Fundamental J. Thermal Science and Engineering, Vol. 1, Issue 2, 2011, Pages 73-81
Published online at http://www.frdint.com/

A DISCUSSION OF ENDOREVERSIBLE ENGINES AT MAXIMUM OUTPUT

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Abstract

A well-known formula for the cycle efficiency of endoreversible (semi-ideal) engines at maximum power output was obtained by several authors using different approaches, but other results shown in these studies are not always in agreement. A brief review of this body of work suggests that the so-called finite-time thermodynamics approach, for which the value of maximum isentropic power remains indeterminate, is not consistent with engines operating at a constant mass flow rate of working fluid. Predictive formulas are further tested with a well documented and carefully instrumented experimental Ocean Thermal Energy Conversion (OTEC) plant. With small temperature differences, OTEC cycles can be well approximated as endoreversible engines. The accuracy and practicality of one set of formulas for working fluid temperatures and isentropic power is clearly demonstrated.

1. Introduction

In the field of power plant thermodynamics, idealized cycles are useful because they lend themselves to relatively simple optimizations. Carnot engines represent ideal systems producing net work as a fluid exchanges heat isothermally with hot

Keywords and phrases: endoreversible (semi-ideal) thermal engines, Ocean Thermal Energy Conversion (OTEC).

Received October 26, 2011

and cold reservoirs at temperatures T_1 and T_2, respectively, and undergoes isentropic transformations through the remainder of the cycle. For real engines, heat transfer in finite times can only take place if there are temperature differences between the working fluid and the reservoirs. The concept of endoreversible engines allows such temperature separations. The working fluid is still assumed to operate in a reversible manner, but irreversibilities occur through heat transfer with the external reservoirs. The working fluid temperatures vary between $T_{2w} \geq T_2$ and $T_{1w} \leq T_1$.

Most real power plants follow Rankine cycles, where the two isothermal sections are replaced by two isobaric sections. For practical reasons, the pumping of a liquid, rather than a two-phase mixture, and the expansion of a superheated vapor through the turbine, are sought just outside the saturation dome. Most heat is exchanged to effect phase changes, however, and for common working fluids, this occurs isothermally. Hence, a semi-ideal endoreversible engine may be a good approximation of a real Rankine cycle, especially if the temperature differences involved are not too great.

Operational power maximization represents the simplest type of performance optimization for a power plant since cost issues that are essential in actual project evaluation are not considered. Moreover, the physical parameters involved in design optimization have been set, and the operator of the plant is dealing with given hardware. Endoreversible systems have been analyzed before from the point of view of maximum power output. The pioneering work of Chambadal [1] and Novikov [2] focused of the performance of nuclear plants in the particular situation when there is no limitation on available cooling fluid $(T_{2w} = T_2)$. Curzon and Ahlborn [3] considered a more general case, although heat fluxes in the isothermal steps of the cycle were still assumed to be proportional to the temperature differences in the heat exchangers, i.e., $\dot{Q}_1 = \alpha(T_1 - T_{1w}) = \alpha x$ and $\dot{Q}_2 = \beta(T_{2w} - T_2) = \beta y$. By definition, $\dot{Q}_i$ $(i = 1, 2)$, x and y are positive. In these early studies, a remarkable result was obtained for the efficiency $\eta_{\max}$ of such endoreversible systems at maximum output:

$$\eta_{\max} = 1 - \sqrt{\frac{T_2}{T_1}}. \tag{1}$$

This formula, the simplicity of which is reminiscent of Carnot's celebrated limit, has been widely quoted, and generally attributed to Curzon and Ahlborn [3]. De Vos

[4] later extended the analysis when the dependence of $\dot{Q}_i$ $(i = 1, 2)$ on temperature differences in the heat exchangers is not restricted to linear functions. Equation (1) was shown to be recovered in the linear case in a short Appendix. De Vos noted that his derivation seemed simpler than Curzon and Ahlborn's. He apparently did not realize that the two approaches were fundamentally different. In fact, different formulas are shown in the two articles for the intermediate results leading to the cycle efficiency $\eta_{\max}$, such as $x_{\max}$, $y_{\max}$ and the maximum isentropic power $P_{\max}$.

In De Vos' work, we have:

$$x_{\max} = \frac{\beta}{\alpha + \beta}(T_1 - \sqrt{T_1 T_2}), \tag{2}$$

$$y_{\max} = \frac{\alpha}{\alpha + \beta}(\sqrt{T_1 T_2} - T_2), \tag{3}$$

$$P_{\max} = \frac{\alpha\beta}{\alpha + \beta}(\sqrt{T_1} - \sqrt{T_2})^2. \tag{4}$$

The derivation of these results is very simple. From a global enthalpy balance for this steady-state bulk flow cycle, the isentropic power produced is given by. $P_{\text{isen}} = \dot{Q}_1 - \dot{Q}_2 = \alpha x - \beta y.$ Using the definitions of $\dot{Q}_1, \dot{Q}_2,$ as well as the additional relationship $\dot{Q}_1 T_{2w} = \dot{Q}_2 T_{1w}$ derived from internal reversibility (isentropic processes), one of the two unknowns x and y can be eliminated. The necessary condition for P_{isen} to be maximal is found by setting its derivative with respect to the independent variable to zero. Equations (1) through (4) easily follow. The analysis implicitly considers a fixed mass flow rate of working fluid $\dot{m}$ subject to heat fluxes and producing power.

With the same definitions of the heat fluxes $\dot{Q}_1$ and $\dot{Q}_2$, Curzon and Ahlborn [3] used an energy balance for the cycle based on heat and work rather than on heat flux and power. This required the introduction of three additional unknown parameters: the duration t_1 for heat transfer with the hot reservoir, the duration t_2 for heat transfer with the cold reservoir and a factor γ such that the overall cycle takes a time of $\gamma(t_1 + t_2)$, with $\gamma > 1$. With so many unknowns, this so-called finite-thermodynamics approach is inherently more complicated, but equation (1) was obtained. Given intermediate results $x_{\max}$, $y_{\max}$ and $P_{\max}$ also exhibit the same functional dependence on reservoir temperatures as equations (2) through (4). The proportionality coefficients, shown in Table 1, are not the same, however. The maximum isentropic power $P_{\max}$ is not even determined since the coefficient γ remains unknown.

2. Discussion

The first noteworthy point is that there is only one independent operational parameter associated with the working fluid of a simple steady-state engine: the mass flow rate $\dot{m}$ (or any valid substitution thereof). Other operational degrees of freedom are external to the working fluid loop, and are only needed to fix the coefficients α and β. α, for example, may be proportional to the mass flow rate of warming fluid to an evaporator, or to a fuel combustion rate, while β typically would depend on the mass flow rate of cooling fluid to a condenser. This immediately raises questions about the analysis performed by Curzon and Ahlborn [3], since in their expression for the power produced by endoreversible engines, both *x* and *y* remain independent working-fluid parameters.

A thermodynamic cycle analysis can be done either for a given mass flow rate of working fluid subject to heat fluxes and producing power, or for a given mass of working fluid exchanging heat and work with the hot and cold reservoirs [5]. Curzon and Ahlborn chose the latter approach, but as they use heat fluxes $\dot{Q}_1$ and $\dot{Q}_2$ in their analysis, it appears that different masses of working fluid are involved at different stages of the cycle. The amounts of heat exchanged in the evaporator (boiler) and condenser, for example, are given as $\dot{Q}_1 t_1$ and $\dot{Q}_2 t_2$, respectively. In a steady-state cycle with mass flow rate $\dot{m}$, the masses of working fluid in the evaporator (boiler) and condenser would then be $\dot{m}t_1$ and $\dot{m}t_2$, respectively. In other words, different masses are exchanging heat and work with the outside reservoirs in the analysis of Curzon and Ahlborn. If instead one adopts a fixed mass reference such as $\dot{m}t_1$, the heat that it would reject to the cold reservoir in the condenser would be $\dot{Q}_2 t_2$ divided by $\dot{m}t_2$ and multiplied by $\dot{m}t_1$, i.e., $\dot{Q}_2 t_1$. This procedure throughout the whole cycle would correctly yield all the equations used by De Vos [4], since t_1 would appear as a common factor in all heat and work terms. Hence, the introduction of arbitrary finite times is completely unnecessary if mass conservation is strictly enforced, and the derivation of equation (1) by Curzon and Ahlborn [3] is fortuitous.

To illustrate this point in more detail, we consider the experimental open-cycle Ocean Thermal Energy Conversion (OTEC) plant that was operated in Hawaii between 1993 and 1998. OTEC relies on heat engines generally based on a Rankine cycle where the hot reservoir is seawater found in the uppermost layer of tropical oceans, and the cold reservoir is deep seawater pumped to the surface. A detailed discussion of OTEC is beyond the scope of this article and may be found elsewhere [6-8]. The closeness of T_1 and T_2 and the small temperature differences throughout OTEC cycles are expected to make OTEC power plants good approximations of endoreversible engines. Moreover, the unique experimental character of the particular OTEC system under consideration means that it was designed, instrumented and operated under exceptional scrutiny [9-12]. Table 2 shows a few essential design characteristics of this plant optimized for maximum output. In the present context, the last four entries in Table 2 are useful only to relate the isentropic power P_{isen} to the electrical power P_{gen} available at the turbo-generator terminals (gross power): $P_{\text{gen}} = \eta_{\text{gen}}(\eta_{\text{tur}} P_{\text{isen}} - P_{\text{mech}})$.

With the specific enthalpy of seawater $c_p = 4000\,\text{J-kg}^{-1}\text{-K}^{-1}$, we can determine $\alpha = \dot{m}_{ww} c_p (T_1 - T_{ww}) / (T_1 - T_{1w})$ and $\beta = \dot{m}_{cw} c_p (T_{cw} - T_2) / (T_{2w} - T_2)$: $\alpha =$ $2066.67\ \text{k-W-K}^{-1}$ and $\beta = 1503.16\,\text{k-W-K}^{-1}$. Open-cycle OTEC is a special case, conceived by Claude in the 1920s, where the warm seawater continuously produces the working fluid in a vacuum structure [13]. About 0.5% of the warm seawater boils at pressures lower than $3\,\text{kPa}$; this steam drives a low-pressure turbine before condensing when it is mixes with the cold seawater. The cycle is open because no pump brings the condensate back to the evaporator. The amount of power that would ‘close’ this cycle is negligible: for the experimental plant under consideration, the isentropic pumping of 3.5 kg/s of condensed liquid water from $1.4\,\text{kPa}$ to $2.6\,\text{kPa}$ would consume less than 5 W. The isentropic cycle efficiency given by equation (1) is 3.38%, whereas the design value is 3.42%.

Table 3 shows how predicted values of T_{1w}, T_{2w} and P at maximum power output compare with their targets. For the evaporation temperature T_{1w}, the relative accuracy of the value obtained with equation (2) is 0.28% whereas that of the value from Curzon and Ahlborn [3] is 2.10%. For the condensation temperature T_{2w}, the relative accuracy of the value obtained with equation (3) is 0.33% whereas that of the value from Curzon and Ahlborn is 3.58%. All temperature predictions are very accurate. Even though results from Curzon and Ahlborn are not as close to the targets, it would remain difficult to discriminate between models on the basis of these temperatures alone. In fact, Table 1 shows that the closer α and β are, the closer $x_{\max}$ and $y_{\max}$ from both models would be. Equation (4) predicts the maximum isentropic power with a relative accuracy of 0.52%. $P_{\max}$ remains undetermined in Curzon and Ahlborn [3], but with any acceptable value of γ (larger than 1), their proposed formula would completely fail to give the correct power level.

To better appreciate the accuracy and practicality of De Vos' formulas, a special time record of gross power measurements was considered, when a large eddy drifted along the leeward coast of the island of Hawaii on July 23, 1993. This resulted in an abrupt change of about 1°C in the ocean surface temperature T_1 at the plant intake. Figure 1 shows how P_{gen}, at the generator terminals, rapidly tracked the changing ocean thermal resource. Also shown is a curve derived from equation (4). The calculated values rely on measurements of T_1 and T_2. The marked short time scale variations merely reflect fluctuations in the measured seawater volume flow rates that were used in the determination of the coefficients α and β. All other required plant parameters, to specify α and β and relate P_{isen} to P_{gen} (i.e., heat exchanger effectiveness, turbine and generator efficiencies), were set at average experimental values obtained from different, stable time history records. It was established, in particular, that the flash evaporator effectiveness and direct contact condenser effectiveness reached higher values than anticipated through the design process, with 0.92 and 0.98, respectively. The agreement between data and calculations is excellent, even though the OTEC plant was run in a *laissez faire* mode throughout the time period under consideration. In other words, the working fluid operational parameter (vacuum compressor set point controlling steam production) was not adjusted to ensure maximum operational output 'at all times' while ocean temperatures were changing. However, relative power gains from operational adjustment are negligible with a 1°C variation of the thermal reservoir temperatures.

3. Conclusion

A well-known formula for the cycle efficiency of endoreversible (semi-ideal) engines at maximum power output was obtained by several authors using different approaches. Since other results shown in these studies are not in agreement, their derivation was briefly reviewed. It appears that the approach of Curzon and Ahlborn [3] is not consistent with endoreversible engines operating at a constant mass flow rate of working fluid, while the value of maximum isentropic power remains indeterminate. These points were tested by comparing these authors' predictions with those of De Vos [4] for a well documented and carefully instrumented experimental OTEC plant. The accuracy and practicality of De Vos' simple formulas was clearly established.

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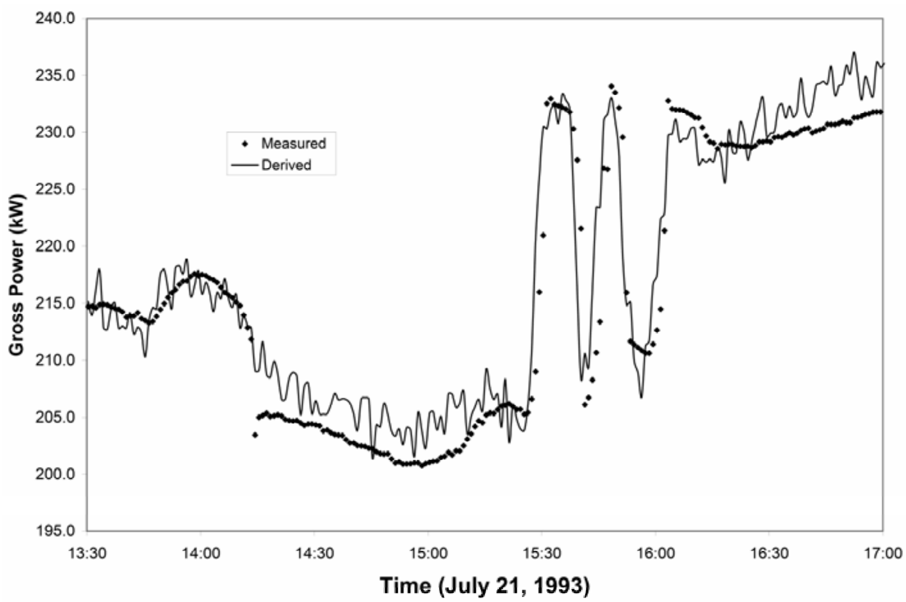

Figure 1. A time history record of electrical power production P_{gen}: measurements and estimates derived from De Vos' formula.

Table 1. Proportionality coefficients for $x_{\max}$, $y_{\max}$ and $P_{\max}$

	$x_{\max}$	$y_{\max}$	$P_{\max}$
De Vos [4]	$\frac{\beta}{\alpha+\beta}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{\alpha+\beta}$
Curzon and Ahlborn [3]	$\frac{\sqrt{\beta}}{\sqrt{\alpha}+\sqrt{\beta}}$	$\frac{\sqrt{\alpha}}{\sqrt{\alpha}+\sqrt{\beta}}$	$\frac{\alpha\beta}{\gamma(\sqrt{\alpha}+\sqrt{\beta})^2}$

Table 2. Design parameters for the experimental open-cycle OTEC plant

Parameter	Value	Definition (for new parameters)
$T_1(°C)$	26	
$T_2(°C)$	6.1	
$\dot{m}_{ww}(\text{kg/s})$	620	mass flow rate of warm seawater in evaporator
$\dot{m}_{cw}(\text{kg/s})$	420	mass flow rate of cold seawater in condenser
$T_{ww}(°C)$	22.5	temperature of seawater at evaporator outlet
$T_{cw}(°C)$	11.2	temperature of seawater at condenser outlet
$T_{1w}(°C)$	21.8	
$T_{2w}(°C)$	11.8	
η_{gen}	0.91	generator efficiency
η_{tur}	0.83	turbine efficiency
$P_{\text{mech}}(\text{kW})$	15	turbine shaft power losses
$P_{\text{gen}}(\text{kW})$	210	gross electrical power

Table 3. Comparison between open-cycle OTEC plant design parameters and predicted values

Parameter	Target	Curzon and Ahlborn [3]	De Vos [4]
$T_{1w}(°C)$	21.8	21.34	21.74
$T_{2w}(°C)$	11.8	11.38	11.76
$P_{\text{isen}}(\text{kW})$	296.45	$149.94/\gamma,\ \gamma > 1$	297.99