^2 n^2}. \tag{34}$$

Discrete total energy eigenvalues E_k of the hydrogen atom in quantum universes from Eqs. (7), (13), (16) were

$$E_k = -\frac{m_{ek} e_k^4}{2\hbar_k^2 n^2} = -\frac{(\alpha^{2k} m_e)\left(\alpha^{-\frac{3}{2}k} e\right)^4}{2(\alpha^{-k}\hbar)^2 n^2} = -\frac{\alpha^{-2k} m_e e^4}{2\hbar^2 n^2}. \tag{35}$$

The term e^4 was replaced in Eq. (35) by its equivalent $e^4 = \alpha^2\hbar^2 c^2$ derived from the fine structure constant Eq. (18) that gave

$$E_k = -\frac{\alpha^{-2k} m_e \alpha^2 \hbar^2 c^2}{2\hbar^2 n^2} = -\alpha^{2-2k}\,\frac{m_e c^2}{2n^2}. \tag{36}$$

Energy eigenvalues E_k; allowed energy levels of the of the ground state of the hydrogen atom in Eq. (32) and Eq. (36) were mathematically checked in Eq. (30) in Subsection 4.1. Table 2 shows the total quantum universes.

Table 2. Calculated approximate values of allowed energy levels for the ground state of the hydrogen atom in quantum universes

Universe k	E_k	Energy level (eV)
4	$-\alpha^{-6}\dfrac{m_e c^2}{2}$	-1.69×10^{18}
3	$-\alpha^{-4}\dfrac{m_e c^2}{2}$	-8.99×10^{13}
2	$-\alpha^{-2}\dfrac{m_e c^2}{2}$	-4.79×10^{9}
1	$-\dfrac{m_e c^2}{2}$	-2.55×10^{5}
0	$-\alpha^{2}\dfrac{m_e c^2}{2}$	-13.6
-1	$-\alpha^{4}\dfrac{m_e c^2}{2}$	-7.25×10^{-4}
-2	$-\alpha^{6}\dfrac{m_e c^2}{2}$	-3.86×10^{-8}
-3	$-\alpha^{8}\dfrac{m_e c^2}{2}$	-2.06×10^{-12}
-4	$-\alpha^{10}\dfrac{m_e c^2}{2}$	-1.10×10^{-16}

The left-hand sides of Eq. (25) derived other ordinary differential equations for the hydrogen atom in quantum universes where the independent variable was time. This gave

$$\frac{i\alpha^{-k}\hbar}{f_k(t)}\frac{df_k(t)}{dt} = E_k. \tag{37}$$

This is the same as

$$\frac{1}{f_k(t)}\frac{df_k(t)}{dt} = -i\alpha^k\,\frac{E_k}{\hbar}. \tag{38}$$

These equations were solved to yield

$$f_k(t) = e^{-i\alpha^k \frac{E_k}{\hbar}t}. \tag{39}$$

Substituting Eq. (35) into Eq. (39) gave $f_k(t)$ for the hydrogen atom.

$$f_k(t) = e^{-i\alpha^k \frac{-\alpha^{2-2k}\frac{m_e c^2}{2n^2}}{\hbar}t} = e^{i\alpha^{2-k}\frac{m_e c^2}{2\hbar n^2}t}. \tag{40}$$

From above, the wave functions $\psi_k(r)$ were the product of $R(r)_{(k)nl}$ Eq. (32) and $Y_l^m(\theta, \phi)$. From Section 3 and above, the wave functions of the hydrogen atoms in quantum universes $\psi_k(r, t)$ were the product of $\psi_k(r)$ and $f_k(t)$ Eq. (40). Thus, $\psi_k(r, t)$ in terms of its three factors were

$$\psi_k(r, t) = \left(\frac{2}{na_k}\right)^{\frac{3}{2}}\left(\frac{(n-l-1)!}{2n[(n+l)!]^3}\right)^{\frac{1}{2}} e^{-\frac{r}{na_k}}\left(\frac{2r}{na_k}\right)^l$$

$$\times L_{n-l-1}^{2l+1}\left(\frac{2r}{na_k}\right)Y_l^m(\theta, \phi)e^{i\alpha^{2-k}\frac{m_e c^2}{2\hbar n^2}t}. \tag{41}$$

Eq. (41) were the wavefunctions of the hydrogen atom in quantum universes $\psi_k(r, t)$. Thus, universe k, principal n, azimuthal l, and magnetic m quantum numbers gave acceptable solutions to Schrödinger equations for the hydrogen atom in quantum universes. And, the quantum numbers of quantum universes met the same definition as the quantum numbers of our quantum universe.

Conservation of energy says, the energy of a particle is equal to its kinetic energy plus its potential energy. In other words, the kinetic energy plus the electrostatic potential energy of the electron of the hydrogen atom is equal to its total energy eigenvalue where the potential energy and total energy eigenvalue are negative. And, total energy eigenvalues are equal to one half potential energies for the hydrogen atom. This meant that the total energy eigenvalues except for sign were equal to the rotational kinetic energies of the electron of the hydrogen atom in quantum universes. This was true whether the electron orbital was circular or not (consequence of the virial theorem [7]). Thus, it was for the hydrogen atom that the absolute values of total energy eigenvalues E_k were equal to rotational kinetic energies of the electron in quantum universes.

$$\mid E_k \mid = \frac{1}{2} m_{ek} v_k^2, \tag{42}$$

where $\mid E_k \mid$ were the absolute value of discrete total energy eigenvalues of the hydrogen atom of quantum universes, discrete masses of the electron of the hydrogen atom in quantum universes were m_{ek}, and discrete tangential velocities of the electron of the hydrogen atom in quantum universes were v_k. Solving for v_k gave

$$v_k = \sqrt{\frac{2\mid E_k \mid}{m_{ek}}} = \alpha^{-k} \sqrt{\frac{2\mid E_k \mid}{m_e}}. \tag{43}$$

Substituting total energy eigenvalues Eq. (36) into Eq. (43) gave

$$v_k = \alpha^{-k} \sqrt{\frac{2\left(\alpha^{2-2k} \dfrac{m_e c^2}{2n^2}\right)}{m_e}} = \alpha^{-k} \sqrt{\alpha^{2-2k} \frac{c^2}{n^2}} = \alpha^{1-2k} \frac{c}{n}. \tag{44}$$

The ratio of the discrete tangential velocities of the electron of the hydrogen atom Eq. (44) to the discrete speeds of light in a vacuum Eq. (9) in quantum universes were

$$\frac{v_k}{c_k} = \frac{\alpha^{1-2k} c}{\alpha^{-2k} cn} = \frac{\alpha}{n}. \tag{45}$$

Thus, the ratio of the tangential velocities of the electron of the hydrogen atom to the speeds of light in a vacuum were α divided by the principal quantum number n for all quantum numbers in all quantum universes.

From Eq. (40) discrete angular frequencies of the wave function of the hydrogen atom in quantum universes were

$$\omega_k' = \alpha^{2-k} \frac{m_e c^2}{2\hbar n^2}, \tag{46}$$

where the discrete angular frequencies of the wave function of the hydrogen atom in quantum universes were ω_k'. And discrete ordinary frequencies, from Eq. (46), of the wave function of the hydrogen atom in quantum universes, were

$$f_k' = \alpha^{2-k} \frac{m_e c^2}{2\pi 2\hbar n^2} = \alpha^{2-k} \frac{m_e c^2}{2hn^2}, \tag{47}$$

where the discrete ordinary frequencies of the wave function of the hydrogen atom in quantum universes were f_k'.

The discrete de Broglie wavelengths [8] of the electron of the hydrogen atom in quantum universes were

$$\lambda_k = \frac{h_k}{p_k} = \frac{h_k}{m_{ek} v_k}, \tag{48}$$

where the discrete de Broglie wavelengths of the electron of the hydrogen atom in quantum universes were λ_k and the momentum of the electron of the hydrogen atom in quantum universes was p_k. Then m_{ek} was replaced by $\alpha^k \hbar^2 / a_k e^2$ derived from Eq. (15), h_k was replaced by $\alpha^{-k} h$ from Eq. (17), and v_k was replaced by $\alpha^{1-2k} \frac{c}{n}$ from Eq. (44) in Eq. (48) that gave

$$\lambda_k = \frac{\alpha^{-k} h}{\alpha^k \dfrac{\hbar^2}{a_k e^2} \alpha^{1-2k} \dfrac{c}{n}} = \frac{h}{\dfrac{\hbar^2}{a_k e^2} \alpha \dfrac{c}{n}} = \frac{2\pi a_k n e^2}{\alpha \hbar c}. \tag{49}$$

Then, because $e^2 = \alpha \hbar c$ derived from Eq. (18), the de Broglie wavelengths of the electron of the hydrogen atom in quantum universes λ_k were

$$\lambda_k = 2\pi a_k n. \tag{50}$$

It was hypothesized that the electron of the hydrogen atom in quantum universes followed a circular orbit. So, the discrete radii of the electron of the hydrogen atom in quantum universes were

$$r_k = a_k n, \tag{51}$$

where the discrete radii of the electron of the hydrogen atom in quantum universes were r_k. Then, it was hypothesized that the wavefunction of the hydrogen atom in quantum universes also followed a circular orbit. And it was hypothesized that discrete radii of the wavefunction of the hydrogen atom in quantum universes were the same as the radii of the electron orbit Eq. (51). Thus,

$$r'_k = a_k n, \tag{52}$$

where the discrete radii of the wave function of the hydrogen atom in quantum universes were r'_k.

Then the discrete tangential velocities of the electron of the hydrogen atom in quantum universes were

$$v_k = \frac{2\pi r_k}{t_k}, \tag{53}$$

where the discrete tangential velocities of the electron of the hydrogen atom in quantum universes were v_k, and the time periods for one rotation of the electron orbit of the hydrogen atom in quantum universes were t_k. Then, because t_k were equal to one divided by the discrete ordinary frequencies of the electron orbit of the hydrogen atom in quantum universes,

$$v_k = 2\pi r_k f_k, \tag{54}$$

where the discrete ordinary frequencies of the electron orbit of the hydrogen atom in quantum universes were f_k. Then,

$$f_k = \frac{v_k}{2\pi r_k}. \tag{55}$$

Substituting Eq. (44) and Eq. (51) into Eq. (55) the discrete ordinary frequencies of the electron orbit were

$$f_k = \frac{\alpha^{1-2k} c}{2\pi a_k n^2}. \tag{56}$$

Now substituting Eq. (15) into Eq. (56) and $e^2 = \alpha\hbar c$ from Eq. (18) into Eq. (15) the discrete ordinary frequencies of the electron orbit became

$$f_k = \frac{\alpha^{1-2k} c m_e e^2}{2\pi\alpha^{-k}\hbar^2 n^2} = \frac{\alpha^{1-2k} c m_e \alpha\hbar c}{2\pi\alpha^{-k}\hbar^2 n^2} = \alpha^{2-k}\,\frac{m_e c^2}{h n^2}. \tag{57}$$

Thus, the discrete ordinary frequencies of the wave function of the hydrogen atom in quantum universes Eq. (47) were half the discrete ordinary frequencies of the electron orbit Eq. (57) of the hydrogen atom in quantum universes.

The discrete tangential velocities of the wave function of the hydrogen atom in quantum universes were

$$v'_k = \frac{2\pi r'_k}{t'_k} = 2\pi a_k n f'_k, \tag{58}$$

where the tangential velocities of the wave function of the hydrogen atom of quantum universes were v'_k, the radii of the wave function r'_k in quantum universes were Eq. (52), the time periods for one rotation of the wave function of the hydrogen atom in quantum universes were t'_k, and t'_k were equal to one divided by the discrete ordinary frequencies f'_k of the wave function orbit of the hydrogen atom in quantum universes. When Eq. (15) and Eq. (47) were substituted into Eq. (58) the latter equations became

$$v'_k = \frac{2\pi\alpha^{-k}\hbar^2 n}{m_e e^2}\frac{\alpha^{2-k} m_e c^2}{2hn^2} = \frac{\alpha^{2-2k}\hbar c^2}{e^2 n}. \tag{59}$$

When $e^2 = \alpha\hbar c$ from Eq. (18) was substituted into Eq. (59) the discrete velocities of the wave function of the hydrogen atom of quantum universes v'_k became

$$v'_k = \frac{\alpha^{2-2k}\hbar c^2}{\alpha\hbar c n} = \alpha^{1-2k}\frac{c}{n}. \tag{60}$$

Thus, the discrete velocities of the wave function of the hydrogen atom Eq. (60) were the same as the discrete velocities of the electron of the hydrogen atom Eq. (44) in quantum universes.

The wave functions Eq. (41) and properties of the hydrogen atom in quantum universes were shown to be real. Thus, wave functions or quantum universe quantum states of all atoms of the chemical elements were real in quantum universes, by virtue of the Schrödinger Eq. (20) for quantum universes. And therefore, quantum universes can exist.

To ensure the wave functions Eq. (41) of the hydrogen atom in quantum universes were correct, they were mathematically checked in Subsection 4.1.

4.1. A mathematical check of the wave functions of the hydrogen atom of quantum universes

Only one radial wave function Eq. (32) could be checked at a time. The radial wave function $R_{(k)10}$ $(k = k,\ n = 1,\ l = 0)$, corresponding to the ground state of the hydrogen atom, was selected to be checked. Thus, the wave functions of the ground state of the hydrogen atom of quantum universes $\psi_{(k)10}$ needed to be looked at. These wave functions were made up of factors $R_{(k)10}$, Y_0^m, $f_k(t)$.

Checking the radial wave functions $R_{(k)10}$, determined from the radial wave functions Eq. (32), were lengthy. The spherical harmonics Y_0^m were not checked because they remained unchanged and were not related to the universe quantum number. The functions of time in quantum universes Eq. (40) were checked once for all quantum numbers.

The ground state radial wave functions $R_{(k)10}$ for the hydrogen atoms were compiled from Eq. (32).

$$R(r)_{(k)10} = \left(\frac{2}{a_k}\right)^{\frac{3}{2}}\left(\frac{1}{2}\right)^{\frac{1}{2}} e^{-\frac{r}{a_k}} L_0^1\left(\frac{2r}{a_k}\right) = \frac{2}{a_k^{\frac{3}{2}}} e^{-\frac{r}{a_k}} L_0^1\left(\frac{2r}{a_k}\right). \tag{61}$$

And, the associated Laguerre polynomial Eq. (33) with $n = 1$ and $l = 0$ of degree zero were

$$L_0^1\left(\frac{2r}{a_k}\right) = \sum_{i=0}^{0} \frac{(-1)^0 [1!]^2 \left(\frac{2r}{a_k}\right)^0}{0!\,(0)!\,(1)!} = 1. \tag{62}$$

Thus, substituting Eq. (62) into the ground state radial wave functions Eq. (61), we had

$$R_{(k)10} = \frac{2e^{-\frac{r}{a^k}}}{a_k^{\frac{3}{2}}}. \tag{63}$$

Bohr radii of quantum universes Eq. (15) were substituted into ground state radial wave functions Eq. (63) to put them in terms of classical physical constants.

$$R_{(k)10} = \frac{2e^{\frac{-r}{\alpha^{-k}\frac{\hbar^2}{m_e e^2}}}}{\left(\alpha^{-k}\frac{\hbar^2}{m_e e^2}\right)^{\frac{3}{2}}} = \frac{2}{\alpha^{-\frac{3}{2}k}\left(\frac{\hbar^2}{m_e e^2}\right)^{\frac{3}{2}}} e^{\frac{-r}{\alpha^{-k}\frac{\hbar^2}{m_e e^2}}}$$

$$= \frac{2\alpha^{\frac{3}{2}k} m_e^{\frac{3}{2}} e^3}{\hbar^3} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}. \tag{64}$$

Then, radial wave functions Eq. (64) derived from Eq. (32) were shown to be solutions of the radial type Eq. (30). This was accomplished in three steps. Radial wave function Eq. (64) were substituted into the left-hand sides of the radial type Eq. (30) to obtain left hand side expressions. Radial wave functions Eq. (64) were substituted into the right-hand sides of the radial type Eq. (30) to obtain right hand side expressions. When the two expressions matched the radial wave functions Eq. (64) were solutions.

Eq. (64) were substituted into the left-hand sides of radial type Eq. (30) that had l equal to zero and the expressions were

$$-\alpha^{-4k}\frac{\hbar^2}{2m_e}\frac{1}{r^2}\left[\frac{d}{dr}\left(r^2\frac{d}{dr}\left[\frac{2\alpha^{\frac{3}{2}k} m_e^{\frac{3}{2}} e^3}{\hbar^3} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}\right]\right)\right]$$

$$-\alpha^{-3k}\frac{e^2}{r}\frac{2\alpha^{\frac{3}{2}k}m_e^{\frac{3}{2}}e^3}{\hbar^3}e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}. \tag{65}$$

These were simplified. The latter expressions became

$$-\frac{\alpha^{-4k}\hbar^2}{2m_e}\frac{1}{r^2}\left[\frac{d}{dr}\left(r^2\frac{d}{dr}\left[\frac{2\alpha^{\frac{3}{2}k}m_e^{\frac{3}{2}}e^3}{\hbar^3}e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}\right]\right)\right]$$

$$-\frac{2\alpha^{-\frac{3}{2}k}m_e^{\frac{3}{2}}e^5}{\hbar^3 r}e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}. \tag{66}$$

Next the first inner derivatives were evaluated and we had

$$-\frac{\alpha^{-4k}\hbar^2}{2m_e}\frac{1}{r^2}\left[\frac{d}{dr}\left(-\frac{2\alpha^{\frac{5}{2}k}m_e^{\frac{5}{2}}e^5}{\hbar^5}r^2 e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}\right)\right]$$

$$-\frac{2\alpha^{-\frac{3}{2}k}m_e^{\frac{3}{2}}e^5}{\hbar^3 r}e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}. \tag{67}$$

Then the second remaining derivatives were evaluated and the latter expressions became

$$-\frac{\alpha^{-4k}\hbar^2}{2m_e}\frac{1}{r^2}\left[-\frac{4\alpha^{\frac{5}{2}k}m_e^{\frac{5}{2}}e^5}{\hbar^5}e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}r+\frac{2\alpha^{\frac{7}{2}k}m_e^{\frac{7}{2}}e^7}{\hbar^7}r^2 e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}\right]$$

$$-\frac{2\alpha^{-\frac{3}{2}k}m_e^{\frac{3}{2}}e^5}{\hbar^3 r}e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}. \tag{68}$$

The two terms in the brackets were multiplied by the initial terms and the latter expressions were

$$\frac{2\alpha^{-\frac{3}{2}k} m_e^{\frac{3}{2}} e^5}{\hbar^3 r} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} - \frac{\alpha^{-\frac{1}{2}k} m_e^{\frac{5}{2}} e^7}{\hbar^5} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}$$

$$- \frac{2\alpha^{-\frac{3}{2}k} m_e^{\frac{3}{2}} e^5}{\hbar^3 r} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} . \tag{69}$$

Because the first and third terms were equal except for sign, the left-hand sides of radial type Eq. (30) in terms of classical physical constants were

$$- \frac{\alpha^{-\frac{1}{2}k} m_e^{\frac{5}{2}} e^7}{\hbar^5} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} . \tag{70}$$

Next the right-hand sides of the radial type Eq. (30) were addressed. When Eq. (64) and Eq. (36) were substituted into the right-hand sides of radial type Eq. (30) with n equal to one and l equal to zero, these expressions became

$$- \alpha^2 \alpha^{-2k} \frac{m_e c^2}{2} \frac{2\alpha^{\frac{3}{2}k} m_e^{\frac{3}{2}} e^3}{\hbar^3} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} . \tag{71}$$

When Eq. (18) squared was substituted for α^2 in Eq. (71) we had

$$- \left(\frac{e^2}{\hbar c}\right) \alpha^{-2k} \frac{m_e c^2}{2} \frac{2\alpha^{\frac{3}{2}k} m_e^{\frac{3}{2}} e^3}{\hbar^3} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}}$$

$$= -\alpha^{-2k} \frac{m_e e^4}{2\hbar^2} \frac{2\alpha^{\frac{3}{2}k} m_e^{\frac{3}{2}} e^3}{\hbar^3} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} . \tag{72}$$

When terms were simplified, the latter expressions became

$$- \frac{\alpha^{-\frac{1}{2}k} m_e^{\frac{5}{2}} e^7}{\hbar^5} e^{\frac{-\alpha^k m_e e^2 r}{\hbar^2}} . \tag{73}$$

Thus, it was seen that the left-hand sides of the radial type Eq. (30), expressions Eq. (70), matched the right-hand sides of the radial type Eq. (30), expressions Eq. (73). Therefore, ground state radial wave functions Eq. (64) and the total energy eigenvalue Eq. (36) of the hydrogen atom were correct for principal quantum number one, azimuthal quantum number zero, and any universe quantum number.

The functions of time Eq. (40) were checked as solutions to their source Eq. (38) for all quantum numbers. Eq. (36) and Eq. (40) were substituted into source Eq. (38) and we had

$$\frac{1}{e^{i\alpha^{2-k}\frac{m_e c^2}{2\hbar n^2}t}}\frac{d\left[e^{i\alpha^{2-k}\frac{m_e c^2}{2\hbar n^2}t}\right]}{dt} = -i\alpha^k\frac{-\alpha^{2-2k}\frac{m_e c^2}{2n^2}}{\hbar}. \tag{74}$$

This is the same as

$$\frac{1}{e^{i(\alpha)^2\alpha^{-k}\frac{m_e c^2}{2\hbar n^2}t}}\frac{d\left[e^{i(\alpha)^2\alpha^{-k}\frac{m_e c^2}{2\hbar n^2}t}\right]}{dt} = -i\alpha^k\frac{-(\alpha)^2\alpha^{-2k}\frac{m_e c^2}{2n^2}}{\hbar}. \tag{75}$$

When Eq. (18) was substituted for α in $(\alpha)^2$ in Eq. (75) we had

$$\frac{1}{e^{i\left(\frac{e^2}{\hbar c}\right)^2\alpha^{-k}\frac{m_e c^2}{2\hbar n^2}t}}\frac{d\left[e^{i\left(\frac{e^2}{\hbar c}\right)^2\alpha^{-k}\frac{m_e c^2}{2\hbar n^2}t}\right]}{dt} = -i\alpha^k\frac{-\left(\frac{e^2}{\hbar c}\right)^2\alpha^{-2k}\frac{m_e c^2}{2n^2}}{\hbar}. \tag{76}$$

This can be simplified to

$$\frac{1}{e^{i\frac{\alpha^{-k}m_e e^4}{2\hbar^3 n^2}t}} \frac{d\left[e^{i\frac{\alpha^{-k}m_e e^4}{2\hbar^3 n^2}t}\right]}{dt} = i\alpha^{-k}\frac{m_e e^4}{2\hbar^3 n^2}. \quad (77)$$

Derivatives were taken on the left-hand sides of the latter equations and we had

$$i\alpha^{-k}\frac{m_e e^4}{2\hbar^3 n^2} = i\alpha^{-k}\frac{m_e e^4}{2\hbar^3 n^2}. \quad (78)$$

The left-hand sides of Eq. (38) matched the right-hand sides and hence Eq. (40) checked for all quantum numbers.

It was now shown that ground state wave functions of the hydrogen atom $\psi_{(k)10m} = R_{(k)10}Y_0^m f_k(t)$ were correct for principal quantum number one, azimuthal quantum number zero, any magnetic quantum number, and any universe quantum number. This means the wave functions for the ground state of the hydrogen atom Eq. (41) in quantum universes were correct and real. Thus, wave functions or quantum universe quantum states of quantum systems of the chemical elements were real in quantum universes by virtue of the Schrödinger Eq. (20). And, it was validated that quantum universes can exist.

5. Atomic Particle Transitions of Quantum Universes

An atomic particle transition of the hydrogen atom would be a change of the electron, from one energy level to another, along with a change in the discrete mass and size of the electron and proton. It changes the electron to a vastly different discrete energy level. An atomic particle transition of the hydrogen atom would produce a hydrogen atom that was a scale model of itself. It would appear discontinuous as the electron "jumps" from one energy level to another. An atomic particle transition of the hydrogen atom would change the quantum universe quantum number, the quantum universe state, and the quantum universe of the hydrogen atom.

An atomic particle transition and/or an atomic electron transition of the hydrogen atom would result in the emission or absorption of a discrete photon. Because total energy needs to be conserved, the discrete energy of a photon emitted or absorbed would be equal to the difference in total energy eigenvalues Eq. (36). Thus,

$$E_{i/f} = -\alpha^{2-2k_i} \frac{m_e c^2}{2n_i^2} - \left[-\alpha^{2-2k_f} \frac{m_e c^2}{2n_f^2} \right]$$

$$= \alpha^2 \frac{m_e c^2}{2} \left[\frac{\alpha^{-2k_f}}{n_f^2} - \frac{\alpha^{-2k_i}}{n_i^2} \right], \tag{79}$$

where the energy of the photon emitted or absorbed from the electron of the hydrogen atom in quantum universes was $E_{i/f}$, k_i was the initial universe quantum number of the atomic particle transition of the hydrogen atom, k_f was the final universe quantum number of the atomic particle transition of the hydrogen atom, n_i was the initial principal quantum number of the atomic electron transition of the hydrogen atom, n_f was the final principal quantum number of the atomic electron transition of the hydrogen atom, i/f represents the initial i and final f universe or universe quantum number to which something refers in an atomic electron transition and/or an atomic particle transition. If the energy was positive the photon was emitted and conversely if the energy was negative the photon was absorbed.

An electron of a hydrogen atom would absorb or emit electromagnetic energy in a discrete packet or photon that had a definite energy. Table 3 shows the magnitude of the discrete energy of a photon Eq. (79) that would be required to be emitted or absorbed when the ground state of the hydrogen atom jumped between quantum universes.

Table 3. Calculated approximate values of the magnitude of the discrete energy of a photon that would be required to be emitted or absorbed by the electron of the ground state of the hydrogen atom that jumped between quantum universes

Universe k	$E_{i/f}$ absorbed/emitted	Magnitude of $E_{i/f}$ (eV)
0/4	$\alpha^{-6}\,\frac{m_e c^2}{2}$	1.69×10^{18}
0/3	$\alpha^{-4}\,\frac{m_e c^2}{2}$	8.99×10^{13}
0/2	$\alpha^{-2}\,\frac{m_e c^2}{2}$	4.79×10^{9}
0/1	$\frac{m_e c^2}{2}$	2.55×10^{5}
0/−1	$\alpha^{2}\,\frac{m_e c^2}{2}$	13.6
−1/−2	$-\,\alpha^{4}\,\frac{m_e c^2}{2}$	7.25×10^{-4}
−2/−3	$-\,\alpha^{6}\,\frac{m_e c^2}{2}$	3.86×10^{-8}
−3/−4	$-\,\alpha^{8}\,\frac{m_e c^2}{2}$	2.06×10^{-12}

The large amount of photon energy emitted when the ground state of the hydrogen atom jumped to a positive quantum universe, as shown in Table 3, was noted. When the ground state of the hydrogen atom jumped from quantum universe number zero to quantum universe number one the photon energy emitted by the electron was approximately equal to one half the equivalent energy of the rest mass of the electron, $m_e c^2 / 2$, as shown in Table 3. This photon energy emitted was approximately equal to the magnitude of the allowed energy level of the ground state of the hydrogen atom for quantum universe number one, $-m_e c^2 / 2$, as shown in Table 2.

Since the photon absorbed or emitted by the electron of the hydrogen atom that jumped between different energy levels would require a definite energy, the photon absorbed or emitted must have a definite wavelength. Wavelength is found from the Planck-Einstein relation $E = h\nu$ [9] and the formula $f = c/\lambda$. That gives us

$$\lambda = c/f = c/\frac{E}{h} = \frac{hc}{E}, \tag{80}$$

where frequency is ν or f, wavelength is λ, and the energy of a photon is E. Because E is positive, and $E_{i/f}$ could be negative, it was necessary for E to be equal to the absolute value of Eq. (79). Thus, Eq. (80) in quantum universes was

$$\lambda_{i/f} = \frac{h_{k_i} c_{k_i}}{|E_{i/f}|} = \frac{\alpha^{-k_i} h \alpha^{-2k_i} c}{\left| \alpha^2 \frac{m_e c^2}{2} \left[\frac{\alpha^{-2k_f}}{n_f^2} - \frac{\alpha^{-2k_i}}{n_i^2} \right] \right|}$$

$$= \frac{2\alpha^{-2-3k_i} h}{m_e c \left| \left[\frac{\alpha^{-2k_f}}{n_f^2} - \frac{\alpha^{-2k_i}}{n_i^2} \right] \right|}, \tag{81}$$

where k was k_i, $\lambda_{i/f}$ was the wavelength of the photon absorbed or emitted by the electron of the hydrogen atom in quantum universe k_i, $h_{k_i} = \alpha^{-k_i} h$ was Eq. (17) where k was k_i, $c_{k_i} = \alpha^{-2k_i} c$ was Eq. (9) where k was k_i and $\left| E_{i/f} \right|$ was the absolute value of the energy of the photon, defined by Eq. (79) that was absorbed or emitted. Table 4 shows the discrete wavelength Eq. (81) of the photon whose energy would be absorbed or emitted by the electron of the ground state of the hydrogen atom when the hydrogen atom jumped between energy levels.

Table 4. Calculated approximate values of the discrete wavelength of the photon whose energy would be absorbed or emitted by the electron of the ground state of the hydrogen atom that jumped between quantum universes.

Universe k	$\lambda_{i/f}$	Wavelength (m)
0/4	$2\alpha^6 h/m_e c$	7.34×10^{-25}
0/3	$2\alpha^4 h/m_e c$	1.38×10^{-20}
0/2	$2\alpha^2 h/m_e c$	2.58×10^{-16}
0/1	$2h/m_e c$	4.85×10^{-12}
0/−1	$2\alpha^{-2} h/m_e c$	9.10×10^{-8}
−1/−2	$2\alpha^{-1} h/m_e c$	6.64×10^{-10}
−2/−3	$2h/m_e c$	4.85×10^{-12}
−3/−4	$2\alpha h/m_e c$	3.54×10^{-14}

The discrete approximate wavelength of the photon absorbed or emitted when the ground state of the hydrogen atom jumped between quantum universe numbers zero and one, of 4.85×10^{-12} meters $(6.19 \times 10^{19}$ Hz) as shown in Table 4, would be an X-ray. The discrete approximate wavelength of the photon absorbed or emitted when the ground state of the hydrogen atom jumped between quantum universes numbers zero and negative one, of 9.10×10^{-8} meters $(3.30 \times 10^{15}\ \text{Hz})$ as shown in Table 4, would be ultraviolet light.

It was hypothesized that multiple atomic particle transitions would produce a reaction engine (see Section 7), producing thrust by ejecting photon relativistic mass rearward, in accordance with Newton's third law. The reaction engine thrust could be used to propel a vehicle. This propulsion system operates on hydrogen fuel. Then the exhaust would be hydrogen transitioned into quantum universe k_f. The concept provides energetic economic ecological electromagnetic energy that propagates at the speed of light in a vacuum c_{k_i}. This theoretical engine would make present engines obsolete. Using a multiple simultaneous atomic particle transitions concept, it was hypothesized that this vehicle could be transitioned between consecutive quantum universe quantum states. Once it was possible to transition a self-contained vehicle up or down one quantum universe quantum state it would automatically be possible to transition up or down to the next quantum universe quantum state. This happens because what works in one quantum universe quantum state, works relatively the same in another quantum universe quantum state. This would provide, among other things, the availability of velocity in upper quantum universe quantum states described by Eq. (3) and acceleration in upper quantum universe quantum states described by Eq. (84) (see Section 6).

6. Physical Laws of Quantum Universes

From Eqs. (2), (3) and the fact that velocities are displacements divided by the corresponding time periods, the following equations were written for quantum universes:

$$\Delta t_k = \frac{\Delta x_k}{v_k} = \frac{\alpha^{-k}\Delta x}{\alpha^{-2k}v} = \frac{\alpha^{-k}\Delta x}{\alpha^{-2k}\frac{\Delta x}{\Delta t}} = \alpha^k \Delta t, \tag{82}$$

where discrete time periods in quantum universes were Δt_k and the time period in our quantum universe was Δt. This was interpreted to mean that a time period depended on the particular universe quantum number of the quantum system involved. In other words, a time period, as demonstrated by the time between ticks of a clock was determined by the universe quantum number of the physical system of the clock. A positive universe quantum number would hasten time and a negative universe quantum number would dilate time in a discrete manner.

Frequency is one divided by the time period. Thus, from Eq. (82), discrete frequencies of quantum universes were

$$f_k = \frac{1}{\Delta t_k} = \frac{1}{\alpha^k \Delta t} = \alpha^{-k}\frac{1}{\Delta t} = \alpha^{-k} f, \tag{83}$$

where discrete frequencies of quantum universes were f_k, frequency in our quantum universe was f, and the period of frequency f was Δt. Discrete frequencies would jump significantly between quantum universes. This rationalizes the significantly different discrete time periods in quantum universes.

Acceleration is the derivative of velocity; thus, substituting Eq. (3) and the equivalent $dt_k = \alpha^k dt$ from Eq. (82) gave

$$a_k = \frac{d}{dt_k}v_k = \frac{d}{\alpha^k dt}\alpha^{-2k}v = \alpha^{-3k}\frac{d}{dt}v = \alpha^{-3k}a, \tag{84}$$

where accelerations in quantum universes were a_k and acceleration in our quantum universe was a.

Atomic density of matter is mass per unit volume; thus, from Eqs. (2), (5)

$$\rho_k = \frac{m_k}{V_k} = \frac{\alpha^{2k} m}{\alpha^{-3k} \Delta x^3} = \alpha^{5k} \rho, \tag{85}$$

where discrete atomic densities in quantum universes were ρ_k, atomic density in our quantum universe was ρ, and discrete volumes in quantum universes were V_k. This is a huge difference between quantum universes.

When masses of particles Eq. (6) and the speeds of light in a vacuum Eq. (9) in quantum universes were substituted into the equations for mass-energy equivalence [10] in quantum universes,

$$E_k = m_k c_k^2 = \alpha^{2k} m (\alpha^{-2k} c)^2 = \alpha^{-2k} m c^2 = \alpha^{-2k} E, \tag{86}$$

where discrete equivalent energies of particles in quantum universes were E_k, discrete equivalent energy of a particle in our quantum universe was E, discrete masses of subatomic particles in quantum universes were m_k, and the mass of a subatomic particle in our quantum universe was m. Thus, a particle of an upper quantum universe had less discrete mass but more discrete equivalent energy. Particles of a lower quantum universe had more discrete mass but less discrete equivalent energy.

7. Electromagnetic Waves and Photons of Quantum Universes

Discrete ordinary frequencies of electromagnetic waves and photons in quantum universes were given by Eq. (83). It was hypothesized that the electromagnetic waves with discrete ordinary frequencies f_k were of the same nature as the electromagnetic waves with ordinary frequency f. The "same nature" meant that if an electromagnetic wave with ordinary frequency f exhibited properties like those of a radio wave, light wave, microwave, X-ray, or gamma ray then the electromagnetic waves with discrete ordinary frequencies f_k exhibited the same properties in its own quantum universe. Thus, interactions of electromagnetic waves and quantum systems were relatively similar no matter which quantum universe you were in except for time.

Wavelength is equal to the speed of light divided by its frequency of oscillation. When Eqs. (9), (83) were substituted into this equation for quantum universes we had

$$\lambda_k = \frac{c_k}{f_k} = \frac{\alpha^{-2k} c}{\alpha^{-k} f} = \alpha^{-k}\lambda, \tag{87}$$

where discrete wavelengths of the electromagnetic wave in quantum universes were λ_k, and the wavelength of an electromagnetic wave in our quantum universe was λ. It was hypothesized that electromagnetic waves and photons of quantum universes, related by the fine structure constant to other physical properties besides frequency, were of the same nature. Thus, the electromagnetic waves of quantum universes with discrete wavelengths λ_k would be of the same nature as the electromagnetic waves of our quantum universe with wavelength λ.

Momentum of a photon in quantum universes, derived from the rearranged energy-momentum relation [11] that had zero rest mass, would be

$$p_k = E_K / c_k = \frac{\alpha^{-2k} E}{\alpha^{-2k} c} = p, \tag{88}$$

where μ for E and c were negative two as the Gaussian units are the same as Eqs. (86), (9), momentum of a photon in quantum universes were p_k, momentum of a photon in our quantum universe was p, discrete energy of a photon in quantum universes were E_k, and the energy of a photon was E in our quantum universe. It was hypothesized in quantum universes that the relativistic masses of photons m_{relk} in quantum universes equaled $\alpha^{2k} m_{rel}$ in quantum universes, the same as masses of subatomic particles in quantum universes Eq. (6). Thus,

$$m_{relk} = \alpha^{2k} m_{rel}, \tag{89}$$

where discrete relativistic masses of photons in quantum universes were m_{relk} and the relativistic mass of a photon was m_{rel} in our quantum universe.

And, it was hypothesized that the momentum of a photon, using its relativistic mass, was the same as the momentum of a subatomic particle; that was equal to its mass multiplied by its velocity. Then, substituting Eq. (9) and Eq. (89) into the equation for the momentum of a photon in quantum universes gives

$$p_k = m_{relk} c_k = \alpha^{2k} m_{rel} \alpha^{-2k} c = m_{rel} c = p, \tag{90}$$

where momentum of a photon in quantum universes were p_k, and momentum of a photon in our quantum universe was p. Thus, photons of the same nature would be of the same momentum and vice versa. It was noted that this result was the same as that determined by Eq. (88).

Eq. (90) was substituted into the energy-momentum relation [11] for quantum universes that had zero rest mass, that gave

$$E_k = p_k c_k = m_{relk} c_k^2 = \alpha^{2k} m_{rel} (\alpha^{-2k} c)^2 = \alpha^{-2k} m_{rel} c^2 = \alpha^{-2k} E, \tag{91}$$

where discrete energy of photons in quantum universes were E_k and energy of a photon in our quantum universe was E. Thus, were the equations $E_k = m_{relk} c_k^2$ Eq. (91) for photons symbolically the same as $E_k = m_k c_k^2$ Eq. (86) for particles. The photons of quantum universes with discrete energies E_k would be of the same nature as a photon of our quantum universe with energy E.

Formulas for relativistic masses of photons in quantum universes, were found by equating Eq. (91), immediately above, to the Planck-Einstein relation $E = h\nu$ [9] in quantum universes. Thus,

$$m_{relk} = \frac{h_k f_k}{c_k^2} = \frac{\alpha^{-k} h \alpha^{-k} f}{\alpha^{-4k} c^2} = \alpha^{2k} \frac{hf}{c^2} = \alpha^{2k} m_{rel}, \tag{92}$$

where frequency is ν discrete frequencies in quantum universes were f_k, and frequency in our quantum universe was f. Thus, photons of quantum universes with discrete relativistic masses m_{relk} would be of the same nature as a photon of our quantum universe with relativistic mass m_{rel}.

It was hypothesized that a photon reaction engine, that runs on hydrogen, could be obtained in quantum universes by harnessing the energy of photons emitted from the electrons of hydrogen atoms during multiple atomic particle transitions.

Eq. (79) was substituted for E into the rearranged Planck-Einstein relation $E = hf$ [9] in quantum universes where the frequency f was the frequency $f_{i/f}$. Then the frequency $f_{i/f}$ of the emitted or absorbed photon of an atomic particle transition and/or an atomic electron transition of the hydrogen atom in quantum universes was

$$f_{i/f} = \frac{|E_{i/f}|}{h_{k_i}} = \frac{|E_{i/f}|}{\alpha^{-k_i} h} = \alpha^{2+k_i} \frac{m_e c^2}{2h} \left| \left[\frac{\alpha^{-2k_f}}{n_f^2} - \frac{\alpha^{-2k_i}}{n_i^2} \right] \right|, \tag{93}$$

where k was k_i.

To determine thrust of a hydrogen photon reaction engine the rate of photons emitted from the engine must be specified. The frequency of the emitted photons $f_{i/f}$ was selected for this rate. Thus, the electromagnetic wave or thrust would be continuous. For a body whose mass is constant, Newton's second law of motion can be expressed as $F = ma$. The equation for thrust of a hydrogen photon reaction engine in quantum universes was derived from this law. Thus, the thrust of the hydrogen photon reaction engine emitting $f_{i/f}$ photons per second was

$$T_{i/f} = F_{i/f} = M_{relk_i}\alpha_{k_i} = \frac{f_{i/f}m_{rel\,k_i}f_{i/f}D_{k_i}}{(f_{i/f}t_{i/f})^2} = \frac{m_{rel\,k_i}}{t_{i/f}}\frac{D_{k_i}}{t_{i/f}}$$

$$= \dot{m}_{relk_i}v_{k_i} = \dot{m}_{relk_i}c_{k_i}, \qquad (94)$$

where $T_{i/f}$ was the thrust caused by a stream of photons emitted from the photon reaction engine, at frequency $f_{i/f}$, for one second in quantum universe k_i, $F_{i/f}$ was the total force caused by a stream of photons emitted from the photon reaction engine, at frequency $f_{i/f}$, for one second in quantum universe k_i, M_{relk_i} was the total photon relativistic mass emitted by the reaction engine in one second in quantum universe k_i, a_{k_i} was the hypothetical acceleration of the photons emitted for one second from the reaction engine in quantum universe k_i, m_{relk_i} was the relativistic mass of a single photon being emitted from the reaction engine in quantum universe k_i, D_{k_i} was the distance a single emitted photon would radiate during one rotation of the electron of the hydrogen atom in quantum universe k_i, $t_{i/f}$ was the reciprocal of $f_{i/f}$, equal to the time period for one photon to be emitted from the reaction engine in quantum universe k_i, $\dot{m}_{relk_i}$ was the relativistic mass flow rate of photons being emitted from the reaction engine at frequency $f_{i/f}$ for one second in quantum universe k_i, and v_{k_i} was the velocity of photons emitted from the reaction engine at frequency $f_{i/f}$ for one second in quantum universe k_i.

From Eq. (94) the relativistic mass flow rate of the photons being emitted from the reaction engine in quantum universes was

$$\dot{m}_{relk_i} = \frac{m_{relk_i}}{t_{i/f}} = m_{relk_i} f_{i/f}, \tag{95}$$

where the time period $t_{i/f}$ was equal to one divided by the frequency $f_{i/f}$ of the emitted photons for one second in quantum universe k_i. Substituting Eq. (92) in quantum universes, where f equals $f_{i/f}$ and k equals k_i and Eq. (93) into Eq. (95) the relativistic mass flow rate of photons emitted from the reaction engine was

$$\begin{aligned} \dot{m}_{relk_i} &= \alpha^{2k_i} \frac{hf_{i/f}}{c^2} \alpha^{2+k_i} \frac{m_e c^2}{2h} \left| \left[\frac{\alpha^{-2k_f}}{n_f^2} - \frac{\alpha^{-2k_i}}{n_i^2} \right] \right| \\ &= \alpha^{2+3k_i} \frac{f_{i/f} m_e}{2} \left| \left[\frac{\alpha^{-2k_f}}{n_f^2} - \frac{\alpha^{-2k_i}}{n_i^2} \right] \right|. \end{aligned} \tag{96}$$

Substituting Eq. (93) into Eq. (96) the relativistic mass flow rate $\dot{m}_{relk_i}$ of the photons emitted from the reaction engine was then

$$\begin{aligned} \dot{m}_{relk_i} &= \alpha^{2+3k_i} \frac{\alpha^{2+k_i} \frac{m_e c^2}{2h} \left| \left[\frac{\alpha^{-2k_f}}{n_f^2} - \frac{\alpha^{-2k_i}}{n_i^2} \right] \right|}{2} \left| \left[\frac{\alpha^{-2k_f}}{n_f^2} - \frac{\alpha^{-2k_i}}{n_i^2} \right] \right| \\ &= \alpha^{4+4k_i} \frac{m_e^2 c^2}{4h} \left| \left[\frac{\alpha^{-2k_f}}{n_f^2} - \frac{\alpha^{-2k_i}}{n_i^2} \right] \right|^2. \end{aligned} \tag{97}$$

Substituting Eq. (97) and Eq. (9) where k was k_i into Eq. (94) the hydrogen photon reaction engine thrust was

$$T_{i/f} = \alpha^{4+4k_i} \frac{m_e^2 c^2}{4h} \left| \left[\frac{\alpha^{-2k_f}}{n_f^2} - \frac{\alpha^{-2k_i}}{n_i^2} \right] \right|^2 \alpha^{-2k_i} c$$

$$= \alpha^{4+2k_i} \frac{m_e^2 c^3}{4h} \left| \left[\frac{\alpha^{-2k_f}}{n_f^2} - \frac{\alpha^{-2k_i}}{n_i^2} \right] \right|^2, \tag{98}$$

where $T_{i/f}$ was the thrust caused by a stream of photons emitted from the hydrogen photon reaction engine, at frequency $f_{i/f}$, for one second in quantum universe k_i. The number $f_{i/f}$ was the number of hydrogen atoms transitioned in one second to obtain the thrust generated for one second in quantum universe k_i.

Power is work divided by time. Work is force multiplied by distance. Distance divided by time is velocity. Thrust has the same units as force. Thus, the power of the hydrogen photon reaction engine $P_{i/f}$ for one second was

$$P_{i/f} = \frac{W_{i/f}}{f_{i/f} t_{i/f}} = \frac{F_{i/f} f_{i/f} D_{k_i}}{f_{i/f} t_{i/f}} = F_{i/f} v_{k_i} = T_{i/f} c_{k_i}, \tag{99}$$

where $P_{i/f}$ was the power of the hydrogen photon reaction engine radiating photons, at frequency $f_{i/f}$, for one second in quantum universe k_i and $W_{i/f}$ was the total work done by the photon reaction engine radiating photons, at frequency $f_{i/f}$, for one second in quantum universe k_i. Then substituting Eq. (98) and Eq. (9) where k was k_i into Eq. (99), the power of a hydrogen photon reaction engine was

$$P_{i/f} = \alpha^{4+4k_i} \frac{m_e^2 c^3}{4h} \left| \left[\frac{\alpha^{-2k_f}}{n_f^2} - \frac{\alpha^{-2k_i}}{n_i^2} \right] \right|^2 \alpha^{-2k_i} c$$

$$= \alpha^{4} \frac{m_e^2 c^4}{4h} \left| \left[\frac{\alpha^{-2k_f}}{n_f^2} - \frac{\alpha^{-2k_i}}{n_i^2} \right] \right|^2. \tag{100}$$

The number $f_{i/f}$ was the number of hydrogen atoms transitioned in one second to obtain the power $P_{i/f}$ generated for one second in quantum universe k_i.

The i/f ratio of a hydrogen photon reaction engine needs to be specified to determine the engines thrust. The approximate comparison of the thrust of a theoretical hydrogen photon reaction engine, where the i/f ratio was $0/2$, with a solid-propellant rocket engine in the vacuum of outer space, was as follows:

The photon energy emitted by a mole $(1.01\,\text{g})$ of hydrogen, where i/f was $0/2$ from Table 3, assuming 100% efficiency, would be approximately

$$E_{0/2} = N_A \alpha^{-2} \frac{m_e c^2}{2}, \tag{101}$$

where $E_{0/2}$ was the photon energy of a hydrogen photon reaction engine and N_A is Avogadro's number. The thrust of the hydrogen photon engine in the vacuum of outer space, from a rearranged Eq. (99), where the dimension of power is energy divided by time, substituting Eq. (101), would be

$$T_{0/2} = \frac{P_{0/2}}{c_0} = \frac{E_{0/2}}{tc} = \frac{N_A \alpha^{-2} m_e c}{2t}, \tag{102}$$

where $T_{0/2}$ was the thrust of the hydrogen photon reaction engine for $1.01 \times 10^{-3}\,\text{kg}$ of transitioned hydrogen atoms, and t was time.

Thrust of a solid-propellant rocket engine in the vacuum of outer space, assuming 100% efficiency, equals mass flow rate multiplied by the exhaust velocity. From this equation the thrust of a solid propellant rocket engine would be

$$T = \dot{m}V_e = \frac{mV_e}{t}, \tag{103}$$

where T is the thrust of a solid-propellant rocket engine, $\dot{m}$ is the mass flow rate, V_e is the exhaust velocity, m is the mass of the solid-propellant, and t is time.

The ratio of the two thrusts, Eq. (102) divided by Eq. (103), where the mass of the transitioned hydrogen atoms and solid-propellant was the same $1.01 \times 10^{-3}\,\mathrm{kg}$, would be approximately

$$\frac{T_{0/2}}{T} = \frac{N_A \alpha^{-2} m_e c}{2mV_e} = 4.75 \times 10^5, \tag{104}$$

where N_A is 6.02×10^{23}, α^{-2} is 137^2, the mass of m_e is $9.11 \times 10^{-31}\,\mathrm{kg}$, c is 3.00×10^8 m/s, m is $1.01 \times 10^{-3}\,\mathrm{kg}$, and V_e is the velocity of exhaust equal to 7200 mph (3219 m/s) for a high-end solid-propellant rocket.

Thus, the thrust of the hydrogen photon reaction engine would be approximately 4.75×10^5 times greater than the thrust of the solid-propellant rocket engine where the mass of the transitioned hydrogen was equal to the mass of the solid-propellant. $T_{0/1}/T$ would reduce this ratio by α^{-2} to 25.3 times greater.

8. Consequences of Einstein's Special Theory of Relativity in Quantum Universes

Lorentz factors in quantum universes were found by substituting instantaneous velocities Eq. (3) and speeds of light in a vacuum Eq. (9) into the formula for the Lorentz factor [12].

$$\gamma_k = \frac{1}{\sqrt{1 - v_k^2/c_k^2}} = \frac{1}{\sqrt{1 - (\alpha^{-2k}v)^2/(\alpha^{-2k}c)^2}}$$

$$= \frac{1}{\sqrt{1 - v^2/c^2}} = \gamma, \tag{105}$$

where Lorentz factors in quantum universes were γ_k and the Lorentz factor in our quantum universe was γ. Thus, the Lorentz factor would be a dimensionless physical factor that would not depend on the universe quantum number. This means that when a moving quantum system underwent a change in its universe quantum number or multiple simultaneous atomic particle transitions, its dimensionless Lorentz factor would remain the same.

The equation for relativistic mass-energy equivalence is given from Albert Einstein's special theory of relativity [13] as,

$$E = m_{rel}c^2 = \frac{1}{\sqrt{1 - v^2/c^2}}\, mc^2 = \gamma mc^2, \tag{106}$$

where E is the relativistic equivalent energy of a particle, m_{rel} is the relativistic mass of a particle in our quantum universe, and m is the mass of a particle in our quantum universe. Substituting Eqs. (6), (9), (105) into Eq. (106) in quantum universes, relativistic mass-energy equivalence in quantum universes were

$$E_k = \gamma_k m_k c_k^2 = \gamma\alpha^{2k} m(\alpha^{-2k}c)^2 = \alpha^{-2k}\gamma mc^2$$

$$= \alpha^{-2k} \frac{1}{\sqrt{1 - {v_k}^2/{c_k}^2}}\, mc^2, \tag{107}$$

where E_k were discrete relativistic equivalent energy of a particle in quantum universes. Thus, velocity of a particle in a quantum universe would be limited to below the speed of light in a vacuum for its universe quantum number.

Relativistic time dilation indicates that, for an observer in an inertial frame of reference, a clock that is moving relative to them will measure a difference in elapsed time less than a clock that is at rest in its frame of reference. When time periods of quantum universes Eq. (82) were inserted for a classical time period the concept of relativistic time dilation $(\Delta t' = \gamma \Delta t)$ [14] was written in quantum universes as

$$\Delta t'_k = \gamma \Delta t_k = \alpha^k \gamma \Delta t, \tag{108}$$

where relativistic time periods in quantum universes, for quantum systems traveling at nonzero velocities in quantum universes, were $\Delta t'_k$. Relativistic time in quantum universes would be dilated or hastened.

Relativistic frequency is one divided by the relativistic time period. Thus, from Eq. (108), the relativistic frequencies of quantum universes were

$$f'_k = \frac{1}{\Delta t'_k} = \alpha^{-k} \frac{1}{\gamma \Delta t} = \alpha^{-k} \frac{f}{\gamma}, \tag{109}$$

where f'_k were relativistic frequencies in quantum universes. It was hypothesized that relativistic time periods, relativistic frequencies, and their relationships applies to quantum systems of quantum universes. Thus, time would be a property of a quantum system, where a quantum universe time period or quantum universe relativistic time period was related to the electron orbital frequencies of the atoms of the quantum system.

Relativistic length contraction is the phenomenon where the length of a moving object undergoes a contraction along the dimension of motion as seen from the stationary reference frame. When displacements in quantum universes Eq. (2) were substituted for the classical displacement

Δx the concept of relativistic length contraction $(\Delta x' = \Delta x/\gamma)$ [14] was written in quantum universes as

$$\Delta x'_k = \frac{\Delta x_k}{\gamma} = \alpha^{-k} \frac{\Delta x}{\gamma}, \tag{110}$$

where relativistic lengths along the dimension of motion in quantum universes, of quantum systems traveling at nonzero velocities in quantum universes, were $\Delta x'_k$. Thus, relativistic length in quantum universes would be contracted or extended in quantum universes. But as shown in the next paragraph, length contraction or extension in quantum universes would be part of size contraction or inflation.

The equations developing the properties of the Bohr radius of the hydrogen atom in quantum universes were nonrelativistic. It was hypothesized that the relativistic Bohr radius of the hydrogen atom at rest in our quantum universe was

$$a'_0 = a_0/\gamma', \tag{111}$$

where the relativistic Bohr radius of the hydrogen atom at rest in our quantum universe was a'_0 and γ' was the Lorentz factor determined by the velocity of the electron in the hydrogen atom at rest in our quantum universe. When Eq. (14) was substituted into Eq. (111),

$$a'_0 = \frac{\hbar^2}{m_e e^2 \gamma'}. \tag{112}$$

Because the relativistic mass of the electron was the relativistic mass of a particle, the relativistic mass of the electron in the hydrogen atom in our quantum universe was determined from Eq. (106) that gave

$$m_{erel} = \gamma m_e, \tag{113}$$

where the relativistic mass of the electron of the hydrogen atom in our quantum universe was m_{erel}, the rest mass of the electron in the hydrogen atom in our quantum universe was m_e, and the Lorentz factor γ was due to the velocity of the hydrogen atom in our quantum universe. Thus, substituting the relativistic mass of the electron Eq, (113) for m_e in Eq. (112),

$$\dot{a}_0' = \frac{\hbar^2}{\gamma m_e e^2 \gamma'}, \tag{114}$$

where the relativistic Bohr radius of the hydrogen atom in motion in our quantum universe was $\dot{a}_0'$. Substituting Eqs. (7), (13), (16) into Eq. (114) in quantum universes the relativistic Bohr radii of the hydrogen atom in motion in quantum universes were

$$\dot{a}_0' = \frac{\hbar_k{}^2}{\gamma m_{ek} e_k{}^2 \gamma'} = \alpha^{-k} \frac{\hbar^2}{\gamma m_e e^2 \gamma'}, \tag{115}$$

where the relativistic Bohr radii of the electron orbits of the hydrogen atom in motion in quantum universes were $\dot{a}_0'$. It was hypothesized from similarity to Eq. (115) that relativistic radii of the electron orbits of all atoms in motion in quantum universes were

$$\dot{r}_{relk} = \alpha^{-k} \frac{\dot{r}}{\gamma}, \tag{116}$$

where $\dot{r}_{relk}$ were relativistic radii, from the nuclei to the electrons, of atoms in motion in the kth quantum universe and $\dot{r}^{\cdot}$ were the same relativistic radii with the atoms at rest in our quantum universe. It was hypothesized, based on Eq. (116) and a similar relationship for the displacements of subatomic particles, that atoms of a quantum system of quantum universes would remain scale models of themselves under relativistic conditions. Then, when α^{-k}/γ in Eq. (116) equaled one there would be no change in the size of a quantum system, from the size at rest in our quantum universe, when a quantum system traveled at this discrete speed way beyond the speed of light in a vacuum c. And, if α^{-k}/γ in Eq. (116) equaled one, then $\alpha^{k}\gamma$ in Eq. (108) would equal one, and there would be no change in the size or time of the quantum system, from the size or time at rest in our quantum universe, when a quantum system traveled at this discrete speed way beyond the speed of light in a vacuum c.

It might be interesting to know, at what discrete speed of a quantum system relativistic phenomenon is eliminated. From Eq. (105) the Lorentz factor for a quantum system in motion in quantum universes was

$$\gamma = \frac{1}{\sqrt{1 - {v_k}^2/{c_k}^2}} = \frac{1}{\sqrt{1 - \beta^2}}, \tag{117}$$

where $\beta = v_k/c_k$. From this it was determined that beta was

$$\beta = \frac{v_k}{c_k} = \sqrt{1 - \left(\frac{1}{\gamma}\right)^2}. \tag{118}$$

When relativistic phenomenon, involving a quantum system, is eliminated, α^{-k}/γ of Eq. (116) would equal one, and gamma would be equal to α^{-k}. So, for a quantum system in quantum universes

$$\beta = \frac{v_k}{c_k} = \sqrt{1 - \left(\frac{1}{\alpha^{-k}}\right)^2}. \tag{119}$$

For quantum universe number one α^{-k} would be approximately 137.04. Then beta for a quantum system in quantum universe number one would be approximately

$$\beta = \frac{v_1}{c_1} = \sqrt{1 - \left(\frac{1}{137.04}\right)^2}, \tag{120}$$

where v_1 would be the velocity of a quantum system in quantum universe number one. Numerically the velocity of the quantum system in quantum universe number one would be approximately

$$v_1 = .99997c_1. \tag{121}$$

Thus, nonrelativistic travel in quantum universe number one must be extremely close to the quantum universe's speed of light in a vacuum c_1. Nonrelativistic travel in greater upper quantum universes would require the velocity to be much closer to the speed of light in a vacuum c_k.

9. Conclusion

The purpose of this work was to investigate if particles could travel faster than the speed of light in a vacuum c, without violating special relativity. It is concluded that there could be particles in the vacuum of space in upper quantum universe states, that could travel faster than the speed of light in a vacuum c, that do not violate the theory of special relativity. However, the velocity for a particle is limited to below the speed of light in a vacuum c_k of its quantum universe. Discrete speeds of light in a vacuum increase with greater universe quantum numbers, and decrease with lessor universe quantum numbers. The relative velocity of a particle is commensurate with the relative mass of a particle in quantum universes. The relative velocity of a photon is commensurate with the relative relativistic mass of a photon in quantum universes.

This work has provided the first plausible theory of multiple universes named quantum universes. Limitless quantum universes would comprise everything that can exist; the entirety of space, matter, energy, time, and the physical constants and laws that describe them. Each quantum universe would be made up entirely and exclusively of its own discrete quantum universe quantum states of quantum systems.

Discrete wave functions and discrete properties of the hydrogen atom in quantum universes, were analytically derived. And, the wave functions of the hydrogen atom of quantum universes were shown to be valid, by a mathematical check of the ground state of the hydrogen atom in quantum universes. Thus, it is concluded that Schrödinger equations of quantum universes can solve quantum systems of quantum universes.

The hydrogen atom can be transitioned between quantum universe quantum states by an atomic particle transition. After the transition, the hydrogen atom is a scale model of its self. In this process a photon is emitted or absorbed whose energy level, frequency, and wavelength were determined.

The equations that relate the wavelength of an electromagnetic wave to its frequency in quantum universes were determined.

Powerful ecological economic hydrogen photon engines are theoretically possible that operate on the principle of multiple atomic particle transitions.

It is concluded that there are discrete speeds, closely approaching an upper quantum universe's speed of light in a vacuum c_k, where it is theoretically possible to travel with no change in size or time from that at rest in our universe.

It is concluded that time is a property of a quantum system, where a quantum universe time period or quantum universe relativistic time period is related to the electron orbital frequencies of the atoms of the quantum system. Time passes very fast in upper rarer less massive quantum universes. Time passes very slowly in lower denser more massive quantum universes.

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