Fundamental Journal of Modern Physics
ISSN: 2249-9768
Volume 19, Issue 1, 2023, Pages 107-148
This paper is available online at http://www.frdint.com/
Published online March 31, 2023

# INCONSISTENCY BETWEEN THE EINSTEIN EQUATION AND $E = mc^2$, THE REPULSIVE GRAVITATION AND THE QUESTION OF BLACK HOLES

**C. Y. LO**

Applied and Pure Research Institute
15 Walnut Hill Rd., Amherst, NH 03031
USA

**Abstract**

The Einstein equation shows that the electromagnetic energy is incompatible with mass since the electromagnetic energy-momentum tensor has trace zero and thus cannot change the Rici curvature $R$. Hence, general $E = mc^2$ and the Einstein equation are inconsistent. Moreover, experiments show that a piece of heated-up metal has a reduced weight, instead of increased weight as Einstein predicted. Thus, the existence of repulsive gravity is confirmed. This implies that Penrose's 1965 proof, based on gravity is always attractive, for the existence of black holes is invalid. Physicists, in particular the Wheeler School, fail to see the nonexistence of dynamic solutions for the Einstein equation due to mathematical deficiency and inadequacy in physics.

Keywords and phrases: $E - mc^2$, charge-mass interaction, repulsive gravitation, black holes.

Received January 23, 2023; Accepted February 8, 2023

"*Unthinking respect for authority is the greatest enemy of truth*" - A. Einstein

## 1. Introduction

It was believed that the Einstein's theory would be completely correct because it predicts the bending of light rays [1]. Moreover, it also obtained the remaining perihelion of Mercury [2]. However, A. Gullstrand [3] pointed out that Einstein failed to derive this rigorously because he failed to show the existence of a bounded dynamic solution for the Einstein equation that is necessary for the perturbation. Thus, Einstein's general relativity is defective.

The formula $E = mc^2$ and the non-linear Einstein equation were considered as the major achievements of Einstein. In fact, there are monumental buildings on $E = mc^2$ in Beijing, Taipei, Berlin, and etc. This is due to that physicists all over the world failed to see that the two claims are actually not consistent (see Section 2). We shall show that while Einstein is a genius in some physical intuition, he can be defective in some simple logical consistency. For instance, Einstein's covariance principle is proven invalid by explicit examples [4].

Although the bending light rays made Einstein famous, the formula $E = mc^2$ exposed the shortcomings of his theory. Einstein predicted the weight of a piece of metal increases as the temperature increases. Experiments [5, 6] show, however, that the weight is reduced (see Appendix A). This error is due to that Einstein overlooked the existence of repulsive gravitation because Einstein incorrectly believed the general validity of $E = mc^2$ [7].

It follows that a repulsive gravitational force is discovered and Einstein's notion of gravitational mass is proven invalid. Thus, the 1965 proof of Penrose [8] on the existence of black holes is invalid since he used a false assumption. Apparently, the 2020 Nobel Committee for Physics was not aware of the existence of repulsive gravitation [7].

## 2. The Inconsistency between the Einstein Equation and the Formula $E = mc^2$

Many physicists seem to choose ignoring the experiments that would show the existence of repulsive gravitation. This is not due to a lack of funds (because an electronic scale with an accuracy of $10^{-4}$g costs less than $ \$ 500.00$) but due to such a repulsive force seem to imply the failure of string theory since it could not produce such a force and also the proof of Penrose [8] on the existence of black holes is based on an invalid assumption.

Note that the photonic energy is the sum of related electromagnetic wave energy $E_e$ and the gravitation wave energy $E_g$ [9, 10] instead of just the electromagnetic energy as Einstein assumed. Thus, although the photonic energy is equivalent to mass, this would mean $E_e + E_g = mc^2$ instead of $E_e = mc^2$. However, the conversion of energies is still possible and energy conservation can be maintained.

In fact, the general validity of the formula $E = mc^2$ is actually an error in Einstein's theory, because it is theoretically inconsistent with the non-linear Einstein equation,

$$G_{\mu\nu} = R_{\mu\nu} - (1/2)g_{\mu\nu}R = -KT_{\mu\nu}, \qquad (1)$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Rici curvature tensor, $g_{\mu\nu}$ is the space-time metric, $R = g^{\mu\nu}R_{\mu\nu}$ and $T_{\mu\nu}$ is the sum of energy-momentum tensors, and $K$ is the coupling constant [11].

Einstein has mistaken that all energies are essentially the same. However, in fact, there are at least two kinds of energy: one kind is associated with an energy-momentum tensor with a zero trace such as the electromagnetic energy, and another kind is associated with a non-zero trace energy-momentum tensor such as mass. These two kinds of energy, however, are not equivalent with each other according to general relativity.

To see the inconsistence, we need to do only some simple algebra. From Einstein equation (1), we have

$$R = Kg^{\mu\nu}T_{\mu\nu} \quad \text{because} \quad g^{\mu\nu}g_{\mu\nu} = 4. \tag{2}$$

Since the electromagnetic energy-momentum tensor $T(E)_{\mu\nu}$ is traceless ($g^{\mu\nu}T(E)_{\mu\nu} = 0$), electromagnetic energy cannot affect the Rici curvature $R$. Thus the electromagnetic energy $E_e$ cannot be equivalent to mass since the mass can affect $R$. This conclusion is independent of whether the coupling constants have the same sign.

Note that the validity of eq. (2) depends only on the static Einstein equation. Naturally, the problem would only be the inadequately verified $E = mc^2$. Since eq. (2) was first derived by Einstein himself in his book, "The Meaning of Relativity" [11], overlooking this inconsistency is clearly his oversight. Thus, the invalidity of $E = mc^2$ is not only due to an existence of the repulsive gravitation [7].

There are videos that Einstein explained incorrectly why $E = mc^2$. However, the electromagnetic energy is the first energy that associated with a traceless energy-momentum tensor, and there is no proof for the general formula $m = E / c^2$. In fact, Einstein [12] tried in 1905-1909 to give a general proof for this formula, but failed. This should be expected from formula (2). Thus, Einstein actually did not understand his own theory well.

Einstein has mistaken that mass and energy are always equivalent as shown in Einstein's wrong personal video! In fact, Einstein has made other errors on the question of energy. For example, Einstein assumed that the photonic energy is the energy of a massless particle, but failed to see such an assumption alone is not compatible with the energy momentum tensor of an electromagnetic wave in Maxwell's theory.

Interestingly, a counter example on his prediction is provided by Einstein himself [3]. He [3] claimed, "An increase of $E$ in the amount of energy must be accompanied by an increase of $E / c^2$ in the mass (although $m = E / c^2$ actually has never been proven). I can easily supply energy to the mass - for instance, if I heat it by ten degrees. So why not measure the mass increase or weight increase connected with this change? The trouble here is that in the mass increase the enormous factor $c^2$ occurs in denominator of the fraction. In such a case the increase is too small to be measured directly; even with the most sensitive balance." Thus, Einstein failed to see his error on energy.

However, Einstein did not show it with a real experiment because of technology limitation of his time, but his favorite thought experiment. Experimentally, however, an increase of temperature for a metal not only did not produce an increase of weight, but on the contrary a measurable reduction of weight [5, 6]. Moreover, there are other examples of weight reduction due to added energy [7, 13] such as a charged metal ball and a charged capacitor. Thus, $E = mc^2$ has been proven as not generally true and the blind faith toward Einstein is broken.

The real problem is that Einstein had developed a blind faith on his physical intuition since his successive special relativity. Another example is that he assumed that the photon is massless although this alone is inconsistent with the energy-momentum tensor of an electromagnetic wave. Moreover, his notion of gravitational mass is due to his incorrect physical intuition because he overlooked an existence of the repulsive gravitational force [7].

## 3. The Weight Reduction of Metal as its Temperature Increases

Based on the unproven speculation $E = mc^2$, Einstein [3] claimed in 1946 that the weight of a metal piece would increase as its temperature increases. However, in 2003 experiments of A. L. Dmitriev, E. M. Nikushchenko and V. S. Snegov [5] (see Appendix A) and in 2010 of L. Z. Fan, J. S. Feng and Q. W. Liu [6] show that the weights of metals actually are reduced as the temperature increases. Thus, $E = mc^2$ is not experimentally generally valid.

However, since Einstein assumed the inertial mass is equivalent to the gravitational mass, in general relativity weight and inertial mass are indistinguishable although Einstein [11] noted that the inertial mass of an object is related to its resistance to acceleration and its weight is related to the attraction to Earth. However, Einstein was able to identify them because *the existence of repulsive gravitational force has not been recognized* [7, 13]. Now, since an existence of repulsive gravitational force has been confirmed, mass and weight actually cannot be identified.

Thus, mass and weight are different, and there is a growth of repulsive gravitational force as the temperature increases. This would be in agreement with the philosophy of Lao Tze about 2,500 years ago. He claimed that for any existing force, there is another force against it such that matter would not be over concentrated. Thus, the existence of a repulsive gravitational force would be appropriate, according to Lao Tze.

However, we must be able to distinguish mass and gravity with experiments. To prove the existence of such a repulsive force the experiment must involve both mass acceleration and gravitational attraction. For such a purpose, the measurement of the period $T$ of a pendulum would be appropriate. The first approximation of a formula for the period $T$ of a pendulum is as follows [14]:

$$T \approx 2\pi\sqrt{l/g}, \tag{3}$$

where $l$ is the length of the pendulum and $g$ is the gravitational acceleration. Thus, the mass change of the pendulum would not change its period $T$, but if the $g$ changes, the period $T$ of the pendulum will be changed.

Thus, a reduction of the mass or gravity can be distinguished by using it as a pendulum. It has been verified by Liu [15] that the mass is essentially the same as Einstein [3] and Lo [16] predicted, but the period $T$ is extended after heated-up. Lo [17] also verified with a torsion balance scale that the lead balls have reduced gravitation after heated-up. However, Einstein had never aware of the existence of a repulsive gravitational force before [7, 13].

Experiments show that the weight reduction of a metal piece as its temperature increases is a reduction of gravitation, but not of mass. Since a small amount of mass $\Delta m$ is equivalent to a large amount of energy $\Delta E = \Delta mc^2$ as the atomic bomb verified. However, there is no evidence for a lost of energy when the weight is reduced. Thus, the repulsive gravitational force must exist and assuming the existence of gravitational mass [10, 11] is incorrect.

Thus, based on assuming gravity was always attractive, Penrose's 1965 proof [8] for the existence of black holes is invalid(11) although the 2020 Nobel Committee [18] awarded a prize to Penrose. He and his colleagues probably have never seen a weight reduction experiment as the temperature increases. Moreover, members of the 2020 Nobel Committee, consist of essentially theorists, also have not encountered such a weight reduction experiment.

Although Einstein's notion of gravitational mass is incorrect, Einstein's assumption of the equivalence between inertial mass and gravitational mass is still valid if electromagnetism is not involved. Then, the net result is only restricting general relativity to the massive case.

## 4. The Repulsive Gravitational Force, the Reissner-Nordstrom metric and the Necessary Extension

Although we have proved that existence of the repulsive gravitational force from experiments that show the weight reduction of metal as its temperature increases, we have not shown how the repulsive gravitational force is generated. We must able to explain how such a weight reduction would happen. We must also able to explain why a charged capacitor or a charged particle would lead to a reduction of weight [7].

Now, we address the nature of repulsive gravitation with some details. In 1916, a charge-mass repulsive force was derived from the Reissner-Nordstrom metric for a particle with charge $q$ and mass $M$ [19] as follows:

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2, \qquad (4)$$

(with $c = 1$) where $r$ is the radial distance (in terms of the Euclidean-like structure [20]) from the particle center. In metric (4), the gravity components generated by electricity have not only a very different radial coordinate dependence but also a different sign that makes a new repulsive gravitational force [7].

Thus, $q^2 / 2r^2$ is the *repulsive gravitational potential*. This repulsion implies that the basic assumption for black holes is invalid because gravity is not always attractive. However, for an elementary charged particle, the repulsive gravitational force would be very small. This is why Maxwell overlooked the repulsive force. However, a similar metric can be derived for a charged ball. The only changes are that $r$ becomes $R$ the distance from the center of the ball, and $q$ becomes $Q$ the total charge of the ball [21]. Thus, for a charged ball with a sufficiently large $Q$ the repulsive gravitational force can be macroscopically observed.

Now, the existence of repulsive gravitational force is very clear from existing experiments. It was a puzzle why a charged particle is always massive. Now we start to see some light on this. Thus, general relativity has not been fully tested, and supplemental experiments are desired. However, due to many theorists, in particular the string theorists cannot explain the repulsive gravitational force, many take the attitude of ignoring such a subject.(13)

In fact, nothing have been derived from metric (4) until 1997 [22] because theorists did not acknowledge the repulsive gravitational force. This is mainly due to that Einstein believed in general validity of $E = mc^2$. Moreover, in 2003 theorists such as Herrera et al. [23] argued that $M$ in metric (4) could involve the electric energy. Thus, no net repulsive force could be generated.

They considered the mass $M$ would include the electric energy, i.e., $M = m(r_0) + q^2 / r_0$ where $m(r_0)$ is the mass of the particle and $q^2 / r_0$ is the electric energy of the particle outside the radius $r_0$ of the particle. Thus, in net effect, there would be no repulsive gravitation since

$$\frac{1}{2}\frac{\partial}{\partial r}\left[1 - \frac{2M}{r} + \frac{q^2}{r^2}\right] = \left(M - \frac{q^2}{r}\right)\frac{1}{r^2}$$

$$= \left(m(r_0) + q^2\left(\frac{1}{r_0} - \frac{1}{r}\right)\right)\frac{1}{r^2} > 0. \qquad (5)$$

The attraction would increase as the charge $q$ increased [7]. (What fundamentally wrong is that the electric energy is not equivalent to mass because the electric energy is spread extensively over a large area). On the other hand, if the mass $M$ is the inertial mass of the particle, the weight of a charged metal ball would be reduced [7].

In 2005, Tsipenyuk and Andreev [24] discovered that a charged metal ball becomes lighter in weight, but they did not know why because repulsive gravitation was not included in Einstein's general relativity.

However, Lo [7] pointed out that this experiment confirms the existence of repulsive gravitation and $E = mc^2$ may not be valid. The crucial point is, as shown in metric (4), that the charge would create a repulsive gravitational force, which is: 1) proportional to the square of the particle charge, and 2) diminished as $1/r^3$. These two characteristics are also supported by the repulsive gravitational force generated by a charged capacitor [7].

Thus, the experiments on two metal balls [24] support the conclusion that the mass $M$ does not include electric energy since a charged ball has a reduced weight and the *repulsive gravitational potential* $q^2/2r^2$ is confirmed. It will be shown that such a force leads to the necessity to extend the theoretical framework of general relativity.

Nevertheless, as many other physicists, ‘t Hooft [25] has mistakenly assumed that $m = E/c^2$ was universally true due to inadequate understanding of Newtonian mechanics and special relativity. Note that if the mass of an electron includes all the electronic energy of an electron, then the Newton's law, $F = m_e a$ (where $F$ is the force acting on the electron, $m_e$ is the mass of the electron, and $a$ is the acceleration of the electron) would not be valid. Because of special relativity, for the electric energy at different distance, it requires to react at different times.

Moreover, since F. Wilzcek [26] used $E = mc^2$ for the asymptotic freedom without any justification, his proof is still incomplete. For this, I have asked Prof. Wilzcek and he agrees that $E = mc^2$ may not always be valid. This shows that Wilzcek has a far better understanding of physics than t' Hooft.

To see the necessity to extend general relativity, we consider the force on a test particle with mass $m$,

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0, \qquad ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta. \tag{6}$$

Let us consider only the static case. For a test particle $p$ with mass $m$ at $r$, the force on $p$ is

$$-m\frac{M}{r^2}+m\frac{q^2}{r^3} \tag{7a}$$

in the first-order approximation because of $g^{rr}\approx -1.$ Thus, the second term is a repulsive force.

If the particles are at rest, then the force acting on the charged particle $P$ has the same magnitude

$$\left(m\frac{M}{r^2}-m\frac{q^2}{r^3}\right)\hat{r}, \quad \text{where } \hat{r} \text{ is a unit vector} \tag{7b}$$

because the action and reaction forces are equal but in opposite directions. However, for the charged particle with mass $M$ if a metric is calculated according to the potential generated by $p$ of mass $m$, only the first term is obtained.

Therefore, it is necessary then to have a repulsive force with the coupling $q^2$ to the charged particle $P$ in a gravitational field generated by mass $m$. Thus, force (7b) to particle $P$ is beyond the framework of gravitation + electromagnetism. As predicted by Lo, Goldstein and Napier [27], general relativity would lead to the necessity of its extension to a five-dimensional theory. Moreover, their five-dimensional theory is a natural candidate. This is similar to the addition of displacement current in Maxwell's theory.

The repulsive force in metric (4) comes from electric energy [7]. An immediate question would be whether such a charge-mass repulsive force $mq^2/r^3$ is subjected to electromagnetic screening. This force, being independent of a charge sign, should not be subjected to such screening. This has been verified by experiments. Moreover, an existence of the repulsive force $mq^2/r^3$ means also that Maxwell's theory is actually inadequate.

Note that this force can be considered a result of $q^2$ interacting with a field created by the mass $m$ [27]. Thus, such a field is independent of electromagnetism and is beyond general relativity, and the need for unification is established. To test this, one can measure whether there is such a repulsive force outside a charged capacitor. Thus, because of the repulsive gravitational force, general relativity must be extended to a five-dimensional space.

The existence of such a repulsive force has been preliminary verified by weighing charged capacitors because a charged capacitor has been observed to have less weight although the distance dependency cannot be verified with such experiments [28, 29]. In contrast, according to existing Einstein theory, the capacitor after charging should become heavier. In fact, general relativity has not been well understood starting from 1916 even by Einstein.

A new explanation would be that there is a neutral weak force due to the mass-charge interaction, which is repulsive and reduces faster than a Newtonian force. Then, at a very long distance, the net effect may appear as a constant additional weak force that observations suggest. Recently, based on general relativity, a very weak repulsive neutral force of charge-mass interaction has been derived [28, 29]. This infinitesimal weak force would produce the anomaly since it reduces faster than the Newtonian force. Moreover, the repulsive fifth force satisfies the overall requirements according to the data [30].

In the past, a discovery of a new force inevitably leads to new developments of technology. However, Einstein's general relativity seems to have little direct influence on modern life because his theory does not provide an improved means for the creation and the detection of phenomena related to gravity. Nevertheless, this will be changed with the discovery of the charge-mass interaction.

This fifth force is related to the local concentration of charges $Q$; and thus the sensitivity of mass detection can be manipulated with a charged capacitor. Moreover, since this force cannot be screened, it would be a powerful tool for the detection of structures of massive and less massive objects in the industry such as mining. This repulsive gravity has a very different dependence to the distance. Thus, it would be a very useful additional tool for passive detections. Moreover, we should be aware that the repulsive gravitation must be considered for space traveling.

## 5. The Einstein Equation has no Bounded Dynamic Solution

Another problem in general relativity is that the Einstein equation has no bounded dynamic solution [31]. Thus, the Einstein equation must be modified to deal with the dynamic problems. This error was started by Einstein at the beginning of his theory. As pointed out by Gullstrand, Einstein failed to show that he can rigorously derive the remaining perihelion of Mercury [2]. This is why Einstein obtained a Nobel Prize from his work on the photoelectric effect instead of general relativity as what had been expected.

In Einstein's calculation, as shown by Weinberg [32], the remaining perihelion of Mercury is based on the second order perturbation for the orbit of a massive particle. In the real situation in the past, however, the remaining perihelion of Mercury is based on after the calculation from the perturbations of all the planets. The problem later discovered is that there is no bounded dynamic solution for the Einstein equation [31]. D. Hilbert seemed to have also found out this earlier, and thus he gracefully turned all the credits of deriving the field equation to Einstein [2].

To make physicists easier to understand, we shall use an example to illustrate the non-existence of dynamic solutions instead of a mathematical logic proof, and that the Einstein equation and its linearization are unrelated equations [33]. This will be illustrated with the metric of Bondi, Pirani and Robinson [34],

$$ds^2 = e^{2\varphi}(d\tau^2 - d\xi^2) - u^2[\cosh 2\beta(d\eta^2 + d\zeta^2) + \sinh 2\beta \cos 2\theta(d\eta^2 - d\zeta^2) - 2\sinh 2\beta \sin 2\theta d\eta d\zeta], \qquad (8a)$$

where $\varphi$, $\beta$ and $\theta$ are functions of $u( = \tau - \xi)$. It satisfies the differential equation (i.e., their Eq. [2.8]),

$$2\varphi = u(\beta'^2 + \theta'^2 \sinh^2 2\beta) \qquad (8b)$$

which is a special case of $G_{\mu\nu} = 0$. They claimed this is a wave form of a distant source. The metric is irreducibly unbounded due to the factor $u^2$. Linearization does not make sense since $u$ is not bounded.

When gravity is absent, it is necessary to have $\varphi = \sinh 2\beta = \sin 2\theta = 0$. These reduce (8a) to

$$ds^2 = (d\tau^2 - d\xi^2) - u^2(d\eta^2 + d\zeta^2). \qquad (8c)$$

However, this metric is not equivalent to the flat metric. Thus, (8c) violates the principle of causality.

Now, let us consider the linearized $G_{\mu\nu} = 0$. Since for a massive source, the Einstein equation is

$$G_{\mu\nu} \equiv R_{\mu\nu} - (1/2)g_{\mu\nu}R = -KT(m)_{\mu\nu}, \qquad (9)$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci curvature tensor, $T(m)_{\mu\nu}$ is the energy-stress tensor for massive matter, and $K( = 8\pi Gc^{-2}$ and $G$ is the Newtonian coupling constant) is the coupling constant [11]. Thus, for the harmonic gauge, the linearized equation is

$$\frac{1}{2}\partial^c\partial_c\bar{\gamma}_{\mu\nu} = -KT(m)_{\mu\nu},$$

$$\text{where } \bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}(\eta^{cd}\gamma_{cd}) \qquad (10a)$$

and

$$\bar{\gamma}_{\mu\nu}(x^i, t) = -\frac{K}{2\pi}\int \frac{1}{R} T_{\mu\nu}[y^i, (t-R)]d^3 y,$$

$$\text{where } R^2 = \sum_{i=1}^{3}(x^i - y^i)^2. \tag{10b}$$

Thus, for $T(m)_{\mu\nu} = 0$, we have $\gamma_{\mu\nu} = 0$, and the linearized eq. (10) has a bounded solution, but the equation $G_{\mu\nu} = 0$ violates the principle of causality because the energy-momentum of a gravitational wave cannot be zero.

Note that the dynamic solution (8) is unbounded, and therefore the linearization is not a valid mathematical operation. Einstein wondered why his equation does not have a gravitational wave solution. This is due to that linearization is not a valid mathematical operation since there is no bounded solution. As we shall show, this mistake was also made by the Wheeler School in part 2 of their book "Gravitation" [19] because *their inadequate training in pure mathematics and physics*.

However, in 1981, Schoen and Yau, based on their positive mass theorem [35], claimed that the Einstein theory is almost perfect. Their position is wrongly strengthen by Witten [36].(1) However, they did not provide a dynamic example to support their claim. Nevertheless, Christodoulou(2) and Klainerman [37] of the Princeton University falsely claimed in 1993 that they have constructed dynamic solutions for the non-linear Einstein equation; and then the attitude of the 1993 Nobel Committee changed.

However, their claim is proven false since I showed in 1993 at a conference in Hong Kong [38] that to have a dynamic solution the Einstein equation must be modified. This led me to a meeting with S. T. Yau and S. Y. Cheng in Yau's office in the Hong Kong Chinese University.

In that meeting, Yau was convinced that to have a dynamic solution, the non-linear Einstein equation must be modified. To challenge me, Yau even claimed that no physical theory is completely self-consistent.(3) This resulted in Yau's claim of loss earlier interest [38] on their work.

Moreover, as shown by my later paper [39], *Christodoulou and Klaineman have never completed a construction of any dynamic solution*. This also confirmed the invalidity of unique coupling sign for dynamic problems and thus the space-time singularity theorems are irrelevant to physics, and Yau's positive mass theorem [35] is misleading.(4)

Moreover, Yau failed to inform Prof. Chern that their paper dedicated to him is actually invalid. He should have admitted their error publicly after Prof. Chern passed away because their error is in their theorem. Thus, it would be impossible to keep their error as a secret for long. Besides, I cannot be silent on this forever because that would damage the reputation of Pui-Ching Middle School (we went to the same school and trained by the same teacher).

Einstein was unable to see that he actually failed to derive the remaining perihelion of Mercury as Gullstrand [40] pointed out, but he was aware of his inadequacy in mathematics. What he had hoped for is that the mathematicians would help. However, unexpectedly because mathematicians such as Hawking, Penrose and Yau do not understand physics adequately, they not only failed to help but also made the failure deeper and more misleading. Thus, physicists actually have little choice, but to improve their own ability in mathematics.

Moreover, the work of mathematician S. T. Yau [35] is worse because it prevented the progress of general relativity by misleading physicists to believe incorrectly that it was perfect. Thus, without any exception, everybody made such a mistake in general relativity. It seems the only exception is mathematician D. Hilbert [2], who avoided such a mistake by turning all credits on the equation to Einstein.

On the other hand, Yau could have found out his mistakes earlier if he was careful enough tried to provide an explicit solution to support their theorem because there is no dynamic solution that can satisfy their requirements. Thus, as Prof. Peter C. Sarnak of the Institute for Advanced Study [41] pointed out the mathematicians of the Fields Medal, who awarded Yau in 1982 and Witten in 1990 a prize,(5) do not understand general relativity.

In the proof of positive mass theorem [35], Schoen and Yau assumed that all the physical solutions of the Einstein equation would be subjected to the requirements of asymptotically flat without a proof. This turns out to be incorrect. Thus, Yau failed to see such requirements eliminate a whole class of important solutions because their requirements cannot be met even for a two-body problem [31].

Their boundary condition on the physical space-time is that it should be asymptotically flat [35], i.e.,

$$g_{ij} = \delta_{ij} + O(r^{-1}). \tag{11}$$

It seems that their motivation comes from the linearized equation. But they do not know that for the dynamic case, the non-linear Einstein equation and its linearization are not compatible [33].

However, Yau did not seem to be aware whether or not a physical requirement is valid also depends on the nonlinear Einstein equation. For instance, in an explicit calculation of a two-body problem, due to the Einstein equation's deficiency, the requirement of asymptotically flat (11) cannot be satisfied [31, 42]. The net result is that the solution to a two-body problem is excluded. Thus, what remains are the gravity of a single mass such as the Schwarzschild solution, the harmonic solution, and the Kerr solution, etc.

Had they tried to obtain a solution for a two-body problem, they would have found that it is impossible to satisfy conditions (11) for the non-linear Einstein equation. In effect, the boundary conditions actually excluded an important class of dynamic problems, and then they mistakenly claimed the result of remaining trivial problems as the general results of the theory [43]. Also, the theorem is stated deceptively, and thus can be easily misunderstood. Thus, the theorem of Schoen and Yau prevented the progress in relativity for about 40 years!(6)

If one relies on others to obtain the physical condition, one can easily make the same mistake as Yau if one does not use explicit solutions to check the results. In fact, mathematicians such as M. Atiyah(7) and L. D. Faddeev(8) had made such errors. In particular, Faddeev's "natural" definition of energy has no valid basis in physics since no bounded dynamic solution has ever been produced. *They are just too eager to believe that Einstein was right*. An answer for this problem is to find a bounded dynamic solution for the Einstein equation, but this is impossible [43].

Thus, the misleading theorem of Scheon and Yau was cited as the main reason to award the Fields Medal to Yau (1982) and Witten (1990). Schoen and Yau also failed to consider the fact that, in the literature, the Einstein equation has no dynamic solution, which is bounded. Thus, they failed to see that the asymptotically flat condition implies the exclusion of the dynamic solutions, instead of including them. The proof for the nonexistence of a bounded dynamic solution was published in 2000 [31], about 20 years later.

Note that D. Hilbert [2] had made a mistake in approving Einstein's calculation of the perihelion of Mercury because he was not aware that it requires a bounded solution of the many-body problem. In fact, theorists such as Yau [35], Witten [36], Christodoulou [37], Hawking [44] and Penrose [45] make the same error of defining a set of solutions that include no dynamic solutions.(4) Thus, their errors were not discovered by the physics community [22].

Moreover, the same erroneous work [37] was cited in awarding the

2011 Shaw Prize to Christodoulou. This mistake came from an accumulation of long-standing errors, often resulting from an insufficient understanding of the principle of causality as Einstein [2] did. There have been doubts about the validity of general relativity, but we were unsure because of the lack of experimental evidence. Now, we have clear and simple evidence [5, 6].

However, as suggested by Einstein's remark, to obtain a dynamic solution for the Einstein equation, modifications to the source tensor are necessary. Based on Einstein's radiation formula, which is supported by the Taylor-Hulse experiment,(9) a theory is developed with the theoretical framework of general relativity [42]. The rate-of-energy loss formula of Einstein equation is

$$-\frac{dE}{dt} = \frac{G}{45}(\dddot{q}_{kj}\dddot{q}^{kj}) > 0. \tag{12}$$

The modified equation is

$$G_{\mu\nu} = R_{\mu\nu} - (1/2)g_{\mu\nu}R = -K[T_{\mu\nu}(m) - t_{\mu\nu}(g)] = -KT_{\mu\nu}, \tag{13}$$

where $T_{\mu\nu}(m)$ is the stress tensor for massive matter and $t_{\mu\nu}(g)$ is for the gravitational field energy. Now, it is clear that if the stress tensor $T_{\mu\nu}$ is zero in vacuum, then it is impossible to have gravitational radiation since $t_{\mu\nu} = 0$.

Note that eq. (13) extends the suggestion of Lorentz (1916) [46] and Levi-Civita (1917) [47]. Just as the modified equation in reference [10], eq. (13) also has non-unique coupling signs. Now, it is clear that the implicit assumption of unique coupling signs is the cause of space-time singularities of the theorems of Hawking and Penrose.

The most important conclusion is that Einstein's radiation formula requires modification of his field equation. Now, eq. (10) is the first order approximation of eq. (9), and is called the Maxwell-Newton

Approximation [42] that produces Newton's law. The verification of Einstein's radiation formula can be considered the strongest evidence supporting the Maxwell-Newton Approximation.

However, investigation on strong gravity can be done only after the exact form of $t_{\mu\nu}(g)$ is determined. Since Einstein's radiation formula is well supported by observation, it is expected eq. (8) would give more accurate physical descriptions for verified predictions due to there is a gravity energy-stress tensor $t_{\mu\nu}(g)$. However, this modification is not exactly complete since eq. (5) is only an approximation.

The non-linear Einstein equation is incompatible with the physical notion that a wave carries away energy-momentum. For dynamic problems, the linear field equation is independent of, and furthermore incompatible with the non-linear Einstein equation. The linear equation, as a first-order approximation, requires the existence of a wave such that it must be related to the dynamics of the source.

However, due to neglecting these crucial associations, unphysical solutions were mistaken as gravitational waves by theorists such as Bondi, Pirani and Robinson [34], who do not understand the principle of causality. Thus, it was concluded that, as Einstein and Rosen [48] suggested, a gravitational wave solution for their 1915 equation does not exist. They should not have such a conclusion since the photons requires the existence of gravitational waves [9, 10].

Due to inadequacy in mathematics, physicists in particular the Wheeler School [19] incorrectly believed, even in violation of the principle of causality, that a wave solution for $G_{\mu\nu} = 0$ such as their metric (35.29) as follows:

$$ds^2 = c^2dt^2 - dx^2 - L^2(e^{2\beta}dy^2 + e^{-2\beta}dz^2),$$

where $L = L(u)$, $\beta = \beta(u)$, $u = ct - x$, and $c$ is the light speed.

Nevertheless, they [19] even claimed that this solution has an approximate solution, their (35.32), obtained invalidity with the linearization of the above metric [49] is

$$ds^2 = c^2dt^2 - dx^2 - (1 + 2\beta)dy^2 - (1 - 2\beta)dz^2.$$

They incorrectly believed that a linearization always produces an approximation [19]. Thus, the Wheeler School often made errors in physics without knowing them because of their inadequacy in mathematics.

Consequently, they also wrongly used the prestige of the Princeton University to promote their errors on dynamic solutions. They [19] insist on that Christodoulou and Klanerman had constructed dynamic solutions. However, it was found that they actually have not completed any meaningful construction [39].

The Wheeler School also give Pauli's [50] misleading version [51] and the abandoned incorrect 1911 *assumption* [52] *on the equivalence of Newtonian gravity and acceleration as references for Einstein's equivalence principle*, but ignoring both crucial work of Einstein, i.e., references [11], and related theorems [53]. This strange behavior clearly indicates that they want to discredit Einstein's equivalence principle for some unknown reason. However, Lo [54] published an effective defense for Einstein's equivalence principle.

The real reason for the attacks from the Wheeler School, as admitted by Ohanian and Ruffini [55], was due to they suspected that Einstein's equivalence principle would be inconsistent with their notion of black holes. However, the real problem for black holes is the existence of the repulsive gravitational force, which the Wheeler School (and also the Nobel committee) failed to anticipate because they incorrectly believed the general validity of $E = mc^2$.

Because of inadequacy in physics, Wald abandoned the equivalence principle but adapted the invalid covariance principle. Yau misleadingly claims that general relativity is almost perfect, but failed to provide a dynamic solution to support their positive mass theorem [35] since this is impossible [43]. These errors show that physicists must improve their skill in mathematics. And mathematicians should improve their physics.

The space-time singularity theorems of Hawking and Penrose are irrelevant to physics. The assumption of unique coupling sign leads to the non-existence of a bounded dynamic solution even for a two-body problem [31]. Note that they did not prove their assumption is valid in Physics. They only claim such an assumption would be "reasonable".

## 6. The Weight Reduction of a Charged Capacitor

The repulsive gravity was first discovered from a charged capacitor that has a reduced weight. Moreover, after being charged with a high voltage (about 40 kilovolts), without a continuous supply of electric energy, the lifter (a light capacitor) is able to lift its own weight plus a payload hovering over Earth. Also, a lifter could work by charging the wire to either a positive or a negative potential. It has been determined that the lift is not due to ion wind effects [56, 57]. Thus, the lift is generated by changing something inside the lifter with a high voltage.

In a charged capacitor, the only change is the state of motion of some electrons that have become statically concentrated instead of moving in orbits. Since such a repulsive force did not appear before, it is clear that such a force was canceled out by the force created by the motion of the electrons. In other words, the repulsive force generated by the charges of protons and the electrons was canceled by the force generated by moving electrons.

This repulsive force, however, cannot be proportional to the charge density. We have equal numbers of negatively charged electrons and positively charged protons with equal charge. This would lead to the cancellation of the forces generated by particle charges. However, if such a force is proportional to the charge density square, these two kinds of forces would be added together instead of canceled out. Moreover, since the lifter has a limited height, one should expect that this repulsive gravitational force would diminish faster than the gravitational force.

If we assume that the force is proportional to mass, the static charge-mass interaction would be a repulsive force between particles with charge density $D_q$ and another particle of mass $m$ would have the following form:

$$F_r \approx Km\, D_q^{\,2} \big/ r^n, \text{ where } n > 2, \tag{14}$$

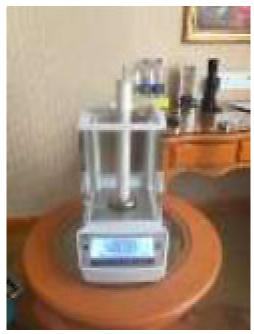
$r$ is the distance between the two particles, and $K$ is the coupling constant. In formula (14) the coupling constant $K$ and $n$ the power of $r$ can be determined by experiments. The simplest case would be $n = 3$.

Note that the electric forces have been cancelled out because there are equal numbers of positive and negative charged particles. The results are that the charged capacitors have reduced weight. If the lift force is large enough, it will hover over the Earth [56, 57] since the repulsive gravitation force reduces faster than the gravitational force.

According to general relativity, the magnetic energy would lead to an attractive force from a current toward a mass [58]. Due to a charged capacitor having reduced weight, it is necessary to have the current-mass interaction canceled out by the effect of the charge-mass interaction. Thus, this would solve a puzzle, i.e., why a charged capacitor exhibits the charge-mass repulsive force.

The existence of such a current-mass attractive force has been discovered by Martin Tajmar and Clovis de Matos [59] from the European Space Agency. Martin et al. found that a spinning ring of superconducting material increases its weight more than expected. However, according to quantum theory, spinning superconductors should produce a weak magnetic field. Thus, they also measured the current-mass interaction to the Earth!

This would generate a force perpendicular to the current. Such a directional dependence of weight is a new phenomenon that verifies the existence of the current-mass interaction. Since the additional weight from a current-mass interaction is directional, the weight of a magnet would depend on the direction. To verify this, we weight the magnets. However, a problem is that an accurate electronic scale would be affected by the magnet to be weighted.



To eliminate the influence of the magnet to the electronic scale, we use a long paper tube to raise the platform to very high, as shown in the above picture. In our case, a paper tube of two feet long would be sufficient. We measure each rectangular magnets in 6 positions and our data confirm that the weight of a magnet is indeed directional [60]. Thus by careful analysis, one sees the existence of the current-mass attractive force.

The experiment data of T. Musha [61, 62] for the charged capacitor show that the repulsive gravitational force would be proportional to the potential square, $V^2$ where $V$ is the electric potential difference of the capacitor ($Q = CV$, $C$ is the capacitance and $Q$ is the charge). Thus, the charge density square in heuristic eq. (14) is correct. Moreover, the lifter's hovering shows that the repulsive force would diminish faster than the gravity. However, even if the $1/r^3$ factor in the repulsive force is verified, the calculation still depends on detailed modeling [63].

The weight reduction effect for charged capacitors is not directional, and it stays if the potential is stable. This was verified by Liu [15] with the rolled-up capacitors. This current-mass interaction also explains the phenomenon that it takes time for a discharged capacitor to recover its weight. A discharged capacitor needs time to dissipate the heat from discharging, and the motions of its charges would accordingly recover to the previous state [64].

However, we are not yet ready to derive this current-mass force explicitly. Nevertheless, we may assume that, for a charged capacitor, the resulting force is the interaction of the net macroscopic charges with the mass. Because the atomic electrons are different for different atoms, the weight reduction processes are also different. This is also why the weight reductions of the heated-up metals are different.

Thus, there are three factors that determine the weight. They are: (1) the mass of the matter, (2) the charge-mass repulsive force, and (3) the attractive current-mass force. For a piece of a heated-up metal, the current-mass attractive force due to orbital electrons is reduced, but the charge-mass repulsive force increases. Hence, a net result is a reduction of weight [6]. However, the string theorists are still unable to address the repulsive gravitation.[13]

It was reported by a British reporter that some Buddhist monks can hover on earth. This was considered as a nonsense in the past because it is against Newton's law. Now, Newton's observation is clearly incomplete. Their hovering on earth is simply an experimental fact [56, 57].

## 7. Discussions and Conclusions

In conclusion, the $E = mc^2$ is not generally valid. Moreover, its invalid application is the implicit cause of some Einstein's errors in general relativity, including Einstein's failure in his unification. Experiments show a piece of metal would reduce its weight as its temperature increase. Thus, it is clear Einstein's theory has made the wrong prediction. Moreover, this leads to the existence of repulsive gravitation confirmed and thus gravity is not always attractive as Penrose believed. Moreover, the existence of repulsive gravitation means Einstein's notion of gravitational mass is invalid. Thus, general relativity needs a major modification and extension.

The existence of repulsive gravitation not only put a big question on the existence of black holes. It also asks why the string theory cannot produce a repulsive gravitation. The string theorists must answer this question to maintain its claim of a theory for every force of the future. If they cannot give a satisfactory answer, the five-dimensional theory started by Kaluza and extended by Lo, Goldstein and Napier [27] would deserve further investigation.

However, relativists, in particular the Wheeler school, failed to see these problems. Thus, many believed that the non-linear Einstein equation had bounded dynamic solutions although they did not get one. Nevertheless, such a violation can be demonstrated with explicit examples such as the metric of Bondi, Pirani and Robinson [34] for $G_{\mu\nu} = 0.$ The principle of causality requires that the energy-momentum tensor cannot be zero in vacuum.

One may ask what makes Lo the discoverer of the repulsive gravitation since it was first derived from the Reissner-Nordstrom metric (1916) and he also was not the first who did the experiment that confirms the reduction of weight as temperature increases. Nevertheless, Einstein's theory led to the misinterpretation of the mass $M$ in metric (4) [6] and thus failed to see the existence of repulsive gravitation. Again, it was Einstein's theory that led many to blindly believe in $E = mc^2$ and thus wrongly concluded that the mass was reduced for weight reduction.

However, Lo was the first who questioned the validity of $E = mc^2$ [22] and established a five-dimensional theory of five variables [27]. These make him question the correctness of Einstein. In addition, Lo is also inspired by Lao Tze on the possibility of existence of repulsive gravitation. Therefore, after a careful analysis of the experiments, Lo discovered the repulsive gravitation.

Gravitational waves are detected although we do not know how a gravitational wave is generated except through the linearized equation because Einstein and Rosen [48] has concluded that the Einstein equation cannot generate gravitational waves. However, we should find the generating equation in the future since the existence of gravitation wave is guaranteed by the existence of photons [9, 10].

Einstein often regarded some partial successes as fully correctness. For example, he did not know that the energy of photons must include also the energy of the related gravitational wave [9, 10]. However, the photons require the existence of the gravitational wave. By adding a photonic tensor with an antigravity coupling to the source, a successful modification [9, 10] is obtained for the gravitational wave from an electromagnetic wave.

Obviously, general relativity is applicable not only to large-scale problems since general relativity provides the foundation for the notion of photons. Einstein was unaware that the energy-momentum tensor of massless particles alone is not compatible with the energy-momentum tensor of the electromagnetic wave. The invalid claim of Hawking and Penrose reflects only that their space-time singularity theorems are irrelevant to physics.

Moreover, although Schoen and Yau dedicated their paper to Prof. Chern in 1979 [65], they did not inform him that their theorem is actually incorrect in 1993. Thus, Yau has a honesty problem in sciences. Also, they should not misled the physical community for 40 years by implicitly implying that they have dynamic solutions.

One may wonder why the experiments cannot settle all errors. However, first some of these issues cannot be settled directly by only experiments. An example is the space-time singularity theorems. Nevertheless, these theorems still can be settled indirectly because the unique coupling sign leads to the non-existence of the dynamic solutions [31]. Unfortunately, this is beyond the ability of theorists such as Hawking and Penrose.

To accommodate the repulsive gravitational force, one must extend general relativity to a five-dimensional theory. Ironically assuming the general validity of $E = mc^2$ is a reason for Einstein's failure in his conjecture. Hawking declared in his book, "A Brief History of Time" that he believes in $E = mc^2$. Perhaps, this is why he never has any verifiable results. Also, a strong repulsive gravitational force can make matter hovering on earth.

It is a good lesson for physicists that in general relativity everybody, including Einstein, failed in mathematics. This teaches us that sometimes we must learn pure mathematics well to do physics well. Also, since the linearized equation is derived from the non-linear Einstein equation, it is also difficult to decide whether the linearized equation is valid except through comparison with experiments.

However, the Hulse-Taylor experiments on binary pulsars suggest that the gravitational wave does exist [42]. Thus it is possible to show the validity of the linearized equation, independent of the Einstein equation. Based on Einstein's equivalence principle, it has been shown that the linearized equation with massive sources is valid independently. Such an equation is called the Maxwell-Newton Approximation in general relativity [42].

As a pioneer in a new field, Einstein once said "I am neither especially clever nor especially gifted. I am only very very curious." In my opinion, Einstein is clever and gifted, but his curiosity and imagination are still insufficient to have led him to wonder why gravity seems to be always attractive. Moreover, Einstein is often made careless mistakes in simple logic such as the photons consisting of only electromagnetic energy.

Einstein had also said "The most important thing is to not stop questioning." A problem is that Einstein questioned everybody, but seldom to question himself. In other words, like many great men, Einstein only paid lip service to self-criticism. From this paper, it is clear that if he had questioned himself more often, he would have made fewer mistakes. Einstein is an example, as the Bible claimed, that nobody is perfect!

Note that Lao Tze of 2,500 years ago, anticipated the existence of repulsive gravitational force, said that for any force, there must be another force against it! Above all, as philosopher Hu Shih said, we must be careful in our proof even though we are allowed to have bold assumptions. However, many theorists, including Einstein, sometimes have only the bold assumptions, but forget to have a necessary careful proof.

A Problem of black holes is that it has no experimental support other than speculations. Even the theoretical simulation is questionable. Thus, a discovery of the repulsive gravitational force raised the question on its existence. Thus, the award of a Nobel Prize to Penrose would be a good lesson to remember. There are so many errors such as the non-existence of dynamic solutions.

In addition, an interesting question is why a charged particle always massive. Now, we have a partial answer because a charge has the repulsive gravitational force. Thus, if a charged particle is not massive, a charged particle would not be able to group with massive particles. It seems that it is the mass let them stay together.

Einstein followers are probably the most conservative in theory because of their inadequacy in mathematics. Since they cannot really tell what is right or wrong with their own judgment, it is safe for them to stay with published results. Such an attitude is probably responsible for errors such as Yau's [35] preserved for a long time. Hence, the most effective way to show their errors is through experiments or asking for explicit supporting evidence.

In my opinion, the most outstanding relativist is Zhou Pei-Yuan, who first pointed out the covariance principle is invalid [66]. However, the Chinese Academic of Sciences failed to recognize his contribution to general relativity. Traditionally, most of the Chinese in pure sciences would likely take, as C. N. Yang, on the side of foreign authorities (Einstein in this case). This made scientists such as Zhou exceptionally valuable.

Zhou's ability to find the truth and his courage to tell the truth against the wrong majority should be a model for all of us to follow. On the other hand, C. N. Yang has never been against the majority. Also, as pointed out by S. Weinberg [67], Yang's understanding of non-abelian gauge was incomplete. Moreover, Yang-Mills is not alone. (In January 1954, R. Shaw [68] wrote a Ph.D. thesis similar to the Yang-Mills theory, but does not want to publish it.)

A big surprise is that Einstein had made so many serious errors, which were not discovered. Since the principle of causality is violated and hence the Einstein equation must be modified to deal with the dynamic problems. Unfortunately, many physicists failed to see these. Also, the photons include not only the electromagnetic wave energy, but also the energy of related gravitational wave [9, 10]. To accommodate the repulsive gravitational force, general relativity must be extended to a five-dimensional theory.

It is hoped that new findings would help gravitation to have some important progress on the repulsive gravitation in new area of applications such as new technology in detection and space traveling and etc.

## Acknowledgments

The author is grateful to Dr. S. Hockfield, the 16th President of MIT, for her supports. Many thanks to Prof. S. Weinberg, who taught us general relativity at MIT, and Prof. I. Halperin, who taught us mathematics at Queen's University. Special thanks are to Prof. P. Morrison of MIT for his guidance on gravitation theory and A. Napier of Tufts University for discussions on repulsive gravitation, and to the Physics Dept. of Tufts University for their assistance in conducting the experiments on the weight reduction of heated-up metals. The author is grateful to G. P. Wood and S. Holcombe for their comments. The author is grateful to the support of Innotec Design Inc., U. S. A., the Chan Foundation, and the Szecheon Co., Hong Kong, China.

## Appendix A: Experiments that Show the Weight of Metal is Reduced as the Temperature Increases

I found experiments that show Einstein wrong actually have been done because the meanings of these experiments were not understood then. They are: 1) the weight reduction of a charged metal ball done by Tsipenyuk and Andreev; 2) The weight reduction of charged capacitor done by scientists from several countries, including the US, Japan, China, etc. [6, 7]; 3) The weight reduction of heated-up metals done by scientists from Russia and China.

Here, we present the experiment done by Dmitriev, Nikushchenko and Snegov [5] in 2003 that a piece of heated-up brass has reduced weight. Their results can be shown in the following figures.

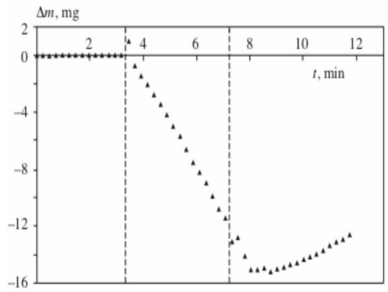



**Figure 1.** Change in mass of a brass rod mounted in an open holder. Ultrasound frequency is 131.25 kHz. The dashed lines indicate the moments when the ultrasound was switched on and off.

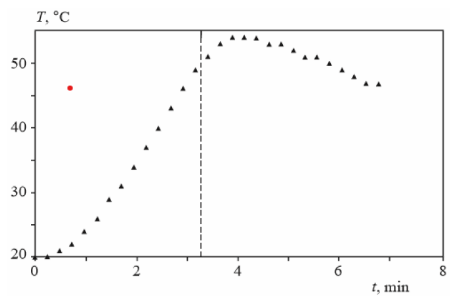



**Figure 2.** Time dependence of the temperature of a part of the surface of an ultrasonically heated brass rod (open holder). Ultrasound frequency is 131.28 kHz. The dashed line indicate the moment when the ultrasound was switched off.

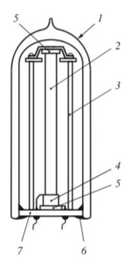



**Figure 3.** Arrangement of the air tight container: 1) Dewar vessel; 2) metal rod; 3) holder pillar (textolite cloth-based laminate); 4) piezoelectric transducer; 5) foam plastic spacers; 6) cold weld; 7) holder base (ebonite).

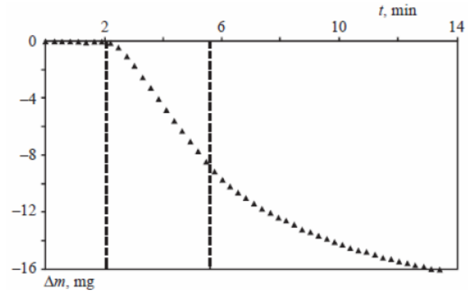



**Figure 4.** Change in mass of a brass rod mounted in a closed Dewar vessel. Ultrasound frequency is 131.27 kHz. The dashed lines indicate the moments when the ultrasound was switched on and off.

Figure 1 shows the change of weight for the brass rod mounted in an open holder. Figure 2 shows the time dependence of the temperature of a part of the surface of an ultrasonically heated brass rod (open holder). Figure 3 shows the arrangement in an air-tight container. Figure 4 shows the change of weight for the brass rod in a closed Dewar vessel. It separates the influence of outside heat. The brass rod weighs $58.5\,\mathrm{g}$ before heating, with a length of $140.0\,\mathrm{mm}$ and a diameter of $8.0\,\mathrm{mm}$. These figures show that the Dewar vessel is not essential for the weight experiment.

Dmitriev et al. [5] pointed out, “It is well known that the temperature regimes play an important role when weighing with high accuracy. The basic reasons for temperature influencing the results of such measurements are thermal expansion of the bodies, temperature changes in the magnetization of the weighed sample, adsorption of moisture by the surface of the sample (a change in the buoyancy), thermal convection of the air near the surface of the sample, the influence of the heated sample on the balance mechanism (through thermal radiation, heat conduction, or convection). These factors are quite well known in modern measurement technology and their contribution to the results of measuring the mass of samples can be estimated quantitatively.” So, they are confident with good reasons that their measured result of a reduction of weight as temperature increased is correct.

However, they misinterpreted the reduction of weight as a reduction of mass because there is no repulsive gravitation in Einstein's theory. More important, it has been verified by Lo [17] with a torsion balance scale that the lead balls have reduced gravitation after heated-up. This confirms the reduction of gravity. In 2010, Fan Liangzao, Feng Jinsong and Liu Wu Qing [6] measured another set of metals, their weight also reduced as the temperature increased, but they also mistaken these as a reduction of mass. Recently H. Y. Woo [69] has measured a number of metals after heated-up, and conclude also that the repulsive gravitational force does exist.

## Appendix B: The Principle of Causality in Physics

Physics is essentially a science for causality. There are two aspects in causality: its relevance and its time ordering. In time ordering, a cause event must happen before its effects. This is further restricted by relativistic causality that no cause event can propagate faster than the light speed in the vacuum. The time-tested assumption that phenomena can be explained in terms of identifiable causes will be called the principle of causality. This is the basis of relevance for all scientific investigations.

Causality means causes will lead to consequences. It should be emphasized that the principle assumed:

(1) From the consequences that causes must exist even we do not know what they are.

(2) The partial consequences of the cause are identified even its full consequences remain to be known.

Then, we can use such partial consequences as requirements to decide whether a solution or even an equation is valid in physics. This might often provide crucial steps to solve a problem correctly. For example, this is how the modified Einstein equation [9, 10] for the electromagnetic wave as a source was modified.

Thus, this principle implies that any parameter in a solution for physics must be related to some physical causes. Moreover, the principle of causality implies that a weak source would produce a weak gravity. Here this principle will be elucidated first in connection with symmetries of a field, and the boundedness of a field solution. Although this principle alone cannot derive a field equation or its solution, it can help determine whether they are valid in physics. This has made a difference between a successful or a failure in the investigation of gravity [9, 10, 13, 42].

In practice, when the considered field is absent, physical properties are ascribed to the space-time as in a "normal" state. For example, the electromagnetic field is zero in a normal state. Then, any deviation from the normal state must have physically identifiable causes. Thus, the principle of causality implies that the symmetry must be preserved if no cause breaks it. The implication of causality to symmetry has been used in deriving the inverse square law from Gauss's law. The normal state of a space-time metric is the flat metric in special relativity. Thus, if a metric does not possess a symmetry, then there must be a physical cause(s) which has broken such a symmetry. For a spherically symmetric mass, causality requires that the metric is spherically symmetric and asymptotically flat. Also, a weak cause can lead to only weak gravity. Thus, Einstein's weak gravity is a consequence of causality.

However, the physical cause(s) should not be confused with the mathematical source term in the field equation. In general relativity, the cause of gravity is the physical matter itself, but not its energy tensors in the source term of Einstein's field equation. The energy-stress tensors (for example, the perfect fluid model) may explicitly depend on the metric. Since nothing should be a cause of itself, such a source tensor does not represent the cause of a metric. For the accompanying gravitational wave of an electromagnetic wave, the physical cause is the electromagnetic wave. Thus, one should not infer the symmetries of the metric based on the source term instead of its causes.

Moreover, inferences based on the source term can be misleading because it may have higher symmetries than those of the cause and the metric. For instance, a transverse electromagnetic plane-wave is not rotationally invariant with respect to the $z$-direction of propagation. But the related electromagnetic energy-stress tensor component $T(E)_{tt}$ for a circularly polarized wave is. Such an assumption violates causality and results in theoretical difficulties.

The Einstein equation did not have a bounded dynamic solution is due to its violation of causality. In the Einstein equation the left side is the Einstein tensor $G_{\mu\nu}$ and the right side are the energy-momentum tensors. For the dynamic case, the energy-momentum tensor of the gravitational waves should have been included. In particular, because energy must be transferred in vacuum, the energy momentum tensor cannot be zero. Therefore, for the dynamic case, the Einstein equation violates the principle of causality and thus has no bounded dynamic solution.

The modified Einstein equation (12) can have a dynamic solution because the missing energy-momentum tensor has been added back. A common problem of the dynamic solutions is that they are not bounded in amplitude. For instance, Yau [35] believes that a physical solution should be bounded. However, he fails to find a bounded dynamic solution to support his requirements [43].

Classical electrodynamics implies that the flat metric is an accurate approximation. The principle of causality is not violated because the electromagnetic energy is not related to a source of electromagnetic wave. However, this required boundedness for a dynamic gravitational field without the gravitational wave energy in the source does not satisfy the principle of causality because gravitational wave energy is also related to the source of gravitation.

Many theorists and journals do not understand the principle of causality adequately. The Physical Review accepted an unbounded solution as valid in physics. As well, the Royal Society (London) accepted Hawking, even though the space-time singularity theorems violate the principle of causality. The claims of Hawking and Penrose, as a rule, are not verifiable. To avoid mistakes, these journals should study the principle of causality very thoroughly.

From the view point of the principle of causality the formula $E = mc^2$ could be different from the formula $m = E/c^2$. If the first formula means that for any mass $m$, it can be equivalent to the energy $mc^2$, and this is proven by special relativity. However, if the second formula means that for any energy $E$, it corresponds to a mass $E/c^2$. Then, the second formula could be different from the first.

The second formula is incorrect if the energy $E$ is the electromagnetic energy. Thus, we must be careful and understand the formula correctly. Now, we all know even Einstein made a mistake in understanding of the formula.

## Endnotes

(1) E. Witten who incorrectly believes that self-consistent is most important in physics. Then Einstein's thought experiment would always be considered as valid. But, a self-consistent theory can still be in disagreement with experiments. He also failed to see that the Einstein equation has no dynamic solutions.

(2) D. Christodoulou, Ph.D. (1971) in Physics, Princeton University (advisor John Wheeler) who falsely claimed to have constructed dynamic solutions for the Einstein equation. He is probably the only member of the US Academy of Sciences (2012) who was accepted because he had made

big mistakes in general relativity. He also received honours that would witness the incompetence of the US Academy of Sciences.

(3) In a way, Yau's claim is not entirely without any justification. For instance, in Maxwell's theory, the same charges would repel each other. Thus, this would be against the existence of a charged electron.

(4) To help physicists, mathematicians must understand adequate physics; otherwise they may make errors as Hawking, Penrose [8, 45], Yau [35], or Witten [36] did.

(5) At the time of awarding the Fields Medal, Yau was affiliated with the Institute for Advanced Study at Princeton. Note that in 1990, a Fields medal was awarded again to E. Witten of the Institute for Advanced Study. Thus, the Fields Medal has made the same mistake twice.

(6) In fact, their positive mass theorem is stated very deceptively, and thus is widely misunderstood.

(7) Michael Francis Atiyah has been a leader of the Royal Society with a long list of honours. However, Peter C. Sarnak, Chairman of the 2011 Shaw Prize Committee found out that Atiyah does not understand relativity.

(8) Ludwig D. Faddeev, the Chairman of the Fields Medal Committee, did not see that the so-called 'natural definition of energy' is invalid. He failed to see that Yau's theory excludes all the two-body problems.

(9) The calculation of gravitational radiation was incorrect and need correction. This is why Taylor failed to answer the inquiry from P. Morrison of MIT [70]. Thus, the Nobel Prize Committee had made mistakes before [70].

(10) This is an example that Misner, Thorn and Wheeler made a mistake in mathematics without knowing it.

(11) Penrose did not understand the principle of causality and also failed to recognize the repulsive gravitation.

(12) It is a surprise that Einstein actually did not understand Maxwell's theory well.

(13) The string theory needs to go into a very high dimensional space. Moreover, the greatest achievement of string theory seems only that it could reproduce the out-dated Einstein equation.

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